International Correspondence Schools, Scranton, Pa.

I. C. S.
REFERENCE LIBRARY

A SERIES OF TEXTBOOKS PREPARED FOR THE STUDENTS OF THE INTERNATIONAL CORRESPONDENCE SCHOOLS AND CONTAINING IN PERMANENT FORM THE INSTRUCTION PAPERS, EXAMINATION QUESTIONS, AND KEYS USED IN THEIR VARIOUS COURSES

ARITHMETIC
MENSURATION AND USE OF LETTERS IN FORMULAS
PRINCIPLES OF MECHANICS
MACHINE ELEMENTS
MECHANICS OF FLUIDS
STRENGTH OF MATERIALS
ELEMENTS OF ELECTRICITY AND MAGNETISM
HEAT AND STEAM

SCRANTON
INTERNATIONAL TEXTBOOK COMPANY
1

Printed in the United States.

22942

PREFACE

Formerly it was our practice to send to each student entitled to receive them a set of volumes printed and bound especially for the Course for which the student enrolled. In consequence of the vast increase in the enrolment, this plan became no longer practicable and we therefore concluded to issue a single set of volumes, comprising all our textbooks, under the general title of I. C. S. Reference Library. The students receive such volumes of this Library as contain the instruction to which they are entitled. Under this plan some volumes contain one or more Papers not included in the particular Course for which the student enrolled, but in no case are any subjects omitted that form a part of such Course. This plan is particularly advantageous to those students who enroll for more than one Course, since they no longer receive volumes that are, in some cases, practically duplicates of those they already have. This arrangement also renders it much easier to revise a volume and keep each subject up to date.

Each volume in the Library contains, in addition to the text proper, the Examination Questions and (for those subjects in which they are issued) the Answers to the Examination Questions.

In preparing these textbooks, it has been our constant endeavor to view the matter from the student's standpoint, and try to anticipate everything that would cause him trouble. The utmost pains have been taken to avoid and correct any and all ambiguous expressions—both those due to faulty rhetoric and those due to insufficiency of statement or explanation. As the best way to make a statement, explanation, or description clear is to give a picture or a

diagram in connection with it, illustrations have been used almost without limit. The illustrations have in all cases been adapted to the requirements of the text, and projections and sections or outline, partially shaded, or full-shaded perspectives have been used, according to which will best produce the desired results.

The method of numbering pages and articles is such that each part is complete in itself; hence, in order to make the indexes intelligible, it was necessary to give each part a number. This number is placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number, it is preceded by a section mark (§). Consequently, a reference, such as § 3, page 10, can be readily found by looking along the inside edges of the headlines until § 3 is found, and then through § 3 until page 10 is found.

INTERNATIONAL CORRESPONDENCE SCHOOLS

CONTENTS

ARITHMETIC.

(PART 1.)

DEFINITIONS.

1. Arithmetic is the art of reckoning, or the study of numbers.

2. A **unit** is *one*, or a single thing, as *one*, *one* bolt, *one* pulley, *one* dozen.

3. A **number** is a unit, or a collection of units, as *one*, *three* engines, *five* boilers.

4. The **unit of a number** is one of the collection of units which constitutes the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar, of *one hundred* bolts is *one* bolt.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *grate bars*, five *dollars*, ten *pounds*.

6. An **abstract number** is a number that is not applied to any object or quantity, as *three*, *five*, *ten*.

7. Like numbers are numbers which express units of the *same kind*, as six *days* and ten *days*, two *feet* and five *feet*.

8. Unlike numbers are numbers which express units of *different kinds*, as ten *months* and eight *miles*, seven *wrenches* and five *bolts*.

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) by words; (2) by figures; (3) by letters.

10. Notation is the art of expressing numbers by figures or letters.

11. Numeration is the art of reading the numbers which have been expressed by figures or letters.

12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten different **figures** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher, or zero.*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first character (0) is called **naught, cipher,** or **zero,** and when standing alone has no value.

The other nine figures are called **digits,** and each has a value of its own.

Any whole number is called an **integer.**

13. As there are only ten *figures* used in expressing numbers, each figure must have a different *value* at different times.

14. The value of a figure depends upon its *position* in relation to other figures.

15. Figures have **simple** values, and **local,** or **relative,** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local,** or **relative,** value of a figure is the *increased* value it expresses by having other figures placed on its right.

For instance, if we see the figure 6 standing alone, thus 6

we consider it as *six units*, or simply **six.**

Place another 6 to the *left* of it; thus 66

The original figure is still *six units*, but the second figure is *ten times* 6, or 6 **tens.**

If a third 6 be now placed still one place further to the *left*, it is increased in value *ten times* more, thus making it 6 **hundreds** 666

A fourth 6 would be 6 **thousands** 6666

A fifth 6 would be 6 **tens of thousands,** or **sixty thousand** 66666

A sixth 6 would be 6 **hundreds of thousands** . . 666666

A seventh 6 would be 6 **millions** 6666666

The entire line of seven figures is read *six millions six hundred sixty-six thousands six hundred sixty-six.*

18. The **increased value** of each of these figures is its *local,* or *relative,* value. Each figure is *ten times* greater in value than the one immediately on its *right.*

19. The **cipher** (0) has no value itself, but it is useful in determining the place of other figures. To represent the number *four hundred five*, two digits only are necessary, one to represent *four hundred* and the other to represent *five units;* but if these two digits are placed together, as 45, the 4 (being in the second place) will mean 4 *tens.* To mean 4 *hundreds*, the 4 should have two figures on its right, and a *cipher* is therefore inserted in the place usually given to *tens*, to show that the number is composed of *hundreds* and *units* only, and that there are no *tens. Four hundred five* is therefore expressed as 405. If the number were *four thousand five*, two ciphers would be inserted; thus, 4005. If it were *four hundred fifty*, it would have the *cipher* at the right-hand side to show that there were no *units*, and only *hundreds* and *tens;* thus, 450. *Four thousand fifty* would be expressed 4050.

20. In *reading* figures, it is usual to *point off* the number into groups of three figures each, beginning at the right-hand, or **units,** column, a comma (,) being used to point off these groups.

Billions.			*Millions.*			*Thousands.*			*Units.*		
Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds of Units.	Tens of Units.	Units.
4	3	2,	1	9	8,	7	6	5,	4	3	2

In *pointing off* these figures, begin at the right-hand figure and count—*units, tens of units, hundreds of units;* the next

group of three figures is *thousands;* therefore, we insert a comma (,) before beginning with them. Beginning at the figure 5, we say *thousands*, *tens of thousands*, *hundreds of thousands*, and insert another comma. We next read *millions*, *tens of millions*, *hundreds of millions* (insert another comma), *billions*, *tens of billions*, *hundreds of billions.*

The entire line of figures would be read: *Four hundred thirty-two billions one hundred ninety-eight millions seven hundred sixty-five thousands four hundred thirty-two.* When we thus *read* a line of figures, it is called *numeration*, and if the numeration be changed back to *figures*, it is called *notation.*

For instance, the writing of the following figures,

72,584,623,

would be the notation, and the numeration would be *seventy-two millions five hundred eighty-four thousands six hundred twenty-three.*

21. NOTE.—It is customary to leave the "s" off the words millions, thousands, etc. in cases like the above, both in speaking and writing; hence, the above would usually be expressed seventy-two million five hundred eighty-four thousand six hundred twenty-three.

22. The four fundamental processes of arithmetic are addition, subtraction, multiplication, and division. They are called fundamental processes because all operations in arithmetic are based upon them.

ADDITION.

23. Addition is the *process of finding the sum of two or more numbers.* The sign of addition is +. It is read **plus,** and means *more.* Thus, 5 + 6 is read *5 plus 6*, and means that 5 and 6 are to be added.

24. The sign of equality is =. It is read **equals** or **is equal to.** Thus, 5 + 6 = 11 may be read *5 plus 6 equals 11*, or *5 plus 6 is equal to 11.*

25. *Like numbers* can be added; *unlike numbers* cannot be added. 6 dollars can be added to 7 dollars, and the sum will be 13 dollars; but 6 dollars cannot be added to 7 feet.

26. The following table gives the sum of any two numbers from 1 to 12; it should be carefully committed to memory:

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

27. For *addition*, place the numbers to be added directly under one another, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

When the numbers are thus written, the *right-hand figure of one number is placed directly under the right-hand figure of the one above it, thus bringing units under units, tens under tens, etc.* Proceed as in the following examples:

28. Example.—What is the sum of 131, 222, 21, 2, and 413?

Solution.—

$$\begin{array}{rr} & 131 \\ & 222 \\ & 21 \\ & 2 \\ & 413 \\ \hline \textit{sum} & 789 \end{array} \text{ Ans.}$$

Explanation.—After placing the numbers in proper order, begin at the bottom of the right-hand, or units, column and add; thus, $3 + 2 + 1 + 2 + 1 = 9$, the sum of the numbers in units column. Place the 9 directly beneath as the first, or units, figure in the sum.

The sum of the numbers in the next, or tens, column equals 8 tens, which is the second, or tens, figure in the sum.

The sum of the numbers in the next, or hundreds, column equals 7 hundreds, which is the third, or hundreds, figure in the sum.

The sum, or answer, is 789.

29. Example.—What is the sum of 425, 36, 9,215, 4, and 907?

Solution.—

$$\begin{array}{rr} & 425 \\ & 36 \\ & 9215 \\ & 4 \\ & 907 \\ \hline & 27 \\ & 60 \\ & 1500 \\ & 9000 \\ \hline \textit{sum} & 10587 \end{array} \text{ Ans.}$$

Explanation.—The sum of the numbers in the first, or units, column is 27 units; i. e., 2 tens and 7 units. Write 27 as shown. The sum of the numbers in the second, or tens, column is 6 tens, or 60. Write 60 underneath 27 as shown. The sum of the numbers in the third, or hundreds, column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth, or thousands, column, 9, which represents 9,000.

Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

30. The addition may also be performed as follows:

$$\begin{array}{rr} & 425 \\ & 36 \\ & 9215 \\ & 4 \\ & 907 \\ \hline \textit{sum} & 10587 \end{array} \quad \text{Ans.}$$

EXPLANATION.—The sum of the numbers in units column = 27 units, or 2 tens and 7 units. Write the 7 units as the first, or right-hand, figure in the sum. Reserve the 2 tens and add them to the figures in tens column. The sum of the figures in the tens column plus the 2 tens reserved and carried from the units column is 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 hundreds, or 1 thousand and 5 hundreds. Write down the 5 as the third, or hundreds, figure in the sum and carry the 1 to the next column. $1 + 9 = 10$, which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

31. EXAMPLE.—Add the numbers in the column below :

SOLUTION.—

$$\begin{array}{rr} & 890 \\ & 82 \\ & 90 \\ & 393 \\ & 281 \\ & 80 \\ & 770 \\ & 83 \\ & 492 \\ & 80 \\ & 383 \\ & 84 \\ & 191 \\ \hline \textit{sum} & 3899 \end{array} \quad \text{Ans.}$$

EXPLANATION.—The sum of the digits in the first column equals 19 units, or 1 ten and 9 units. Write the 9 and carry 1 to the next column. The sum of the digits in the second column + the 1 carried from the first column is 109 tens, or 10 hundreds and 9 tens. Write down the 9 and carry the 10 to the next column. The sum of the digits in this column plus the 10 carried from the second column is 38.

The entire sum is 3,899.

32. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column and add the remaining figure or figures to the next column.*

33. Proof.—*To prove addition, add each column from top to bottom. If you obtain the same result as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

34. Find the sum of:

(*a*) 104 + 203 + 613 + 214.	Ans.	(*a*) 1,134.
(*b*) 1,875 + 3,143 + 5,826 + 10,832.		(*b*) 21,676.
(*c*) 4,865 + 2,145 + 8,173 + 40,084.		(*c*) 55,267.
(*d*) 14,204 + 8,173 + 1,065 + 10,042.		(*d*) 33,484.
(*e*) 10,832 + 4,145 + 3,133 + 5,872.		(*e*) 23,982.
(*f*) 214 + 1,231 + 141 + 5,000.		(*f*) 6,586.
(*g*) 123 + 104 + 425 + 126 + 327.		(*g*) 1,105.
(*h*) 6,354 + 2,145 + 2,042 + 1,111 + 3,333.		(*h*) 14,985.

1. A week's record of coal burned in an engine room is as follows: Monday, 1,800 pounds; Tuesday, 1,655 pounds; Wednesday, 1,725 pounds; Thursday, 1,690 pounds; Friday, 1,648 pounds; Saturday, 1,020 pounds. How much coal was burned during the week?

Ans. 9,538 lb.

2. A steam pump, in one hour, pumps out of a cistern 4,200 gallons; in the next hour, 5,420 gallons; and in 45 minutes more an additional 3,600 gallons, when the cistern becomes empty. How many gallons were in the cistern at first? Ans. 13,220 gal.

3. What is the total cost of a steam plant, the several items of expense being as follows: steam engine, \$900; boiler, \$775; fittings and connections, \$225; erecting the plant, \$125; engine house, \$650?

Ans. \$2,675.

SUBTRACTION.

35. In arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is called the **difference,** or **remainder.**

36. The sign of subtraction is —. It is read **minus,** and means *less.* Thus, 12 — 7 is read *12 minus* 7, and means that 7 is to be taken from 12.

37. Example.—From 7,568 take 3,425.

Solution.—

minuend	7 5 6 8
subtrahend	3 4 2 5
remainder	4 1 4 3 Ans.

Explanation.—Begin at the right-hand, or units, column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

38. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are *less* than the figures directly under them in the subtrahend, proceed as in the following example:

Example.—From 8,453 take 844.

Solution.—

minuend	8 4 5 3
subtrahend	8 4 4
remainder	7 6 0 9 Ans.

Explanation.—Begin at the right-hand, or units, column to subtract. We cannot take 4 from 3, and must,

therefore, borrow 1 from 5 in tens column and annex it to the 3 in units column. The 1 ten = 10 units, which added to the 3 in units column = 13 units. 4 from 13 = 9, the first, or units, figure in the remainder.

Since we borrowed 1 from the 5, only 4 remains; 4 from 4 = 0, the second, or tens, figure. We cannot take 8 from 4, so borrow 1 thousand, or 10 hundreds, from 8; 10 hundreds + 4 hundreds = 14 hundreds, and 8 from 14 = 6, the third, or hundreds, figure in the remainder.

Since we borrowed 1 from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder, or answer.

The operation of borrowing is placing 1 before the figure following the one from which it is borrowed. In the above example the 1 borrowed from 5 is placed before 3, making it 13, from which we subtract 4. The 1 borrowed from 8 is placed before 4, making 14, from which 8 is taken.

39. Example.—Find the difference between 10,000 and 8,763.

Solution.—	*minuend*	10000	
	subtrahend	8763	
	remainder	1237	Ans.

Explanation.—In the above example we borrow 1 from the second column and place it before 0, making 10; 3 from 10 = 7. In the same way we borrow 1 and place it before the next cipher, making 10; but as we have borrowed 1 from this column and have taken it to the units column, only 9 remains from which to subtract 6; 6 from 9 = 3. For the same reason we subtract 7 from 9 and 8 from 9 for the next two figures, and obtain a total remainder of 1,237.

40. **Rule.**—*Place the subtrahend (or smaller) number under the minuend (or larger) number, in the same manner as for addition, and proceed as in Arts.* **37, 38,** *and* **39.**

41. **Proof.**—*To prove an example in subtraction, add the subtrahend and the remainder. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of the above example:

subtrahend	8763
remainder	1237
minuend	10000

EXAMPLES FOR PRACTICE.

42. From:

		Ans.
(*a*)	94,278 take 62,574.	31,704.
(*b*)	53,714 take 25,824.	27,890.
(*c*)	71,832 take 58,109.	13,723.
(*d*)	20,804 take 10,408.	10,396.
(*e*)	310,465 take 102,141.	208,324.
(*f*)	(81,043 + 1,041) take 14,831.	67,253.
(*g*)	(20,482 + 18,216) take 21,214.	17,484.
(*h*)	(2,040 + 1,213 + 542) take 3,791.	4.

1. A cistern is fed by two pipes which supply 1,200 and 2,250 gallons per hour, respectively, and is being emptied by a pump which delivers 5,800 gallons per hour. Starting with 8,000 gallons in the cistern, how much water does it contain at the end of an hour? Ans. 5,650 gal.

2. A train in running from New York to Buffalo travels 38 miles the first hour, 42 the second, 39 the third, 56 the fourth, 52 the fifth, and 48 the sixth hour. How many miles remain to be traveled at the end of the sixth hour, the distance between the two places being 410 miles? Ans. 135 mi.

3. On Monday morning a bank had on hand $2,862. During the day $1,831 were deposited and $2,172 drawn out; on Tuesday, $3,126 were deposited and $1,954 drawn out. How many dollars were on hand Wednesday morning? Ans. $3,693.

MULTIPLICATION.

43. To **multiply** a number is to *add* it to itself a certain number of times.

44. **Multiplication** is the process of multiplying one number by another.

The number thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The number which shows how many times the *multiplicand* is to be taken, or the number by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

45. In the following table, the product of any two numbers (neither of which exceeds 12) may be found:

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18
1 times 7 is 7	2 times 7 is. 14	3 times 7 is 21
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36
4 times 1 is 4	5 times 1 is 5	6 times 1 is 6
4 times 2 is 8	5 times 2 is 10	6 times 2 is 12
4 times 3 is 12	5 times 3 is 15	6 times 3 is 18
4 times 4 is 16	5 times 4 is 20	6 times 4 is 24
4 times 5 is 20	5 times 5 is 25	6 times 5 is 30
4 times 6 is 24	5 times 6 is 30	6 times 6 is 36
4 times 7 is 28	5 times 7 is 35	6 times 7 is 42
4 times 8 is 32	5 times 8 is 40	6 times 8 is 48
4 times 9 is 36	5 times 9 is 45	6 times 9 is 54
4 times 10 is 40	5 times 10 is 50	6 times 10 is 60
4 times 11 is 44	5 times 11 is 55	6 times 11 is 66
4 times 12 is 48	5 times 12 is 60	6 times 12 is 72
7 times 1 is 7	8 times 1 is 8	9 times 1 is 9
7 times 2 is 14	8 times 2 is 16	9 times 2 is 18
7 times 3 is 21	8 times 3 is 24	9 times 3 is 27
7 times 4 is 28	8 times 4 is 32	9 times 4 is 36
7 times 5 is 35	8 times 5 is 40	9 times 5 is 45
7 times 6 is 42	8 times 6 is 48	9 times 6 is 54
7 times 7 is 49	8 times 7 is 56	9 times 7 is 63
7 times 8 is 56	8 times 8 is 64	9 times 8 is 72
7 times 9 is 63	8 times 9 is 72	9 times 9 is 81
7 times 10 is 70	8 times 10 is 80	9 times 10 is 90
7 times 11 is 77	8 times 11 is 88	9 times 11 is 99
7 times 12 is 84	8 times 12 is 96	9 times 12 is 108
10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

This table should be carefully committed to memory.

Since 0 has no value, the product of 0 and any number is 0.

46. The sign of multiplication is ×. It is read **times** or **multiplied by.** Thus, 9 × 6 is read *9 times 6*, or *9 multiplied by 6.*

47. It matters not in what order the numbers to be multiplied together are placed. Thus, 6 × 9 is the same as 9 × 6.

48. To multiply a number by one figure only:

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

multiplicand	4 2 5
multiplier	5
product	2 1 2 5 Ans.

EXPLANATION.—For convenience, the multiplier is generally written under the right-hand figure of the multiplicand. On looking in the multiplication table, we see that 5 × 5 is 25. Multiplying the first figure at the right of the multiplicand, or 5, by the multiplier 5, it is seen that 5 × 5 units is 25 units, or 2 tens and 5 units. Write the 5 units in units place in the product and reserve the 2 tens to add to the product of tens. Looking in the multiplication table again, we see that 5 × 2 is 10. Multiplying the second figure of the multiplicand by the multiplier 5, we see that 5 times 2 tens is 10 tens, which, plus the 2 tens reserved, is 12 tens, or 1 hundred and 2 tens. Write the 2 tens in tens place and reserve the 1 hundred to add to the product of hundreds. Again, we see by the multiplication table that 5 × 4 is 20. Multiplying the third, or last, figure of the multiplicand by the multiplier 5, we see that 5 times 4 hundreds is 20 hundreds, which, plus the 1 hundred reserved, is 21 hundreds, or 2 thousands and 1 hundred, which we write in thousands and hundreds places, respectively.

Hence, the product is 2,125.

This result is the same as adding 425 five times. Thus,

	4 2 5
	4 2 5
	4 2 5
	4 2 5
	4 2 5
sum	2 1 2 5 Ans.

EXAMPLES FOR PRACTICE.

49. Find the product of:

(*a*) 61,483 × 6.		(*a*)	368,898.
(*b*) 12,375 × 5.		(*b*)	61,875.
(*c*) 10,426 × 7.		(*c*)	72,982.
(*d*) 10,835 × 3.	Ans.	(*d*)	32,505.
(*e*) 98,376 × 4.		(*e*)	393,504.
(*f*) 10,873 × 8.		(*f*)	86,984.
(*g*) 71,543 × 9.		(*g*)	643,887.
(*h*) 218,734 × 2.		(*h*)	437,468.

1. A stationary engine makes 5,520 revolutions per hour. Running 9 hours a day, 5 days in the week, and 5 hours on Saturday, how many revolutions would it make in 4 weeks? Ans. 1,104,000 rev.

2. An engineer earns $650 a year and his average expenses are $548. How much could he save in 8 years at that rate? Ans. $816.

3. The connection between an engine and boiler is made up of 5 lengths of pipe, three of which are 12 feet long, one 2 feet 6 inches long, and one 8 feet 6 inches long. If the pipe weighs 9 pounds per foot, what is the total weight of the pipe used? Ans. 423 lb.

---

50. To multiply a number by two or more figures:

Example.—Multiply 475 by 234.

Solution.—

multiplicand	4 7 5
multiplier	2 3 4
	1 9 0 0
	1 4 2 5
	9 5 0
product	1 1 1 1 5 0 Ans.

Explanation.—For convenience, the multiplier is generally written under the multiplicand, placing units under units, tens under tens, etc.

We cannot multiply by 234 at one operation; we must, therefore, multiply by the *parts* and then *add* the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the *first partial*

product; 3 times 475 = 1,425, the *second partial product, the right-hand figure of which is written directly under the figure multiplied by*, or 3; 2 times 475 = 950, the *third partial product*, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the *entire product.*

51. Rule.—I. *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will equal the required product.*

52. Proof.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

53. When there is a *cipher* in the *multiplier*, multiply by it the same as with the other figures; since the result will be zero, place a cipher under the cipher in the multiplier and write the next partial product to the left of it, as shown below:

(*a*)	(*b*)	(*c*)	(*d*)
0	2	15	708
× 0	× 0	× 0	× 0
0 Ans.	0 Ans.	0 Ans.	0 Ans.

(*e*)	(*f*)	(*g*)
3114	4008	31264
203	305	1002
9342	20040	62528
62280	120240	3126400
632142 Ans.	1222440 Ans.	31326528 Ans.

EXAMPLES FOR PRACTICE.

54. Find the product of:

(*a*)	3,842 × 26.	(*a*)	99,892.
(*b*)	3,716 × 45.	(*b*)	167,220.
(*c*)	1,817 × 124.	(*c*)	225,308.
(*d*)	675 × 38.	(*d*)	25,650.
(*e*)	1,875 × 33.	(*e*)	61,875.
(*f*)	4,836 × 47.	(*f*)	227,292.
(*g*)	5,682 × 543.	(*g*)	3,085,326.
(*h*)	3,257 × 246.	Ans. (*h*)	801,222.
(*i*)	2,875 × 302.	(*i*)	868,250.
(*j*)	17,819 × 1,004.	(*j*)	17,890,276.
(*k*)	38,674 × 205.	(*k*)	7,928,170.
(*l*)	18,304 × 100.	(*l*)	1,830,400.
(*m*)	7,834 × 10.	(*m*)	78,340.
(*n*)	87,543 × 1,000.	(*n*)	87,543,000.
(*o*)	48,763 × 100.	(*o*)	4,876,300.

1. If the area of a steam-engine piston is 113 square inches, what is the total pressure upon it when the steam pressure is 85 pounds per square inch? Ans. 9,605 lb.

2. A steam engine which indicated 164 horsepower was found to consume 4 pounds of coal per horsepower per hour. Being replaced by a new engine of the same horsepower, another test was made, which showed a consumption of 3 pounds per horsepower per hour. What was the saving of coal for a year of 309 days, if the engine's average run was 14 hours a day? Ans. 709,464 lb.

3. Two steamers are 7,846 miles apart and are sailing towards each other, one at the rate of 18 miles an hour and the other at the rate of 15 miles an hour. How far apart will they be at the end of 205 hours? Ans. 1,081 mi.

DIVISION.

55. **Division** is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number which shows how many times the divisor is contained in the dividend is called the **quotient.**

56. The sign of division is ÷. It is read **divided by.** 54 ÷ 9 is read *54 divided by 9.* Another way to write 54

divided by 9 is $\frac{54}{9}$. Thus, $54 \div 9 = 6$, or $\frac{54}{9} = 6$. The expression $\frac{54}{9}$ may be read either 54 divided by 9 or 54 *over* 9, the word "over" implying that there is a line between the two numbers indicating that the upper number is to be divided by the lower one. In both of these cases 54 is the *dividend* and 9 is the *divisor*.

Division is the reverse of multiplication.

57. **To divide when the divisor consists of but one figure,** proceed as in the following example:

EXAMPLE.—What is the quotient of $875 \div 7$?

```
                 divisor dividend quotient
SOLUTION.—              7)875(125  Ans.
                          7
                          -
                          17
                          14
                          --
                           35
                           35
                           --
              remainder     0
```

EXPLANATION.— 7 is contained in 8 hundreds, 1 hundred times. Place the 1 as the first, or left-hand, figure of the quotient. Multiply the divisor, 7, by the 1 hundred of the quotient, and place the product, 7 hundreds, under the 8 hundreds in the dividend, and subtract. Beside the remainder, 1, bring down the 7 tens, making 17 tens; 17 divided by $7 = 2$ tens. Write the 2 as the second figure of the quotient. Multiply the divisor, 7, by the 2, and subtract the product from 17. Beside the remainder, 3, bring down the 5 units of the dividend, making 35 units. 7 is contained in 35, 5 times, and 5 is placed in the quotient. 5 times $7 = 35$, which, subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

58. In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

dividend
divisor 7)$8^{1}7^{3}5$
quotient 1 2 5 Ans.

The mental operation is as follows: 7 is contained in 8, 1 time and 1 remainder; 1 placed before 7 makes 17; 7 is contained in 17, 2 times and 3 over; the 3 placed before 5 makes 35; 7 is contained in 35, 5 times. These partial quotients, placed in order as they are found, make the entire quotient, 125.

59. If the divisor consists of *two or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

```
             divisor   dividend     quotient
SOLUTION.—     63)2702826(42902  Ans.
                  252
                  ---
                   182
                   126
                   ---
                    568
                    567
                    ---
                      126
                      126
                      ---
```

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial we must find how many times 63 is contained in 270. 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first, or left-hand, figure in the quotient. Multiply the divisor, 63, by 4, and subtract the product, 252, from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product, 189, is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor, 63, by 2 and subtracting the product, 126, from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568. 6 is contained in 56 about 9 times. Multiply the divisor, 63, by 9 and subtract the product, 567, from 568. The remainder is 1, and

bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

60. Rule.—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right for the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, and to the remainder annex the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

V. *If there be at last a remainder, write it after the quotient, with the divisor underneath.*

61. Proof.—*Multiply the quotient by the divisor and add the remainder, if there be any, to the product. The result will be the dividend.* Thus,

divisor *dividend* *quotient*

63) 4235 ($67\frac{14}{63}$ Ans.
378
———
455
441
———
remainder 14

Proof, *quotient* 67
divisor 63
———
201
402
———
4221
remainder 14
———
dividend 4235

EXAMPLES FOR PRACTICE.

62. Divide the following:

(*a*)	126,498 by 58.	(*a*)	2,181.
(*b*)	3,207,594 by 767.	(*b*)	4,182.
(*c*)	11,408,202 by 234.	(*c*)	48,753.
(*d*)	2,100,315 by 581.	Ans. (*d*)	3,615.
(*e*)	969,936 by 4,008.	(*e*)	242.
(*f*)	7,481,888 by 1,021.	(*f*)	7,328.
(*g*)	1,525,915 by 5,003.	(*g*)	305.
(*h*)	1,646,301 by 381.	(*h*)	4,321.

1. In a mile there are 5,280 feet. How many rails would it take to lay a double row 1 mile long, each rail being 30 feet long? Ans. 352 rails.

2. How many rivets will be required for the longitudinal seams of a cylindrical boiler 20 feet long, the joint being double-riveted, and the rivets being spaced 4 inches apart? Ans. 120 rivets.

NOTE.—First find the length of the boiler in inches.

3. It requires 7,020,000 bricks to build a large foundry. How many teams will it require to draw the bricks in 60 days, if each team draws 6 loads per day and 1,500 bricks at a load? Ans. 13 teams.

NOTE.—Find how many loads 7,020,000 bricks make; then, how many days it will take one team to draw the bricks.

CANCELATION.

63. Cancelation is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

64. The **factors** of a number are those numbers which, when multiplied together, will *equal that number*. Thus, 5 and 3 are the factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

65. A **prime number** is one which cannot be divided by any number except itself and 1. Thus, 2, 3, 11, 29, etc. are prime numbers.

66. A **prime factor** is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its

prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

Numbers are said to be **prime to each other** when no two of them can be divided by any number except 1; the numbers themselves *may* be either prime or composite. Thus, the numbers 3, 5, and 11 are prime to one another; so also are 22, 25, and 21—all three of which are composite numbers.

67. *Canceling equal factors from both dividend and divisor does not change the quotient.*

Canceling a factor in both dividend and divisor is the same as *dividing them both by the same number*, which, by the principle of division, *does not change the quotient.*

Write the numbers forming the dividend above the line, and those forming the divisor below it.

68. Example.—Divide $4 \times 45 \times 60$ by 9×24.

Solution.—Placing the dividend over the divisor, and canceling,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} = \frac{50}{1} = 50. \quad \text{Ans.}$$

Explanation.—The 4 in the dividend and the 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cross off the 4 and write the 1 over it; also cross off the 24 and write the 6 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}} =$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cross off the 60 and write 10 over it; also cross off the 6 and write 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} =$$

Again, 45 in the dividend and 9 in the divisor are divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9

equals 1. Cross off the 45 and write the 5 over it; also cross off the 9 and write the 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} =$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $5 \times 1 \times 10 = 50$; the product of all the uncanceled numbers in the divisor equals $1 \times 1 = 1$.

$$\text{Hence, } \frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} = \frac{1 \times 5 \times 10}{1 \times 1} = 50. \quad \text{Ans.}$$

It is usual to omit the 1's when canceling, instead of writing them as above.

69. Rule.—I. *Cancel the common factors from both the dividend and the divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

70. Divide:

		Ans.	
(*a*)	$14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$.	(*a*)	32.
(*b*)	$3 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 13 \times 10$.	(*b*)	250.
(*c*)	$8 \times 4 \times 3 \times 9 \times 11$ by $11 \times 9 \times 4 \times 3 \times 8$.	(*c*)	1.
(*d*)	$164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$.	(*d*)	48.
(*e*)	$50 \times 100 \times 200 \times 72$ by $1{,}000 \times 144 \times 100$.	(*e*)	5.
(*f*)	$48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$.	(*f*)	105.
(*g*)	$110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$.	(*g*)	42.
(*h*)	$115 \times 120 \times 400 \times 1{,}000$ by $23 \times 1{,}000 \times 60 \times 800$.	(*h*)	5.

ARITHMETIC.

(PART 2.)

FRACTIONS.

REMARK.—If a stick of wood is divided into, say, 12 equal parts, one of these parts is called a twelfth. If we take away 5 of these equal parts, we shall have left 7 parts or 7-twelfths. Since it would be very inconvenient to spell out the names of the number of parts into which an object has been (or is supposed to have been) divided, mathematicians invented, long ago, a kind of a shorthand method of expressing 7-twelfths, 25-forty-fifths, etc., viz.: they wrote the number of the equal parts taken or considered above a horizontal line, and called this number the *numerator;* then they wrote below the horizontal line the number which denoted the number of equal parts into which the object was supposed to be divided, and called it the *denominator.* Hence, instead of writing 7-twelfths, 25-forty-fifths, etc., they wrote $\frac{7}{12}$, $\frac{25}{45}$, etc.; but they read them the same as if they had been written the other way.

1. A **fraction** is *one or more equal parts of a unit. One-half, two-thirds, seven-fifths* are fractions.

2. *Two* numbers are required to express a fraction; one is called the **numerator** and the other the **denominator.**

3. The numerator is placed above the denominator with a line between them, as $\frac{2}{3}$. Here 3 is the denominator, and shows into how many *equal parts* the *unit*, or *one*, is divided. The numerator 2 shows how many of these *equal parts* are

taken or considered. The denominator also indicates the names of the parts.

$\frac{1}{2}$ is read one-half.

$\frac{3}{4}$ is read three-fourths.

$\frac{3}{8}$ is read three-eighths.

$\frac{5}{16}$ is read five-sixteenths.

In the expression "$\frac{3}{4}$ of an apple," the denominator, 4, shows that the apple is cut into 4 *equal parts*, and the numerator, 3, shows that *three of these parts*, or *fourths*, are taken or considered.

If each of the parts, or fourths, of the apple were cut into *two equal pieces*, one of these pieces would be $\frac{1}{2}$ of $\frac{1}{4}$, or $\frac{1}{8}$ of the whole apple, and three of the pieces would be $\frac{3}{8}$ of the apple. Thus, it is evident that the larger the *denominators*, the smaller the parts into which anything is divided and the less the value of the fraction, the numerators being the same. Thus, $\frac{3}{8}$ is less than $\frac{3}{4}$.

4. The **value** of a fraction is the numerator divided by the denominator, as $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

5. The line between the numerator and the denominator means *divided by*, or $\div$.

$\frac{3}{4}$ is equivalent to $3 \div 4$.

$\frac{5}{8}$ is equivalent to $5 \div 8$.

6. The numerator and the denominator of a fraction are called the **terms** of a fraction.

7. The *value* of a fraction whose numerator and denominator are equal is 1.

$\frac{4}{4}$, or four-fourths $= 1$.

$\frac{8}{8}$, or eight-eighths $= 1$.

$\frac{64}{64}$, or sixty-four sixty-fourths $= 1$.

8. A **proper fraction** is a fraction whose numerator is *less* than its denominator. Its value is *less* than 1, as $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

9. An **improper fraction** is a fraction whose numerator is *equal* to or is *greater* than the denominator. Its value is 1 or *more than* 1, as $\frac{4}{4}$, $\frac{9}{8}$, $\frac{42}{32}$.

10. A **mixed number** is a whole number and a fraction united. $4\frac{2}{3}$ is a mixed number and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds*.

REDUCTION OF FRACTIONS.

11. Reduction of fractions is the process of changing their forms without changing their *value.*

12. *A fraction is reduced to higher terms by multiplying both terms of the fraction by the same number.* Thus, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The *value* is not changed, for $\frac{3}{4} = \frac{6}{8}$.

13. *A fraction is reduced to lower terms by dividing both terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}.$$

14. *A fraction is reduced to lowest terms when its numerator and denominator cannot be divided by the same number,* as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{17}$.

15. To reduce a whole number or a mixed number to an improper fraction:

EXAMPLE 1.—How many fourths in 5?

SOLUTION.—Since there are 4 fourths in 1 ($\frac{4}{4} = 1$), in 5 there will be 5×4 fourths, or 20 fourths; i. e., $5 \times \frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE 2.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.— $8 \times \frac{4}{4} = \frac{32}{4}$. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$. Ans.

16. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under the result.*

EXAMPLES FOR PRACTICE.

17. Reduce to improper fractions.

(*a*) $4\frac{1}{8}$.		(*a*) $\frac{33}{8}$.
(*b*) $5\frac{1}{9}$.		(*b*) $\frac{46}{9}$.
(*c*) $10\frac{2}{10}$.	Ans.	(*c*) $\frac{102}{10}$.
(*d*) $37\frac{3}{4}$.		(*d*) $\frac{151}{4}$.
(*e*) $50\frac{4}{5}$.		(*e*) $\frac{254}{5}$.

18. To reduce an improper fraction to a whole or a mixed number:

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.— 4 is contained in 21, 5 times and 1 remaining; as this is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$, is the number. Ans.

19. Rule.—*Divide the numerator by the denominator, the quotient will be the whole number; the remainder, if there be any, will be the numerator of the fractional part, of which the denominator is the same as the denominator of the improper fraction.*

EXAMPLES FOR PRACTICE.

20. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.		(*a*) $24\frac{1}{6}$.
(*b*) $\frac{185}{3}$.		(*b*) $61\frac{2}{3}$.
(*c*) $\frac{701}{6}$.	Ans.	(*c*) $116\frac{5}{6}$.
(*a*) $\frac{149}{3}$.		(*d*) $49\frac{2}{3}$.
(*e*) $\frac{76}{19}$.		(*e*) 4.
(*f*) $\frac{125}{25}$.		(*f*) 5.

21. A **common denominator** of two or more fractions is a number which will contain all the denominators of the fractions without a remainder. The **least common denominator** is the least number that will contain all the denominators of the fractions without a remainder.

22. To find the least common denominator:

EXAMPLE.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

$$
\begin{array}{r|rrrr}
2 & 4, & 3, & 9, & 16 \\
\hline
2 & 2, & 3, & 9, & 8 \\
\hline
3 & 1, & 3, & 9, & 4 \\
\hline
3 & 1, & 1, & 3, & 4 \\
\hline
4 & 1, & 1, & 1, & 4 \\
\hline
 & 1, & 1, & 1, & 1
\end{array}
$$

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. Ans.

EXPLANATION.—Divide each of them by some prime number which will divide at least two of them without a remainder (if possible), bringing down those denominators to the

row below which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 1, 3, 9, 4. Dividing the third row by 3, the result is 1, 1, 3, 4. So continue until the last row contains only 1's. The product of all the divisors, or $2 \times 2 \times 3 \times 3 \times 4 = 144$, is the least common denominator.

23. EXAMPLE.—Find the least common denominator of $\frac{4}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.

SOLUTION.—

$$\begin{array}{r|l} 3 & 9,\ 12,\ 18 \\ \hline 3 & 3,\ \ 4,\ \ 6 \\ \hline 2 & 1,\ \ 4,\ \ 2 \\ \hline 2 & 1,\ \ 2,\ \ 1 \\ \hline & 1,\ \ 1,\ \ 1 \end{array}$$

$3 \times 3 \times 2 \times 2 = 36$. Ans.

24. To reduce two or more fractions to fractions having a common denominator:

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having a common denominator.

SOLUTION.—The common denominator is a number which will contain 3, 4, and 2. The least common denominator is 12, because it is the smallest number which can be divided by 3, 4, and 2 without a remainder.

$$\frac{2}{3} = \frac{8}{12},\ \frac{3}{4} = \frac{9}{12},\ \frac{1}{2} = \frac{6}{12}.$$

Reducing $\frac{2}{3}$, 3 is contained in 12, 4 times. By multiplying both numerator and denominator of $\frac{2}{3}$ by 4, we find

$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$. In the same way we find $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{2} = \frac{6}{12}$.

25. Rule.—*Divide the common denominator by the denominator of the given fraction and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

26. Reduce to fractions having a common denominator:

(*a*)	$\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$.	Ans. (*a*)	$\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*)	$\frac{3}{16}$, $\frac{3}{4}$, $\frac{7}{32}$.	(*b*)	$\frac{6}{32}$, $\frac{24}{32}$, $\frac{7}{32}$.
(*c*)	$\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$.	(*c*)	$\frac{77}{88}$, $\frac{7}{88}$, $\frac{80}{88}$.
(*d*)	$\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{40}$.	(*d*)	$\frac{24}{40}$, $\frac{25}{40}$, $\frac{11}{40}$.
(*e*)	$\frac{4}{10}$, $\frac{6}{40}$, $\frac{9}{20}$.	(*e*)	$\frac{16}{40}$, $\frac{6}{40}$, $\frac{18}{40}$.
(*f*)	$\frac{7}{15}$, $\frac{17}{30}$, $\frac{21}{30}$.	(*f*)	$\frac{14}{30}$, $\frac{17}{30}$, $\frac{21}{30}$.

ADDITION OF FRACTIONS.

27. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent parts of different sizes. Fourths cannot be added to eighths.

The fractions should be reduced to the *least* common denominator, or the *least* number which will contain all the denominators.

28. EXAMPLE.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator*, or the *least number* which will contain all the denominators, is 8.

$$\tfrac{1}{2} = \tfrac{4}{8},\ \tfrac{3}{4} = \tfrac{6}{8},\ \text{and}\ \tfrac{5}{8} = \tfrac{5}{8}.$$

EXPLANATION.—As the denominator tells or indicates the *names* of the parts, the numerators only are added to obtain the total number of parts indicated by the denominator.

$$\tfrac{4}{8} + \tfrac{6}{8} + \tfrac{5}{8} = \frac{4+6+5}{8} = \frac{15}{8} = 1\tfrac{7}{8}. \quad \text{Ans.}$$

29. EXAMPLE 1.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is 16.

$$\begin{array}{rl} 12\tfrac{3}{4} = & 12\tfrac{12}{16} \\ 14\tfrac{5}{8} = & 14\tfrac{10}{16} \\ 7\tfrac{5}{16} = & 7\tfrac{5}{16} \\ \hline \end{array}$$

$$\textit{sum}\ \ 33 + \tfrac{27}{16} = 33 + 1\tfrac{11}{16} = 34\tfrac{11}{16}. \quad \text{Ans.}$$

The sum of the fractions $= \frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $= 34\frac{11}{16}$.

EXAMPLE 2.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

SOLUTION.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$, $3\frac{1}{4} = 3\frac{8}{32}$.

$$\begin{array}{rr} & 17 \\ & 13\tfrac{6}{32} \\ & \tfrac{9}{32} \\ & 3\tfrac{8}{32} \\ \hline \textit{sum} & 33\tfrac{23}{32} \end{array} \quad \text{Ans.}$$

30. **Rule.—I.** *Reduce the given fractions to fractions having the least common denominator and write the sum of the numerators over the common denominator.*

II. *When there are mixed numbers and whole numbers, add the fractions first, and if their sum is an improper fraction, reduce it to a mixed number and add the whole number with the other whole numbers.*

EXAMPLES FOR PRACTICE.

31. Find the sum of:

(*a*)	$\frac{4}{6}, \frac{7}{24}, \frac{5}{8}$.	Ans. (*a*)	$1\frac{7}{12}$.
(*b*)	$\frac{2}{3}, \frac{5}{15}, \frac{24}{45}$.	(*b*)	$1\frac{8}{15}$.
(*c*)	$\frac{1}{2}, \frac{3}{8}, \frac{5}{16}$.	(*c*)	$1\frac{3}{16}$.
(*d*)	$\frac{5}{6}, \frac{11}{72}, \frac{13}{18}$.	(*d*)	$1\frac{51}{72}$.
(*e*)	$\frac{10}{11}, \frac{6}{33}, \frac{28}{66}$.	(*e*)	$1\frac{29}{66}$.
(*f*)	$\frac{23}{45}, \frac{11}{15}, \frac{14}{45}$.	(*f*)	$1\frac{5}{9}$.
(*g*)	$\frac{4}{11}, \frac{7}{22}, \frac{14}{22}$.	(*g*)	$1\frac{7}{22}$.
(*h*)	$\frac{3}{7}, \frac{14}{49}, \frac{2}{7}$.	(*h*)	1.

1. The weights of a number of castings were $412\frac{3}{4}$ pounds, $270\frac{1}{2}$ pounds, 1,020 pounds, $75\frac{1}{4}$ pounds, and $68\frac{1}{2}$ pounds. What was their total weight? Ans. 1,847 lb.

2. Four bolts are required, $2\frac{3}{4}$, $1\frac{7}{8}$, $2\frac{5}{16}$, and $1\frac{13}{32}$ inches long. How long a piece of iron will be required to cut them from, allowing $\frac{3}{4}$ of an inch altogether for cutting off and finishing the ends? Ans. $9\frac{3}{32}$ in.

SUBTRACTION OF FRACTIONS.

32. *Fractions cannot be subtracted without first reducing them to a common denominator.*

EXAMPLE.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The common denominator is 16.

$$\frac{3}{8} = \frac{6}{16}. \quad \frac{13}{16} - \frac{6}{16} = \frac{13-6}{16} = \frac{7}{16}. \quad \text{Ans.}$$

33. EXAMPLE.—From 7 take $\frac{5}{8}$.

SOLUTION.— $1 = \frac{8}{8}$; therefore, $7 = 6 + \frac{8}{8} = 6\frac{8}{8}$; $6\frac{8}{8} - \frac{5}{8} = 6\frac{3}{8}$. Ans.

34. EXAMPLE.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

SOLUTION.—The common denominator of the fractions is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

minuend	$17\frac{18}{32}$
subtrahend	$9\frac{15}{32}$
difference	$8\frac{3}{32}$ Ans.

35. Example.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

Solution.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

minuend	$9\frac{4}{16}$ or	$8\frac{20}{16}$
subtrahend	$4\frac{7}{16}$	$4\frac{7}{16}$
difference	$4\frac{13}{16}$	$4\frac{13}{16}$ Ans.

Explanation.—As the fraction in the subtrahend is greater than the fraction in the minuend, it cannot be subtracted; therefore, *borrow* 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, 8 remains; 4 from $8 = 4$; $4 + \frac{13}{16} = 4\frac{13}{16}$.

36. Example.—From 9 take $8\frac{3}{16}$.

Solution.—

minuend	9 or	$8\frac{16}{16}$
subtrahend	$8\frac{3}{16}$	$8\frac{3}{16}$
difference	$\frac{13}{16}$	$\frac{13}{16}$ Ans.

Explanation.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, borrow 1, or $\frac{16}{16}$, from 9. $\frac{3}{16}$ from $\frac{16}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, only 8 is left. 8 from $8 = 0$.

37. Rule.—I. *Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions and whole numbers separately, and place the remainders side by side.*

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1; reduce it to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

EXAMPLES FOR PRACTICE.

38. Subtract:

(*a*) $\frac{10}{24}$ from $\frac{11}{12}$.		(*a*) $\frac{1}{2}$.	
(*b*) $\frac{7}{14}$ from $\frac{17}{28}$.		(*b*) $\frac{3}{28}$.	
(*c*) $\frac{4}{30}$ from $\frac{5}{10}$.		(*c*) $\frac{11}{30}$.	
(*d*) $\frac{15}{35}$ from $\frac{45}{70}$.	Ans.	(*d*) $\frac{3}{14}$.	
(*e*) $\frac{15}{16}$ from $\frac{57}{48}$.		(*e*) $\frac{1}{4}$.	
(*f*) $13\frac{1}{4}$ from $30\frac{1}{2}$.		(*f*) $17\frac{1}{4}$.	
(*g*) $12\frac{1}{8}$ from 27.		(*g*) $14\frac{7}{8}$.	
(*h*) $5\frac{1}{4}$ from 30.		(*h*) $24\frac{3}{4}$.	

1. An engineer found that he had on hand $48\frac{1}{2}$ gallons of cylinder oil. During the following week he used $\frac{3}{4}$ of a gallon each day for three days, $\frac{7}{8}$ of a gallon on the fourth day, $\frac{15}{16}$ of a gallon on the fifth day, and $\frac{1}{2}$ of a gallon on the sixth day. How much oil remained at the end of the week? Ans. $43\frac{15}{16}$ gal.

2. The main line shaft of a manufacturing plant is run by an engine and waterwheel. A test of the plant showed that the engine was capable of developing $251\frac{4}{5}$ H. P. (horsepower), and the waterwheel, under full gate, $67\frac{1}{2}$ H. P. It was also found that the machinery consumed $210\frac{11}{16}$ H. P., and the friction of the shafting and belting was $32\frac{1}{7}$ H. P. How much power remained unused? Ans. $76\frac{263}{560}$ H. P.

MULTIPLICATION OF FRACTIONS.

39. *In multiplication of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

Multiplying the numerator or dividing the denominator multiplies the fraction.

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $$\frac{3}{4} \times 4 = \frac{3 \times 4}{4} = \frac{12}{4} = 3. \quad \text{Ans.}$$

$$\text{Or, } \frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3. \quad \text{Ans.}$$

40. The word "of" in multiplication of fractions means the same as $\times$, or times. Thus,

$$\frac{3}{4} \text{ of } 4 = \frac{3}{4} \times 4 = 3.$$

$$\frac{1}{8} \text{ of } \frac{5}{16} = \frac{1}{8} \times \frac{5}{16} = \frac{5}{128}.$$

EXAMPLE.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.— $$2 \times \frac{3}{8} = \frac{3 \times 2}{8} = \frac{6}{8} = \frac{3}{4}. \quad \text{Ans.}$$

$$\text{Or, } 2 \times \frac{3}{8} = \frac{3}{8 \div 2} = \frac{3}{4}. \quad \text{Ans.}$$

41. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. Ans.

Or, by cancelation, $\frac{\cancel{4} \times 7}{\underset{4}{\cancel{16}} \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. Ans.

42. EXAMPLE.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

SOLUTION.— $\frac{\cancel{4} \times 3 \times \cancel{16}}{8 \times \cancel{4} \times \underset{2}{\cancel{32}}} = \frac{3}{8 \times 2} = \frac{3}{16}$. Ans.

43. EXAMPLE.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.— $9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$$\frac{39}{4} \times \frac{45}{8} = \frac{39 \times 45}{4 \times 8} = \frac{1755}{32} = 54\frac{27}{32}. \quad \text{Ans.}$$

44. EXAMPLE.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.— $\begin{array}{r} 15\frac{7}{8} \\ 3 \\ \hline 47\frac{5}{8} \end{array}$ or $\begin{array}{l} 15\frac{7}{8} \\ \quad 3 \\ \hline 45 + \frac{21}{8} = 45 + 2\frac{5}{8} = 47\frac{5}{8}. \end{array}$ Ans.

45. **Rule.—I.** *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

II. *To multiply one mixed number by another, reduce them both to improper fractions.*

III. *To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number and add the whole number part to the product of the multiplier and the whole number.*

EXAMPLES FOR PRACTICE.

46. Find the product of:

(*a*) $7 \times \frac{3}{19}$.
(*b*) $14 \times \frac{5}{16}$.
(*c*) $\frac{21}{32} \times \frac{5}{14}$.
(*d*) $\frac{16}{27} \times 4$.
(*e*) $\frac{19}{18} \times 7$.
(*f*) $17\frac{18}{21} \times 7$.
(*g*) $\frac{105}{224} \times 32$.
(*h*) $\frac{15}{28} \times 14$.

Ans.
(*a*) $1\frac{2}{19}$.
(*b*) $4\frac{3}{8}$.
(*c*) $\frac{15}{64}$.
(*d*) $2\frac{10}{27}$.
(*e*) $7\frac{7}{18}$.
(*f*) 125.
(*g*) 15.
(*h*) $7\frac{1}{2}$.

1. A single belt can transmit $107\frac{3}{8}$ horsepower, but as it is desired to use more power, a double belt of the same width is substituted for it. Supposing the double belt to be capable of transmitting $\frac{10}{7}$ as much power as the single belt, how many horsepower can be used after the change? Ans. $153\frac{17}{21}$ H. P.

2. What is the weight of $2\frac{5}{8}$ miles of copper wire weighing $5\frac{3}{4}$ pounds per 100 feet? There are 5,280 feet in a mile. Ans. $796\frac{19}{20}$ lb.

3. The grate of a steam boiler contains $20\frac{1}{2}$ square feet. If the boiler burns $8\frac{3}{10}$ pounds of coal an hour per square foot of grate area and can evaporate $7\frac{1}{2}$ pounds of water an hour per pound of coal burned, how many pounds of water are evaporated by the boiler in 1 hour? Ans. $1,276\frac{1}{8}$ lb.

DIVISION OF FRACTIONS.

47. *In division of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

48. *Dividing the numerator or multiplying the denominator divides the fraction.*

Example 1.—Divide $\frac{6}{8}$ by 3.

Solution.—When *dividing the numerator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6 \div 3}{8} = \tfrac{2}{8} = \tfrac{1}{4}. \quad \text{Ans.}$$

When *multiplying the denominator*, we have

$$\tfrac{6}{8} \div 3 = \frac{6}{8 \times 3} = \tfrac{6}{24} = \tfrac{1}{4}. \quad \text{Ans.}$$

Example 2.—Divide $\frac{3}{16}$ by 2.

Solution.— $$\tfrac{3}{16} \div 2 = \frac{3}{16 \times 2} = \tfrac{3}{32}. \quad \text{Ans.}$$

Example 3.—Divide $\frac{14}{32}$ by 7.

Solution.— $$\tfrac{14}{32} \div 7 = \frac{14 \div 7}{32} = \tfrac{2}{32} = \tfrac{1}{16}. \quad \text{Ans.}$$

49. To **invert** a fraction is to *turn it upside down;* that is, make the numerator and denominator *change places.*

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

50. Example.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

Solution.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$, 3 times, for the denominators are the same, and one numerator is contained in the other 3 times. 2. If we now invert the divisor $\frac{3}{16}$ and multiply, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \cancel{3}} = 3. \quad \text{Ans.}$$

This brings the same quotient as in the first case.

51. Example.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

Solution.—We cannot divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the denominators are not the same; therefore, we must solve as in the second case.

$$\frac{3}{8} \div \frac{1}{4} = \frac{3}{8} \times \frac{4}{1} = \frac{3 \times \cancel{4}}{\underset{2}{\cancel{8}} \times 1} = \frac{3}{2} = 1\tfrac{1}{2}. \quad \text{Ans.}$$

52. Example 1.—Divide 5 by $\frac{10}{16}$.

Solution.— $\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5 \times \frac{16}{10} = \frac{\cancel{5} \times \overset{8}{\cancel{16}}}{\underset{\cancel{2}}{\cancel{10}}} = 8. \quad \text{Ans.}$$

Example 2.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

Solution.— $3\frac{3}{4} = \frac{15}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{15}{4}$ inverted equals $\frac{4}{15}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \cancel{4}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\tfrac{59}{60}. \quad \text{Ans.}$$

53. Rule.—*Invert the divisor and proceed as in multiplication.*

54. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ shows that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\dfrac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\dfrac{3 \times 7}{\frac{8+4}{16}}$ means that 3×7 is to be divided by the value of $\dfrac{8+4}{16}$.

$\dfrac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4} \div \frac{3}{8}$.

It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show as to whether 9 was to be divided by $\frac{3}{8}$ or $\frac{9}{3}$ was to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line is to be divided by all below it.*

55. Whenever an expression like one of the three following ones is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator when so transferred is a multiplier.

1. $\dfrac{\frac{3}{4}}{9} = \dfrac{3}{9 \times 4} = \frac{3}{36} = \frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\dfrac{\frac{3}{4} \times 4}{9 \times 4} = \dfrac{3}{9 \times 4}$, as before.

2. $\dfrac{9}{\frac{3}{4}} = \dfrac{9 \times 4}{3} = 12$. The proof is the same as in the first case.

3. $\dfrac{\frac{5}{9}}{\frac{3}{4}} = \dfrac{5 \times 4}{3 \times 9} = \frac{20}{27}$; for, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\dfrac{\frac{5}{9} \times 9}{\frac{3}{4} \times 9} = \dfrac{5}{\frac{3 \times 9}{4}}$; and $\dfrac{5 \times 4}{\frac{3 \times 9}{4} \times 4} = \dfrac{5 \times 4}{3 \times 9} = \frac{20}{27}$, as above.

This principle may be used to great advantage in cases like $\dfrac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times 4\frac{1}{2} \times 5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\dfrac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times \frac{9}{2} \times \frac{31}{6}}$. Now transferring the denominators of the fractions and canceling,

$$\frac{1 \times 310 \times 27 \times 72 \times 2 \times 6}{40 \times 9 \times 31 \times 4 \times 12} = \frac{1 \times \overset{10}{\cancel{310}} \times \overset{3}{\cancel{27}} \times \overset{\overset{3}{\cancel{6}}}{\cancel{72}} \times \cancel{2} \times \overset{3}{\cancel{6}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{40}} \times \cancel{9} \times \cancel{31} \times \underset{2}{\cancel{4}} \times \cancel{12}}$$

$= \frac{27}{2} = 13\frac{1}{2}$.

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

EXAMPLES FOR PRACTICE.

56. Divide:

(a) 15 by $6\frac{3}{7}$.
(b) 30 by $\frac{6}{8}$.
(c) 172 by $\frac{4}{5}$.
(d) $\frac{14}{18}$ by $1\frac{7}{16}$.
(e) $\frac{103}{6}$ by $14\frac{2}{3}$.
(f) $\frac{142}{27}$ by $17\frac{1}{9}$.
(g) $\frac{14}{18}$ by $\frac{145}{72}$.
(h) $\frac{128}{18}$ by $72\frac{1}{3}$.

Ans.
(a) $2\frac{1}{3}$.
(b) 40.
(c) 215.
(d) $\frac{112}{207}$.
(e) $1\frac{15}{88}$.
(f) $\frac{71}{231}$.
(g) $\frac{56}{145}$.
(h) $\frac{64}{651}$.

1. A $\frac{5}{8}$-inch boiler plate containing 24 square feet of surface weighs $362\frac{4}{10}$ pounds. What is its weight per square foot? Ans. $15\frac{1}{10}$ lb.

2. A certain boiler has $927\frac{1}{2}$ square feet of heating surface, which is equal to 35 times the area of the grate. What is the area of the grate in square feet? Ans. $26\frac{1}{2}$ sq. ft.

3. If the distance around the rim of a locomotive driving wheel is $13\frac{1}{12}$ feet, how many revolutions will the wheel make in traveling 682 feet? Ans. $52\frac{20}{157}$ rev.

ARITHMETIC.

(PART 3.)

DECIMALS.

1. Decimals are *tenth* fractions; that is, the parts of a unit are expressed on the scale of ten, as *tenths*, *hundredths*, *thousandths*, etc.

2. The *denominator*, which is always 10, 100, 1,000, etc., is not expressed, as it would be in fractions, by writing it under the *numerator* with a line between them, as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, but is expressed by placing a *period* (.), which is called a **decimal point,** to the *left* of the *figures of the numerator*, to indicate that the figures on the right form the numerator of a fraction whose denominator is *ten*, *one hundred*, *one thousand*, etc.

3. The *reading* of a decimal number depends upon the number of decimal places in it, i. e., the number of figures to the right of the decimal point.

One decimal place expresses *tenths.*
Two decimal places express *hundredths.*
Three decimal places express *thousandths.*
Four decimal places express *ten-thousandths.*
Five decimal places express *hundred-thousandths.*
Six decimal places express *millionths.*

Thus:

$$
\begin{aligned}
.3 &= \tfrac{3}{10} &&= 3 \text{ tenths.}\\
.03 &= \tfrac{3}{100} &&= 3 \text{ hundredths.}\\
.003 &= \tfrac{3}{1000} &&= 3 \text{ thousandths.}\\
.0003 &= \tfrac{3}{10000} &&= 3 \text{ ten-thousandths.}\\
.00003 &= \tfrac{3}{100000} &&= 3 \text{ hundred-thousandths.}\\
.000003 &= \tfrac{3}{1000000} &&= 3 \text{ millionths.}
\end{aligned}
$$

We see in the above that the *number of decimal places in a decimal equals the number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a decimal point is a whole number. *The decimal point affects only the figures to its right.*

When a whole number and decimal are written together, the expression is a *mixed number.* Thus, 8.12 and 17.25 are mixed numbers.

The relation of decimals and whole numbers to each other is clearly shown by the following table :

9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9
hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.

The figures to the *left* of the decimal point represent *whole numbers ;* those to the *right* are *decimals.*

In both decimals and whole numbers, the *units* place is made the starting point of notation and numeration. The decimals *decrease* on the scale of *ten* to the *right*, and the whole numbers *increase* on the scale of *ten* to the *left.* The *first* figure to the *left* of units is *tens*, and the *first* figure to the *right* of units is *tenths.* The *second* figure to the *left* of

units is *hundreds*, and the *second* figure to the *right* is *hundredths*. The *third* figure to the *left* is *thousands*, and the *third* to the *right* is *thousandths*, and so on, the *whole* numbers on the *left* and the *decimals* on the *right*. The figures equally distant from units place correspond in name. The *decimals* have the ending *ths*, which distinguishes them from *whole* numbers. The following is the numeration of the number in the above table: nine hundred eighty-seven million six hundred fifty-four thousand three hundred twenty-one and twenty-three million four hundred fifty-six thousand seven hundred eighty-nine hundred-millionths.

The decimals increase to the *left* on the scale of *ten*, the same as whole numbers; for, beginning at, say, 4-*thousandths* in the table, the next figure to the left is *hundredths*, which is ten times as great, and the next *tenths*, or ten times the *hundredths*, and so on through both decimals and whole numbers.

4. *Annexing or taking away a cipher at the right of a decimal does not affect its value.*

$.5$ is $\frac{5}{10}$; $.50$ is $\frac{50}{100}$, but $\frac{5}{10} = \frac{50}{100}$; therefore, $.5 = .50$.

5. *Inserting a cipher between a decimal and the decimal point divides the decimal by 10.*

$$.5 = \tfrac{5}{10};\ \tfrac{5}{10} \div 10 = \tfrac{5}{100} = .05.$$

6. *Taking away a cipher from the left of a decimal multiplies the decimal by 10.*

$$.05 = \tfrac{5}{100};\ \tfrac{5}{100} \times 10 = \tfrac{5}{10} = .5.$$

ADDITION OF DECIMALS.

7. The only respect in which addition of decimals differs from addition of whole numbers is in the placing of the numbers to be added.

Whole numbers begin at units and increase on the scale of 10, to the left. Decimals decrease on the scale of 10, to the right. Whole numbers are to the left of the decimal point, and decimals are to the right of it. In whole numbers the *right-hand* side of a column of figures to be added must be in line,

and in decimals the *left-hand* side must be in line, which brings the decimal points directly under one another.

whole numbers	*decimals*	*mixed numbers*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

8. Example.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

Solution.—

```
    242.
       .36
    118.725
      1.005
      6.
    100.1
    -------
sum 468.190  Ans.
```

9. Rule.—*Place the numbers to be added so that the decimal points will be directly under one another. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

10. Find the sum of:

		Ans.
(*a*)	.2143, .105, 2.3042, and 1.1417.	(*a*) 3.7652.
(*b*)	783.5, 21.473, .2101, and .7816.	(*b*) 805.9647.
(*c*)	21.781, 138.72, 41.8738, .72, and 1.413.	(*c*) 204.5078.
(*d*)	.3724, 104.15, 21.417, and 100.042.	(*d*) 225.9814.
(*e*)	200.172, 14.105, 12.1465, .705, and 7.2.	(*e*) 234.3285.
(*f*)	1,427.16, .244, .32, .032, and 10.0041.	(*f*) 1,437.7601.
(*g*)	2,473.1, 41.65, .7243, 104.067, and 21.073.	(*g*) 2,640.6143.
(*h*)	4,107.2, .00375, 21.716, 410.072, and .0345.	(*h*) 4,539.02625.

1. The estimated weights of the parts of a return-tubular boiler were as follows: shell, 3,626 pounds; tubes, 3,564.5 pounds; manhole cover, ring, and yoke, 270.34 pounds; stays, etc., 1,089.4 pounds; steam nozzles, 236.07 pounds; handhole covers and yokes, 120.25 pounds; feedpipe, 34.75 pounds; boiler supports, 350.6 pounds. What was the total estimated weight of the boiler? Ans. 9,291.91 lb.

2. A bill for engine-room supplies had the following items: 1 waste can, $8.30; 20 feet of 4-inch belting, $11.20; 1 pipe wrench, $1.65; 12 pounds of waste, $0.84; 5 gallons of cylinder oil, $8.75; 20 gallons of machine oil, $24. How much did the bill amount to? Ans. $54.74.

SUBTRACTION OF DECIMALS.

11. For the same reason as in addition of decimals, the left-hand figures of decimal numbers are placed in line and the decimal points under each other.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—	*minuend*	.3 0 6 3	
	subtrahend	.1 3 2	
	difference	.1 7 4 3	Ans.

12. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—	*minuend*	7.8 9 5	
	subtrahend	.7 2 5	
	difference	7.1 7 0, or 7.17	Ans.

13. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—	*minuend*	1 1.0 0 0	
	subtrahend	.6 2 5	
	difference	1 0.3 7 5	Ans.

14. **Rule.**—*Place the subtrahend under the minuend, so that the decimal points will be directly under each other. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them and subtract as before.

EXAMPLES FOR PRACTICE.

15. From:

(*a*) 407.385 take 235.0004.		(*a*)	172.3846.
(*b*) 22.718 take 1.7042.		(*b*)	21.0138.
(*c*) 1,368.17 take 13.6817.		(*c*)	1,354.4883.
(*d*) 70.00017 take 7.000017.	Ans.	(*d*)	63.000153.
(*e*) 630.630 take .6304.		(*e*)	629.9996.
(*f*) 421.73 take 217.162.		(*f*)	204.568.
(*g*) 1.000014 take .00001.		(*g*)	1.000004.
(*h*) .783652 take .542314.		(*h*)	.241338.

1. If the temperature of steam at 5 pounds pressure is 227.964 degrees and at 100 pounds pressure is 337.874 degrees, how many degrees hotter is the steam at the higher pressure?

Ans. 109.91 degrees.

2. The outside diameter of $2\frac{1}{2}$-inch wrought-iron pipe is 2.87 inches and the inside diameter is 2.46 inches. How thick is the pipe?

Ans. $.41 \div 2 = .205$ in.

3. In a cistern that will hold 326.5 barrels of water there are 178.625 barrels? How much does it lack of being full?

Ans. 147.875 bbl.

4. A wrought-iron rod is 2.53 inches in diameter. What must be the thickness of metal turned off, so that the rod will be 2.495 inches in diameter?

Ans. $.035 \div 2 = .0175$ in.

MULTIPLICATION OF DECIMALS.

16. In multiplication of decimals, we do not place the decimal points directly under each other as in addition and subtraction. We pay no attention for the time being to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.—

multiplicand	.8 2 5
multiplier	1 3
	2 4 7 5
	8 2 5
product	1 0.7 2 5 Ans.

In this example there are 3 decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product.

17. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.—

multiplicand	4 2 6
multiplier	.0 0 5
product	2.1 3 0, or 2.13 Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

18. It is not necessary to multiply by the ciphers on the *left* of a decimal; they merely *determine the number of decimal places.* Ciphers to the *right* of a decimal should be omitted, as they only make more figures to deal with and do not change the value.

19. Example.—Multiply 1.205 by 1.15.

Solution.—

$$\begin{array}{rr} \textit{multiplicand} & 1.205 \\ \textit{multiplier} & 1.15 \\ \hline & 6025 \\ & 1205 \\ & 1205 \\ \hline \textit{product} & 1.38575 \end{array} \text{ Ans.}$$

In this example there are 3 decimal places in the multiplicand and 2 in the multiplier; therefore, 3 + 2, or 5, decimal places must be pointed off in the product.

20. Example.—Multiply .232 by .001.

Solution.—

$$\begin{array}{rr} \textit{multiplicand} & .232 \\ \textit{multiplier} & .001 \\ \hline \textit{product} & .000232 \end{array} \text{ Ans.}$$

In this example we multiply the multiplicand by the digit in the multiplier, which gives 232 for the product; but since there are 3 decimal places each in the multiplier and multiplicand, we must prefix 3 ciphers to the 232 to make 3 + 3, or 6, decimal places in the product.

21. Rule.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

22. Find the product of:

(*a*)	.000492 × 4.1418.	(*a*)	.0020377656.
(*b*)	4,003.2 × 1.2.	(*b*)	4,803.84.
(*c*)	78.6531 × 1.03.	(*c*)	81.012693.
(*d*)	.3685 × .042.	Ans. (*d*)	.015477.
(*e*)	178,352 × .01.	(*e*)	1,783.52.
(*f*)	.00045 × .0045.	(*f*)	.000002025.
(*g*)	.714 × .00002.	(*g*)	.00001428.
(*h*)	.00004 × .008.	(*h*)	.00000032.

1. The stroke of an engine was found by measurement to be 2.987 feet. How many feet will the crosshead pass over in 600 revolutions? Ans. 3,584.4 ft.

2. If a steam pump delivers 2.39 gallons of water per stroke and runs at 51 strokes a minute, how many gallons of water would it pump in 58½ minutes? Ans. 7,130.565 gal.

3. Wishing to obtain the weight of a connecting-rod from a drawing, it was calculated that the rod contained 294.8 cubic inches of wrought iron, 63.5 cubic inches of brass, and 10.4 cubic inches of Babbitt. Assuming the weight of wrought iron to be .278 pound per cubic inch, of brass .303 pound, and of Babbitt .264 pound, what was the weight of the rod? Ans. 103.94 lb.

DIVISION OF DECIMALS.

23. In division of decimals we pay no attention to the decimal point until after the division is performed. The *number of decimal places in the dividend must equal* (*or be made to equal by annexing ciphers*) *the number of decimal places in the divisor. Divide exactly as in whole numbers. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as are indicated by the remainder.*

EXAMPLE.—Divide .625 by 25.

SOLUTION.—

```
             divisor dividend quotient
               2 5 ) .6 2 5 ( .0 2 5   Ans.
                       5 0
                       ---
                       1 2 5
                       1 2 5
                       -----
        remainder          0
```

In this example there are no decimal places in the divisor and 3 decimal places in the dividend; therefore, there are 3 minus 0, or 3, decimal places in the quotient. One cipher has to be prefixed to the 25 to make the 3 decimal places.

24. Example.—Divide 6.035 by .05.

Solution.—

```
   divisor dividend quotient
       .05)6.035(120.7  Ans.
           5
           -
           10
           10
           --
             35
             35
             --
remainder     0
```

In this example we divide by 5, as if the cipher were not before it. There is 1 more decimal place in the dividend than in the divisor; therefore, 1 decimal place is pointed off in the quotient.

25. Example.—Divide .125 by .005.

Solution.—

```
   divisor dividend quotient
       .005).125(25  Ans.
             10
             --
              25
              25
              --
remainder      0
```

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places and is a whole number.

26. Example.—Divide 326 by .25.

Solution.—

```
   divisor dividend quotient
       .25)326.00(1304  Ans.
           25
           --
            76
            75
            --
             100
             100
             ---
remainder      0
```

In this problem two ciphers were annexed to the dividend, to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

27. EXAMPLE.—Divide .0025 by 1.25.

```
                divisor  dividend  quotient
SOLUTION.—       1.25 ) .00250 ( .002  Ans.
                          250
                         ----
                remainder   0
```

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number, i. e., as 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, i. e., as 125. It is clearly evident that the dividend, 25, will not contain the divisor, 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only 4 decimal places in the dividend; but one cipher was annexed, thereby making $4 + 1$, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the divisor, we must point off $5 - 2$, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

28. **Rule.—I.** *Place the divisor to the left of the dividend and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceeds the number of decimal places in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one whole number by another there be a remainder, the remainder can be placed over the divisor as a fractional part of the quotient; but it is generally better to annex ciphers to the remainder and continue dividing until there are three or four decimal places in the quotient, and*

then if there still be a remainder, terminate the quotient by the plus sign (+), *which shows that it can be carried farther.*

29. EXAMPLE.—What is the quotient of 199 divided by 15?

divisor dividend quotient

SOLUTION.— 15) 199 (13 + $\frac{4}{15}$ Ans.

```
15
--
 49
 45
 --
remainder  4
```

Or, 15) 199.000 (13.266+ Ans.

```
15
--
 49
 45
 --
  40
  30
  --
  100
   90
   ---
   100
    90
    ---
remainder 10
```

$$13\tfrac{4}{15} = 13.266+$$
$$\tfrac{4}{15} = .266+$$

It very frequently happens, as in the above example, that the division will never terminate. In such cases, decide to how many decimal places the division is to be carried and carry the work one place farther. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (–), thus indicating that the quotient is not quite as large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than as indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667–. This remark applies to any other calculation involving decimals, when it is

desired to omit some of the figures in the decimal. Thus, if it is desired to retain three decimal places in the number .2471253, it would be expressed as .247+; if it was desired to retain five decimal places, it would be expressed as .24713−. Both the + and − signs are frequently omitted; they are seldom used outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not quite exact.

EXAMPLES FOR PRACTICE.

30. Divide:

(*a*)	101.6688 by 2.36.	Ans.	(*a*)	43.08.
(*b*)	187.12264 by 123.107.		(*b*)	1.52.
(*c*)	.08 by .008.		(*c*)	10.
(*d*)	.0003 by 3.75.		(*d*)	.00008.
(*e*)	.0144 by .024.		(*e*)	.6.
(*f*)	.00375 by 1.25.		(*f*)	.003.
(*g*)	.004 by 400.		(*g*)	.00001.
(*h*)	.4 by .008.		(*h*)	50.

1. In a steam-engine test of an hour's duration, the horsepower developed was found to be as follows at 10-minute intervals: 25.73, 25.64, 26.13, 25.08, 24.20, 26.7, 26.34. What was the average horsepower? Ans. 25.6886−, average.

NOTE.—Add the different horsepowers together and divide by the number of tests, or 7.

2. There are 31.5 gallons in a barrel. How many barrels are there in 2,787.75 gallons? Ans. 88.5 bbl.

3. A carload of 18.75 tons of coal cost $60.75. How much was it worth per ton? Ans. $3.24 per ton.

4. A keg of $\frac{9}{16}'' \times 1\frac{5}{8}''$ boiler rivets weighs 100 pounds and contains 595 rivets. What is the weight of one of the rivets? Ans. .168+ lb.

TO REDUCE A FRACTION TO A DECIMAL.

31. EXAMPLE 1.— $\frac{3}{4}$ equals what decimal?

SOLUTION.— $4\,)\,\underline{3.0\,0}$

.7 5, or $\frac{3}{4} = .75$ Ans.

EXAMPLE 2.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.—

```
8 ) 7.0 0 0 ( .8 7 5
    6 4
    ---
      6 0
      5 6      or 7/8 = .875.  Ans.
      ---
        4 0
        4 0
        ---
```

32. Rule.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

33. Reduce the following common fractions to decimals:

(*a*) $\frac{15}{32}$.	Ans.	(*a*)	.46875.
(*b*) $\frac{7}{8}$.		(*b*)	.875.
(*c*) $\frac{21}{32}$.		(*c*)	.65625.
(*d*) $\frac{51}{64}$.		(*d*)	.796875.
(*e*) $\frac{4}{25}$.		(*e*)	.16.
(*f*) $\frac{5}{8}$.		(*f*)	.625.
(*g*) $\frac{10}{200}$.		(*g*)	.05.
(*h*) $\frac{4}{1000}$.		(*h*)	.004.

---

34. To reduce inches to decimal parts of a foot:

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in 1 foot, 1 inch is $\frac{1}{12}$ of a foot and 9 inches is $9 \times \frac{1}{12}$, or $\frac{9}{12}$ of a foot. This reduced to a decimal by the above rule shows what decimal part of a foot 9 inches is.

```
1 2 ) 9.0 0 ( .7 5 of a foot.  Ans.
      8 4
      ---
        6 0
        6 0
        ---
```

35. Rule.—I. *To reduce inches to a decimal part of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one, and it is desired to terminate the division at some point, say the fourth decimal place, carry the division one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by 1, omitting the signs + and −.*

EXAMPLES FOR PRACTICE.

36. Reduce to the decimal part of a foot:

(*a*) 3 in.		(*a*) .25 ft.
(*b*) $4\frac{1}{2}$ in.		(*b*) .375 ft.
(*c*) 5 in.	Ans.	(*c*) .4167 ft.
(*d*) $6\frac{5}{8}$ in.		(*d*) .5521 ft.
(*e*) 11 in.		(*e*) .9167 ft.

1. The lengths of belting required to connect three countershafts with the main line shaft were found with the tape measure to be 27 feet 4 inches, 23 feet 8 inches, and 38 feet 6 inches. How many feet of belting were necessary? Ans. 89.5 ft.

2. The stroke of an engine is 14 inches. What is the length of the crank in feet measured from center of shaft to center of crankpin, knowing that length of crank is one-half the stroke? Ans. .5833+ ft.

3. A steam pipe fitted with an expansion joint was found to expand 1.668 inches when steam was admitted into it. How much was its expansion in decimal parts of a foot? Ans. .139 ft.

TO REDUCE A DECIMAL TO A FRACTION.

37. Example 1.—Reduce .125 to a fraction.

Solution.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$. Ans.

Example 2.—Reduce .875 to a fraction.

Solution.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

38. Rule.—*Under the figures of the decimal, place 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

EXAMPLES FOR PRACTICE.

39. Reduce the following to common fractions:

(*a*) .375.		(*a*) $\frac{3}{8}$.	
(*b*) .625.		(*b*) $\frac{5}{8}$.	
(*c*) .3125.		(*c*) $\frac{5}{16}$.	
(*d*) .04.	Ans.	(*d*) $\frac{1}{25}$.	
(*e*) .06.		(*e*) $\frac{3}{50}$.	
(*f*) .75.		(*f*) $\frac{3}{4}$.	
(*g*) .15625.		(*g*) $\frac{5}{32}$.	
(*h*) .875.		(*h*) $\frac{7}{8}$.	

---

40. To express a decimal approximately as a fraction having a given denominator:

EXAMPLE 1.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE 2.—Express .3917 in 12ths.

SOLUTION.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

41. Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

42. Express:

(*a*) .625 in 8ths.		(*a*) $\frac{5}{8}$.	
(*b*) .3125 in 16ths.		(*b*) $\frac{5}{16}$.	
(*c*) .15625 in 32ds.	Ans.	(*c*) $\frac{5}{32}$.	
(*d*) .77 in 64ths.		(*d*) $\frac{49}{64}$.	
(*e*) .81 in 48ths.		(*e*) $\frac{39}{48}$.	
(*f*) .923 in 96ths.		(*f*) $\frac{89}{96}$.	

---

43. The sign for dollars is $. It is read *dollars.* $25 is read *25 dollars.*

Since there are 100 cents in a dollar, 1 cent is 1 one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents *mills*.

Thus, $25.16 is read *twenty-five dollars and sixteen cents;* $25.168 is read *twenty-five dollars sixteen cents and eight mills.*

SIGNS OF AGGREGATION.

44. The **vinculum** ——, **parenthesis** ( ), **brackets** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers which are to be considered together; thus, $13 \times \overline{8-3}$, or $13 \times (8-3)$, shows the 3 is to be taken from 8 before multiplying by 13.

$$13 \times \overline{8-3} = 13 \times 5 = 65. \quad \text{Ans.}$$
$$13 \times (8-3) = 13 \times 5 = 65. \quad \text{Ans.}$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101. \quad \text{Ans.}$$

45. In any series of numbers connected by the signs $+$, $-$, $\times$, and $\div$, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication, the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

46. EXAMPLE.—What is the value of the following expression: $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2}$?

SOLUTION.— $1{,}296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication follows 22; hence, multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to have divided $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to perform *all* the multiplication or division included between the signs + and −, or − and +, before adding or subtracting. Also, had the expression been written $1{,}296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to have multiplied $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multiplication and division signs occurs, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{5}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums, the last expression becomes $24 \times \overline{3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

47. Find the values of the following expressions:

(*a*) $(8 + 5 - 1) \div 4$.
(*b*) $5 \times 24 - 32$.
(*c*) $5 \times 24 \div 15$.
(*d*) $144 - 5 \times 24$.
(*e*) $(1{,}691 - 540 + 559) \div 3 \times 57$.
(*f*) $2{,}080 + 120 - 80 \times 4 - 1{,}670$.
(*g*) $(\overline{90 + 60} \div 25) \times 5 - 29$.
(*h*) $\overline{90 + 60} \div 25 \times 5$.

Ans. $\begin{cases} (a) & 3. \\ (b) & 88. \\ (c) & 8. \\ (d) & 24. \\ (e) & 10. \\ (f) & 210. \\ (g) & 1. \\ (h) & 1.2. \end{cases}$

ARITHMETIC.

(PART 4.)

PERCENTAGE.

DEFINITIONS AND PRINCIPLES.

1. In certain operations, particularly those pertaining to business, it is very convenient to regard the quantity on which we are to operate as being divided into 100 equal parts; thus, instead of using the ordinary fractions $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{7}$, we use the equivalent fractions $\frac{25}{100}$, $\frac{60}{100}$, $\frac{28\frac{4}{7}}{100}$, or their equivalent decimals, .25, .60, .28$\frac{4}{7}$. This practice is a very convenient one in all computations involving United States money, because, since \$1 equals 100 cents, it is easier to comprehend what part of the whole $\frac{35}{100}$ is than some other equivalent fraction, as $\frac{49}{140}$; it is also much easier to compute with fractions whose denominators are 100 than it is to compute with fractions whose denominators are composed of other figures.

2. **Percentage** is a term applied to those arithmetical operations in which the number or quantity to be operated upon is supposed to be divided into 100 equal parts.

3. The term **per cent.** means *by the hundred.* Thus, 8 per cent. of a number means 8 hundredths; i. e., $\frac{8}{100}$, or .08, of that number; 8 per cent. of 250 is $250 \times \frac{8}{100}$, or $250 \times .08 = 20$; 47 per cent. of 75 tons is $75 \times \frac{47}{100} = 75 \times .47 = 35.25$ tons. The statement that the population of a city has increased 22 per cent. in a given time, say from

1880 to 1890, is equivalent to saying that the increase is 22 in every hundred; that is, for every 100 in 1880 there are 22 more, or 122, in 1890.

4. The **sign** of per cent. is %, and is read per cent. Thus, 6% is read *six per cent.*; $12\frac{1}{2}$% is read *twelve and one-half per cent.*, etc.

5. When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

6. The following table will show how any per cent. can be expressed either as a decimal or as a fraction:

1%	.01	$\frac{1}{100}$	$\frac{1}{4}$%	.0025	$\frac{\frac{1}{4}}{100}$ or $\frac{1}{400}$
2%	.02	$\frac{2}{100}$ or $\frac{1}{50}$	$\frac{1}{2}$%	.005	$\frac{\frac{1}{2}}{100}$ or $\frac{1}{200}$
5%	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$1\frac{1}{2}$%	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
10%	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$6\frac{1}{4}$%	$.06\frac{1}{4}$	$\frac{6\frac{1}{4}}{100}$ or $\frac{1}{16}$
25%	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$8\frac{1}{3}$%	$.08\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
50%	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$12\frac{1}{2}$%	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
75%	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$16\frac{2}{3}$%	$.16\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
100%	1.00	$\frac{100}{100}$ or 1	$33\frac{1}{3}$%	$.33\frac{1}{3}$	$\frac{33\frac{1}{3}}{100}$ or $\frac{1}{3}$
125%	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$37\frac{1}{2}$%	$.37\frac{1}{2}$	$\frac{37\frac{1}{2}}{100}$ or $\frac{3}{8}$
150%	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$	$62\frac{1}{2}$%	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$
500%	5.00	$\frac{500}{100}$ or 5	$87\frac{1}{2}$%	.875	$\frac{87\frac{1}{2}}{100}$ or $\frac{7}{8}$

7. The names of the different terms used in percentage are: the *base*, the *rate* or *rate per cent.*, the *percentage*, the *amount*, and the *difference.*

8. The **base** is the number or quantity which is supposed to be divided into 100 equal parts.

9. The **rate per cent.** is that number of the 100 equal parts into which the base is supposed to be divided which is taken or considered. The **rate** is the number of hundredths of the base that is taken or considered. The distinction between the rate per cent. and the rate is this: The *rate per cent.* is always 100 times the *rate.* Thus, 7% of 125 and .07 of 125 amount in the end to the same thing; the former, 7, is the *rate per cent.*—the *number* of hundredths of 125 intended; the latter, .07, is the *rate*, the *part* of 125 that is to be found; 7% is used in *speech*, .07 is the form used in *computation.* So, also, $12\frac{1}{2}\% = .125$, $\frac{1}{2}\% = .005$, $1\frac{3}{4}\% = .0175$. In the table just given, the numbers in the first column are rates per cent.; those in the second column are rates.

10. The **percentage** is the result obtained by multiplying the base by the rate. Thus, 7% of $125 = 125 \times .07 = 8.75$, the percentage.

11. The **amount** is the sum of the base and the percentage.

12. The **difference** is the remainder obtained when the percentage is subtracted from the base.

13. The terms amount and difference are ordinarily used when there is an increase or a decrease in the base. For example, suppose the population of a village is 1,500 and it increases 25 per cent. This means that for every 100 of the original 1,500 there is an increase of 25, or a total increase of $15 \times 25 = 375$. This increase added to the original population gives the amount, or the population after the increase. If the population had decreased 375,

the final population would have been $1,500 - 375 = 1,125$, and this would be the **difference.** The original population, 1,500, is the base on which the percentage is computed; the 25 is the rate per cent., and the increase or decrease, 375, is the percentage. If the base increases, the final value is the amount; and if it decreases, its final value is the difference.

CALCULATIONS INVOLVING PERCENTAGE.

14. From the foregoing it is evident that to find the percentage, the base must be multiplied by the rate. Hence, the following

Rule.—*To find the percentage, multiply the base by the rate.*

EXAMPLE.—Out of a lot of 300 boiler tubes 76% was used in a boiler. How many tubes were used?

SOLUTION.—The rate is .76; the base is 300; hence, the number of tubes used, or the percentage, is by the above rule

$$300 \times .76 = 228 \text{ tubes. Ans.}$$

Expressing the rule as a

Formula, *percentage = base × rate.*

15. When the percentage and rate are given, the base may be found by dividing the percentage by the rate. For, suppose that 12 is 6%, or $\frac{6}{100}$, of some number; then 1%, or $\frac{1}{100}$, of the number, is $12 \div 6$, or 2. Consequently, if $2 = 1\%$, or $\frac{1}{100}$, 100%, or $\frac{100}{100} = 2 \times 100 = 200$. But as the same result may be arrived at by dividing 12 by .06, since $12 \div .06 = 200$, it follows that:

Rule.—*When the percentage and rate are given, to find the base, divide the percentage by the rate.*

Formula, *base = percentage ÷ rate.*

Example.— 76% of a lot of boiler tubes was used in the construction of a boiler. If the number of tubes used was 228, how many tubes were in the lot?

Solution.—Here 228 is the percentage and .76 is the rate; hence, applying the rule,

$$228 \div .76 = 300 \text{ tubes. Ans.}$$

16. When the base and percentage are given, to find the rate, the rate may be found by dividing the percentage by the base. For, suppose that it is desired to find what per cent. 12 is of 200. 1% of 200 is $200 \times .01 = 2$. Now, if 1% is 2, 12 is evidently as many per cent. as 2 is contained times in 12, or $12 \div 2 = 6\%$. But the same result may be obtained by dividing 12, the percentage, by 200, the base, since $12 \div 200 = .06 = 6\%$. Hence,

Rule.—*When the percentage and base are given, to find the rate, divide the percentage by the base, and the result will be the rate.*

Formula, *rate* = *percentage* ÷ *base*.

Example 1.—Out of a lot of 300 boiler tubes 228 were used. What per cent. of the total number was used?

Solution.—Here 300 is the base and 228 is the percentage; hence, applying rule,

$$\text{Rate} = 228 \div 300 = .76 = 76\%. \text{ Ans.}$$

Example 2.—What per cent. of 875 is 25?

Solution.—Here 875 is the base and 25 is the percentage; hence, applying rule,

$$\text{Rate} = 25 \div 875 = .02\tfrac{6}{7} = 2\tfrac{6}{7}\%. \text{ Ans.}$$

Proof.— $875 \times .02\tfrac{6}{7} = 25$.

EXAMPLES FOR PRACTICE.

17. What per cent of:

			Ans.
(*a*)	360 is 90?	(*a*)	25%.
(*b*)	900 is 360?	(*b*)	40%.
(*c*)	125 is 25?	(*c*)	20%.
(*d*)	150 is 750?	(*d*)	500%.
(*e*)	280 is 112?	(*e*)	40%.
(*f*)	400 is 200?	(*f*)	50%.
(*g*)	47 is 94?	(*g*)	200%.
(*h*)	500 is 250?	(*h*)	50%.

18. The amount may be found, when the base and rate are given, by multiplying the base by 1 plus the rate expressed decimally. For, suppose that it is desired to find the amount when 200 is the base and .06 is the rate. The percentage is $200 \times .06 = 12$, and, according to definition, Art. **11**, the amount is $200 + 12 = 212$. But the same result may be obtained by multiplying 200 by $1 + .06$, or 1.06, since $200 \times 1.06 = 212$. Hence,

Rule.—*When the base and rate are given, to find the amount, multiply the base by 1 plus the rate.*

$$\text{Formula, } \textit{amount} = \textit{base} \times (1 + \textit{rate}).$$

EXAMPLE.—If a man earned $725 in a year, and the next year 10% more, how much did he earn the second year?

SOLUTION.—Here 725 is the base and .10 is the rate, and the amount is required. Hence, applying the rule,

$$725 \times 1.10 = \$797.50. \quad \text{Ans.}$$

19. When the base and rate are given, the difference may be found by multiplying the base by 1 minus the rate expressed decimally. For, suppose that it is desired to find the difference when the base is 200 and the rate is 6%. The percentage is $200 \times .06 = 12$; and, according to definition, Art. **12**, the difference $= 200 - 12 = 188$. But the same result may be obtained by multiplying 200 by $1 - .06$, or .94, since $200 \times .94 = 188$. Hence,

Rule.—*When the base and rate are given, to find the difference, multiply the base by 1 minus the rate.*

$$\text{Formula, } \textit{difference} = \textit{base} \times (1 - \textit{rate}).$$

EXAMPLE.—Out of a lot of 300 boiler tubes all but 24% were used in one boiler; how many tubes were used?

SOLUTION.—Here 300 is the base, .24 is the rate, and it is desired to find the difference. Hence, applying the rule,

$$300 \times (1 - .24) = 228 \text{ tubes.} \quad \text{Ans.}$$

20. When the amount and rate are given, the base may be found by dividing the amount by 1 plus the rate. For, suppose that it is known that 212 equals some number

increased by 6% of itself. Then, it is evident that 212 equals 106% of the number (base) that it is desired to find. Consequently, if $212 = 106\%$, $1\% = \frac{212}{106} = 2$, and $100\% = 2 \times 100 = 200 =$ the base. But the same result may be obtained by dividing 212 by $1 + .06$ or 1.06, since $212 \div 1.06 = 200$. Hence,

Rule.—*When the amount and rate are given, to find the base, divide the amount by 1 plus the rate.*

Formula, $base = amount \div (1 + rate)$.

EXAMPLE.—The theoretical discharge of a certain pump when running at a piston speed of 100 feet per minute is 278,910 gallons per day of 10 hours. Owing to leakage and other defects, this value is 25% greater than the actual discharge. What is the actual discharge?

SOLUTION.—Here 278,910 equals the actual discharge (base) increased by 25% of itself. Consequently, 278,910 is the amount and 25% is the rate. Applying rule,

Actual discharge $= 278{,}910 \div 1.25 = 223{,}128$ gal. Ans.

21. When the difference and rate are given, the base may be found by dividing the difference by 1 minus the rate. For, suppose that 188 equals some number less 6% of itself. Then, 188 evidently equals $100 - 6 = 94\%$ of some number. Consequently, if $188 = 94\%$, $1\% = 188 \div 94 = 2$, and $100\% = 2 \times 100 = 200$. But the same result may be obtained by dividing 188 by $1 - .06$, or .94, since $188 \div .94 = 200$. Hence,

Rule.—*When the difference and rate are given, to find the base, divide the difference by 1 minus the rate.*

Formula, $base = difference \div (1 - rate)$.

EXAMPLE 1.—From a lot of boiler tubes 76% was used in the construction of a boiler. If there were 72 tubes unused, how many tubes were in the lot?

SOLUTION.—Here 72 is the difference and .76 is the rate. Applying rule,

$72 \div (1 - .76) = 300$ tubes. Ans.

EXAMPLE 2.—The theoretical number of foot-pounds of work per minute required to operate a boiler feed-pump is 127,344. If 30% of the total number actually required be allowed for friction, leakage, etc., how many foot-pounds are actually required to work the pump?

SOLUTION.—Here the number actually required is the base; hence, 127,344 is the difference and .30 is the rate. Applying the rule,

$$127,344 \div (1 - .30) = 181,920 \text{ foot-pounds. Ans.}$$

22. EXAMPLE.—A certain chimney gives a draft of 2.76 inches of water. By increasing the height 20 feet, the draft was increased to 3 inches of water. What was the gain per cent.?

SOLUTION.—Here it is evident that 3 inches is the amount and that 2.76 inches is the base. Consequently, $3 - 2.76 = .24$ inch is the percentage, and it is required to find the rate. Hence, applying the rule given in Art. **16**,

$$\text{Gain per cent.} = .24 \div 2.76 = .087 = 8.7\%. \text{ Ans.}$$

23. EXAMPLE.—A certain chimney gave a draft of 3 inches of water. After an economizer had been put in, the draft was reduced to 1.2 inches of water. What was the loss per cent.?

SOLUTION.—Here it is evident that 1.2 inches is the difference (since it equals 3 inches diminished by a certain per cent. loss of itself) and 3 inches is the base. Consequently, $3 - 1.2 = 1.8$ inches is the percentage. Hence, applying the rule given in Art. **16**,

$$\text{Loss per cent.} = 1.8 \div 3 = .60 = 60\%. \text{ Ans.}$$

24. To find the gain or loss per cent.:

Rule.—*Find the difference between the initial and the final value; divide this difference by the initial value.*

EXAMPLE.—If a man buys a steam engine for $1,860 and some time afterwards purchases a condenser for 25% of the cost of the engine, does he gain or lose, and how much per cent., if he sells both engine and condenser for $2,100?

SOLUTION.—The cost of the condenser was $\$1,860 \times .25 = \$465$; consequently, the initial value, or cost, was $\$1,860 + \$465 = \$2,325$. Since he sold them for $2,100, he lost $\$2,325 - \$2,100 = \$225$. Hence, applying rule,

$$225 \div 2,325 = .0968 = 9.68\% \text{ loss. Ans.}$$

EXAMPLES FOR PRACTICE.

25. Solve the following:

(*a*)	What is $12\frac{1}{2}\%$ of \$900 ?	Ans.	(*a*)	\$112.50.
(*b*)	What is $\frac{4}{5}\%$ of 627 ?		(*b*)	5.016.
(*c*)	What is $33\frac{1}{3}\%$ of 54 ?		(*c*)	18.
(*d*)	101 is $68\frac{3}{4}\%$ of what number ?		(*d*)	$146\frac{10}{11}$.
(*e*)	784 is $83\frac{1}{3}\%$ of what number ?		(*e*)	940.8.
(*f*)	What % of 960 is 160 ?		(*f*)	$16\frac{2}{3}\%$.
(*g*)	What % of \$3,606 is \$$450\frac{3}{4}$?		(*g*)	$12\frac{1}{2}\%$.
(*h*)	What % of 280 is 112 ?		(*h*)	40%.

1. A steam plant consumed an average of 3,640 pounds of coal per day. The engineer made certain alterations which resulted in a saving of 250 pounds per day. What was the per cent. of coal saved ?
Ans. 7%, nearly.

2. If the speed of an engine running at 126 revolutions per minute should be increased $6\frac{1}{2}\%$, how many revolutions per minute would it then make ? Ans. 134.19 rev.

3. The list price of an engine was \$1,400; of a boiler, \$1,150; and of the necessary fittings for the two, \$340. If 25% discount was allowed on the engine, 22% on the boiler, and $12\frac{1}{2}\%$ on the fittings, what was the actual cost of the plant ? Ans. \$2,244.50.

4. If I lend a man \$1,100, and this is $18\frac{1}{2}\%$ of the amount that I have on interest, how much money have I on interest ? Ans. \$5,945.95.

5. A test showed that an engine developed 190.4 horsepower, 15% of which was consumed in friction. How much power was available for use ? Ans. 161.84 H. P.

6. By adding a condenser to a steam engine, the power was increased 14% and the consumption of coal per horsepower per hour was decreased 20%. If the engine could originally develop 50 horsepower and required $3\frac{1}{2}$ pounds of coal per horsepower per hour, what would be the total weight of coal used in an hour with the condenser, assuming the engine to run full power ? Ans. 159.6 lb.

DENOMINATE NUMBERS.

26. A **denominate number** is a concrete number, and may be either simple or compound; as, 8 quarts; 5 feet 10 inches, etc. Denominate numbers are also called **compound numbers.**

27. A **simple denominate number** consists of units of but one denomination; as, 16 cents; 10 hours; 5 dollars, etc.

28. A **compound denominate number** consists of units of two or more denominations of a similar kind; as, 3 yards 2 feet 1 inch.

29. In *whole numbers* and in *decimals*, the law of increase and decrease is on the scale of 10, but in *compound*, or *denominate*, *numbers* the scale varies.

MEASURES.

30. A **measure** is a *standard unit*, established by *law* or *custom*, by which quantity of any kind is measured. The standard unit of **dry measure** is the Winchester bushel; of **weight**, the pound; of **liquid measure**, the gallon, etc.

31. Measures are of six kinds:

1. Extension.
2. Weight.
3. Capacity.
4. Time.
5. Angles.
6. Money or value.

MEASURES OF EXTENSION.

32. **Measures of extension** are used in measuring lengths, distances, surfaces, and solids.

LINEAR MEASURE.

TABLE.

		Abbreviation.
12 inches (in.)	= 1 foot	ft.
3 feet	= 1 yard	yd.
5.5 yards	= 1 rod	rd.
40 rods	= 1 furlong	fur.
8 furlongs	= 1 mile	mi.

in.	ft.	yd.	rd.	fur.	mi.
36 =	3 =	1			
198 =	$16\frac{1}{2}$ =	5.5 =	1		
7,920 =	660 =	220 =	40 =	1	
63,360 =	5,280 =	1,760 =	320 =	8 =	1

SQUARE MEASURE.

TABLE.

144 square inches (sq. in.)	= 1 square foot	sq. ft.
9 square feet	= 1 square yard	sq. yd.
$30\frac{1}{4}$ square yards	= 1 square rod	sq. rd.
160 square rods	= 1 acre	A.
640 acres	= 1 square mile	sq. mi.

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
1 =	640 =	102,400 =	3,097,600 =	27,878,400 =	4,014,489,600

CUBIC MEASURE.

TABLE.

1,728	cubic inches (cu. in.)	=	1 cubic foot	cu. ft.
27	cubic feet	=	1 cubic yard	cu. yd.
128	cubic feet	=	1 cord	cd.
24¾	cubic feet	=	1 perch	P.

cu. yd.		cu. ft.		cu. in.
1	=	27	=	46,656

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE.

16 ounces (oz.)	=	1 pound	lb.
100 pounds	=	1 hundredweight	cwt.
20 cwt., or 2,000 lb.	=	1 ton	T.

T.		cwt.		lb.		oz.
1	=	20	=	2,000	=	32,000

33. The ounce is divided into halves, quarters, etc. Avoirdupois weight is used for weighing coarse and heavy articles.

LONG TON TABLE.

16 ounces	=	1 pound	lb.
112 pounds	=	1 hundredweight	cwt.
20 cwt., or 2,240 lb.	=	1 ton	T.

34. In all the calculations hereafter, 2,000 pounds will be considered 1 ton, unless the long ton (2,240 pounds) is especially mentioned.

TROY WEIGHT.

TABLE.

24 grains (gr.)	=	1 pennyweight	pwt.
20 pennyweights	=	1 ounce	oz.
12 ounces	=	1 pound	lb.

lb.		oz.		pwt.		gr.
1	=	12	=	240	=	5,760

Troy weight is used in weighing gold and silver ware, jewels, etc. It is used by jewelers.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE.

4 gills (gi.)	=	1 pint	pt.
2 pints	=	1 quart	qt.
4 quarts	=	1 gallon	gal.
31½ gallons	=	1 barrel	bbl.
2 barrels, or 63 gallons	=	1 hogshead	hhd.

hhd.		bbl.		gal.		qt.		pt.		gi.
1	=	2	=	63	=	252	=	504	=	2,016

DRY MEASURE.

TABLE.

2 pints (pt.)	=	1 quart	qt.
8 quarts	=	1 peck	pk.
4 pecks	=	1 bushel	bu.

bu.		pk.		qt.		pt.
1	=	4	=	32	=	64

MEASURE OF TIME.

TABLE.

60 seconds (sec.)	=	1 minute	min.
60 minutes	=	1 hour	hr.
24 hours	=	1 day	da.
7 days	=	1 week	wk.
365 days / 12 months	=	1 common year	yr.
366 days	=	1 leap year.	
100 years	=	1 century.	

NOTE.—It is customary to consider one month as 30 days.

MEASURE OF ANGLES OR ARCS.

TABLE.

60 seconds (″)	=	1 minute	′.
60 minutes	=	1 degree	°.
90 degrees	=	1 right angle or quadrant	∟.
360 degrees	=	1 circle	cir.

$$1 \text{ cir.} = 360° = 21{,}600' = 1{,}296{,}000''$$

MEASURE OF MONEY.

UNITED STATES MONEY.

TABLE.

10 mills (m.)	=	1 cent	ct.
10 cents	=	1 dime	d.
10 dimes	=	1 dollar	$.
10 dollars	=	1 eagle	E.

E.		$		d.		ct.		m.
1	=	10	=	100	=	1,000	=	10,000

MISCELLANEOUS TABLE.

12 things are 1 dozen.
12 dozen are 1 gross.
12 gross are 1 great gross.
2 things are 1 pair.
20 things are 1 score.
1 league is 3 miles.
1 fathom is 6 feet.
1 meter is 39.37 inches.
1 hand is 4 inches.
1 palm is 3 inches.
1 span is 9 inches.
24 sheets are 1 quire.
20 quires, or 480 sheets, are 1 ream.
1 bushel contains 2,150.4 cu. in.
1 U. S. standard gallon (also called a wine gallon) contains 231 cu. in.
1 U. S. standard gallon of water weighs 8.355 pounds, nearly.
1 cubic foot contains 7.481 U. S. standard gallons, nearly.
1 British imperial gallon of water weighs 10 pounds.

It will be of great advantage to the student to carefully memorize all the above tables.

REDUCTION OF DENOMINATE NUMBERS.

35. Reduction of denominate numbers is the process of changing their denomination without changing their value. They may be changed from a higher to a lower denomination or from a lower to a higher—either is reduction. As

2 hours = 120 minutes.
32 ounces = 2 pounds.

36. Principle.—Denominate numbers are changed to lower denominations by *multiplying*, and to higher denominations by *dividing*.

To reduce denominate numbers to lower denominations:

37. EXAMPLE.—Reduce 5 yd. 2 ft. 7 in. to inches.

SOLUTION.—

$$\begin{array}{rlll}
 & \text{yd.} & \text{ft.} & \text{in.} \\
 & 5 & 2 & 7 \\
 & 3 & & \\
\hline
15 & \text{ft.} & & \\
2 & \text{ft.} & & \\
\hline
17 & \text{ft.} & & \\
12 & & & \\
\hline
34 & & & \\
17 & & & \\
\hline
204 & \text{in.} & & \\
7 & \text{in.} & & \\
\hline
211 & \text{in.} & \text{Ans.} &
\end{array}$$

EXPLANATION.—Since there are 3 feet in 1 yard, in 5 yards there are 5×3, or 15, feet, and 15 feet + 2 feet = 17 feet. There are 12 inches in a foot; therefore, $12 \times 17 = 204$ inches, and 204 inches + 7 inches = 211 inches in 5 yards 2 feet 7 inches.

38. EXAMPLE.—Reduce 6 hours to seconds.

SOLUTION.—

$$\begin{array}{rl}
6 & \text{hr.} \\
60 & \\
\hline
360 & \text{min.} \\
60 & \\
\hline
21600 & \text{sec.} \quad \text{Ans.}
\end{array}$$

EXPLANATION.—As there are 60 minutes in 1 hour, in 6 hours there are 6×60, or 360, minutes; as there are no minutes to add, we multiply 360 minutes by 60, to get the number of seconds.

39. In order to avoid mistakes, if any denomination be omitted, represent it by a cipher. Thus, before reducing 3 rods 6 inches to inches, insert a cipher for yards and a cipher for feet, as

rd.	yd.	ft.	in.
3	0	0	6

40. **Rule.**—*Multiply the number representing the highest denomination by the number of units in the next lower required*

to make one of the higher denomination, and to the product add the number of given units of that lower denomination. Proceed in this manner until the number is reduced to the required denomination.

EXAMPLES FOR PRACTICE.

41. Reduce:

(*a*)	4 rd. 2 yd. 2 ft. to feet.	(*a*)	74 ft.
(*b*)	4 bu. 3 pk. 2 qt. to quarts.	(*b*)	154 qt.
(*c*)	13 rd. 5 yd. 2 ft. to feet.	(*c*)	231.5 ft.
(*d*)	5 mi. 100 rd. 10 ft. to feet.	Ans. (*d*)	28,060 ft.
(*e*)	52 hhd. 24 gal. 1 pt. to pints.	(*e*)	26,401 pt.
(*f*)	5 cir. 16° 20′ to minutes.	(*f*)	108,980′.
(*g*)	14 bu. to quarts.	(*g*)	448 qt.

To reduce lower to higher denominations:

42. Example.—Reduce 211 inches to higher denominations:

Solution.—

$$\begin{array}{r} 12\,)\,211 \text{ in.} \\ \hline 3\,)\,17 \text{ ft.} + 7 \text{ in.} \\ \hline 5 \text{ yd.} + 2 \text{ ft.} + 7 \text{ in.} \end{array} \quad \text{Ans.}$$

Explanation.—There are 12 inches in 1 foot; therefore, 211 divided by 12 = 17 feet and 7 inches over. There are 3 feet in 1 yard; therefore, 17 feet divided by 3 = 5 yards and 2 feet over. The last quotient and the two remainders constitute the answer, 5 yards 2 feet 7 inches.

43. Example.—Reduce 14,135 gills to higher denominations.

Solution.—

$$\begin{array}{r} 4\,)\,14135 \phantom{\text{ pt. 3 gi.}} \\ \hline 2\,)\,3533 \text{ pt. 3 gi.} \\ \hline 4\,)\,1766 \text{ qt. 1 pt.} \\ \hline 441 \text{ gal. 2 qt.} \end{array}$$

$$\begin{array}{r} 31.5\,)\,441.0\,(\,14 \text{ bbl.} \\ 315 \phantom{\,(\,14 \text{ bbl.}} \\ \hline 1260 \phantom{\,(\,14 \text{ bbl.}} \\ 1260 \phantom{\,(\,14 \text{ bbl.}} \\ \hline \end{array}$$

Explanation.—There are 4 gills in 1 pint, and in 14,135 gills there are as many pints as 4 is contained times in 14,135, or 3,533 pints and 3 gills remaining. There are 2 pints in 1 quart, and in 3,533 pints there are 1,766 quarts and 1 pint remaining. There are 4 quarts in 1 gallon, and

in 1,766 quarts there are 441 gallons and 2 quarts remaining. There are $31\frac{1}{2}$ gallons in 1 barrel, and in 441 gallons there are 14 barrels.

The last quotient and the three remainders constitute the answer, 14 barrels 2 quarts 1 pint 3 gills.

44. Rule.—*Divide the number representing the denomination given by the number of units of this denomination required to make one unit of the next higher denomination. The remainder will be of the same denomination, but the quotient will be of the next higher. Divide this quotient by the number of units of its denomination required to make one unit of the next higher. Continue until the highest denomination is reached or until there is not enough of a denomination left to make one of the next higher. The last quotient and the remainders constitute the required result.*

EXAMPLES FOR PRACTICE.

45. Reduce to units of higher denominations:

(*a*) 7,460 sq. in.; (*b*) 7,580 sq. yd.; (*c*) 148,760 cu. in.; (*d*) 17,651″; (*e*) 8,000 gi.; (*f*) 36,450 lb.

Ans.
- (*a*) 5 sq. yd. 6 sq. ft. 116 sq. in.
- (*b*) 1 A. 90 sq. rd. 17 sq. yd. 4 sq. ft. 72 sq. in.
- (*c*) 3 cu. yd. 5 cu. ft. 152 cu. in.
- (*d*) 4° 54′ 11″.
- (*e*) 3 hhd. 61 gal.
- (*f*) 18 T. 4 cwt. 50 lb.

ADDITION OF DENOMINATE NUMBERS.

46. As in the case of abstract numbers, denominate numbers may be added, subtracted, multiplied, and divided.

47. EXAMPLE.—Find the sum of 3 cwt. 46 lb. 12 oz.; 8 cwt. 12 lb. 13 oz.; 12 cwt. 50 lb. 13 oz.; 27 lb. 4 oz.

SOLUTION.—

T.	cwt.	lb.	oz.	
0	3	46	12	
0	8	12	13	
0	12	50	13	
0	0	27	4	
1	4	37	10	Ans.

Explanation.—Begin to add at the right-hand column; $4 + 13 + 13 + 12 = 42$ ounces; as 16 ounces make 1 pound, 42 ounces $\div$ 16 = 2 pounds and a remainder of 10 ounces, or 2 pounds and 10 ounces. Place 10 ounces under the ounce column and add 2 pounds to the next, or pound, column. Then, $2 + 27 + 50 + 12 + 46 = 137$ pounds; as 100 pounds make a hundredweight, $137 \div 100 = 1$ hundredweight and a remainder of 37 pounds. Place the 37 under the pound column and add 1 hundredweight to the next, or hundredweight, column. Next, $1 + 12 + 8 + 3 = 24$ hundredweight. 20 hundredweight make a ton; therefore, $24 \div 20 = 1$ ton and 4 hundredweight remaining. Hence, the sum is 1 ton 4 hundredweight 37 pounds 10 ounces.

48. Example.—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 11 in.; 4 yd. 1 ft.?

Solution.—

	rd.	yd.	ft.	in.	
	2	3	2	5	
	6	0	1	10	
	17	0	0	11	
	0	4	1	0	
	26	$3\frac{1}{2}$	0	2	
or	26	3	1	8	Ans.

Explanation.—The sum of the numbers in the first column = 26 inches, or 2 feet and 2 inches remaining. The sum of the numbers in the next column plus 2 feet = 6 feet, or 2 yards and 0 feet remaining. The sum of the numbers in the next column plus 2 yards = 9 yards, or $9 \div 5\frac{1}{2} = 1$ rod and $3\frac{1}{2}$ yards remaining. The sum of the next column plus 1 rod = 26 rods. To avoid fractions in the sum, the $\frac{1}{2}$ yard is reduced to 1 foot and 6 inches, which added to 26 rods 3 yards 0 feet and 2 inches = 26 rods 3 yards 1 foot 8 inches.

49. Example.—What is the sum of 47 ft. and 3 rd. 2 yd. 2 ft. 10 in.?

Solution.—When 47 ft. is reduced, it equals 2 rd. 4 yd. 2 ft., which can be added to 3 rd. 2 yd. 2 ft. 10 in. Thus,

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	6	$1\frac{1}{2}$	1	10	
or	6	2	0	4	Ans.

50. Rule.—*Place the numbers so that like denominations are under one another. Begin at the right-hand column and add. Divide the sum by the number of units of this denomination required to make one unit of the next higher. Place the remainder under the column added and carry the quotient to the next column. Continue in this manner until the highest denomination given is reached.*

EXAMPLES FOR PRACTICE.

51. What is the sum of:

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.?

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2rd. 3 in.; 14 rd. 1 yd. 9 in.?

(*c*) 3 cwt. 46 lb. 12 oz.; 12 cwt. $9\frac{1}{3}$ lb.; $2\frac{1}{4}$ cwt. $21\frac{5}{8}$ lb.?

(*d*) 10 yr. 8 mo. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 5 mo. 4 da.; 17 yr. 17 da.?

(*e*) 17 T. 11 cwt. 49 lb. 14 oz.; 16 T. 47 lb. 13 oz.; 20 T. 13 cwt. 14 lb. 6 oz.; 11 T. 4 cwt. 16 lb. 12 oz.?

(*f*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.?

Ans.
- (*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
- (*b*) 25 mi. 47 rd. 1 ft. 5 in.
- (*c*) 18 cwt. 2 lb. 14 oz.
- (*d*) 77 yr. 8 mo. 1 da.
- (*e*) 65 T. 9 cwt. 28 lb. 13 oz.
- (*f*) 167 sq. yd. 136 sq. in.

SUBTRACTION OF DENOMINATE NUMBERS.

52. Example.—From 21 rd. 2 yd. 2 ft. $6\frac{1}{2}$ in. take 9 rd. 4 yd. $10\frac{1}{4}$ in.

Solution.—

rd.	yd.	ft.	in.	
21	2	2	$6\frac{1}{2}$	
9	4	0	$10\frac{1}{4}$	
11	$3\frac{1}{2}$	1	$8\frac{1}{4}$	Ans.

Explanation.—Since $10\frac{1}{4}$ inches cannot be taken from $6\frac{1}{2}$ inches, we must borrow 1 foot, or 12 inches, from the 2 feet in the next column and add it to the $6\frac{1}{2}$. $6\frac{1}{2} + 12 = 18\frac{1}{2}$. $18\frac{1}{2}$ inches $-$ $10\frac{1}{4}$ inches $= 8\frac{1}{4}$ inches. Then, 0 foot

from the 1 remaining foot = 1 foot. 4 yards cannot be taken from 2 yards; therefore, we borrow 1 rod, or $5\frac{1}{2}$ yards, from 21 rods and add it to 2. $2 + 5\frac{1}{2} = 7\frac{1}{2}$; $7\frac{1}{2} - 4 = 3\frac{1}{2}$ yards. 9 rods from 20 rods = 11 rods. Hence, the remainder is 11 rods $3\frac{1}{2}$ yards 1 foot $8\frac{1}{4}$ inches.

To avoid fractions as much as possible, we reduce the $\frac{1}{2}$ yard to inches, obtaining 18 inches; this added to $8\frac{1}{4}$ inches gives $26\frac{1}{4}$ inches, which equals 2 feet $2\frac{1}{4}$ inches. Then, 2 feet + 1 foot = 3 feet = 1 yard, and 3 yards + 1 yard = 4 yards. Hence, the above answer becomes 11 rods 4 yards 0 feet $2\frac{1}{4}$ inches.

53. Example.—What is the difference between 3 rd. 2 yd. 2 ft. 10 in. and 47 ft.?

Solution.— 47 ft. = 2 rd. 4 yd. 2 ft.

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	0	$3\frac{1}{2}$	0	10	
or		3	2	4	Ans.

To find (approximately) the interval of time between two dates:

54. Example.—How many years, months, days, and hours between 4 o'clock P. M. of June 15, 1868, and 10 o'clock A. M., September 28, 1891?

Solution.—

yr.	mo.	da.	hr.	
1891	9	28	10	
1868	6	15	16	
23	3	12	18	Ans.

Explanation.—Counting 24 hours in 1 day, 4 o'clock P. M. is the 16th hour from the beginning of the day, or midnight. September is the 9th month and June is the 6th month. After placing the earlier date under the later date, subtract as in the previous problems. Count 30 days as 1 month.

55. Rule.—*Place the smaller quantity under the larger quantity, with like denominations under each other. Beginning at the right, subtract successively the number in the subtrahend in each denomination from the one above and place the differences underneath. If the number in the minuend of any*

denomination is less than the number under it in the subtrahend, one unit must be borrowed from the minuend of the next higher denomination, reduced, and added to it.

EXAMPLES FOR PRACTICE.

56. From:

(*a*) 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.
(*b*) 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.
(*c*) 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.
(*d*) 148 sq. yd. 16 sq. ft. 142 sq. in. take 132 sq. yd. 136 sq. in.
(*e*) 100 bu. take 28 bu. 2 pk. 5 qt. 1 pt.
(*f*) 14 mi. 36 rd. 5 yd. 13 ft. 11 in. take 3 mi. 29 rd. 4 ft. 10 in.

Ans.
(*a*) 28 lb. 11 oz. 4 pwt. 14 gr.
(*b*) 22 hhd. 12 gal. 2 qt. 1 pt.
(*c*) 49 T. 3 cwt. 63 lb. 12 oz.
(*d*) 16 sq. yd. 16 sq. ft. 6 sq. in.
(*e*) 71 bu. 1 pk. 2 qt. 1 pt.
(*f*) 11 mi. 7 rd. 5 yd. 9 ft. 1 in.

MULTIPLICATION OF DENOMINATE NUMBERS.

57. EXAMPLE.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by 12.

SOLUTION.—

lb.	oz.	pwt.	gr.	
7	5	13	15	
			12	
89	8	3	12	Ans.

EXPLANATION.— 15 grains × 12 = 180 grains. 180 ÷ 24 = 7 pennyweights and 12 grains remaining. Place the 12 in the grain column and carry the 7 pennyweights to the next column. Now, 13 × 12 + 7 = 163 pennyweights; 163 ÷ 20 = 8 ounces and 3 pennyweights remaining. Then, 5 × 12 + 8 = 68 ounces; 68 ÷ 12 = 5 pounds and 8 ounces remaining. Then, 7 × 12 + 5 = 89 pounds. The entire product is 89 pounds 8 ounces 3 pennyweights 12 grains.

58. Rule.—*Multiply the number representing each denomination by the multiplier, and reduce each product to the next higher denomination, writing the remainders under each denomination, and carry the quotient to the next, as in addition of denominate numbers.*

59. NOTE.—In multiplication and division of denominate numbers, it is sometimes easier to reduce the number to the lowest denomination given before multiplying or dividing, especially if the

multiplier or divisor is a decimal. Thus, in the above example, had the multiplier been 1.2, the easiest way to multiply would have been to reduce the number to grains; then, multiply by 1.2, and reduce the product to higher denominations. For example, 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. 43,047 × 1.2 = 51,656.4 gr. = 8 lb. 11 oz. 12 pwt. 8.4 gr. Also, 43,047 × 12 = 516,564 gr. = 89 lb. 8 oz. 3 pwt. 12 gr., as before. The student may use either method.

EXAMPLES FOR PRACTICE.

60. Multiply:

(*a*) 15 cwt. 90 lb. by 5; (*b*) 12 yr. 10 mo. 3 da. by 14; (*c*) 11 mi. 145 rd. by 20; (*d*) 12 gal. 4 pt. by 9; (*e*) 8 cd. 76 cu. ft. by 15; (*f*) 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

Ans.
- (*a*) 79 cwt. 50 lb.
- (*b*) 179 yr. 9 mo. 12 da.
- (*c*) 229 mi. 20 rd.
- (*d*) 112 gal. 2 qt.
- (*e*) 128 cd. 116 cu. ft.
- (*f*) 48 hhd. 40 gal. 2 qt.

DIVISION OF DENOMINATE NUMBERS.

61. Example 1.—Divide 48 lb. 11 oz. 6 pwt. by 8.

Solution.—

	lb.	oz.	pwt.	gr.	
8)	48	11	6	0	
	6 lb.	1 oz.	8 pwt.	6 gr.	Ans.

Explanation.—After placing the quantities as above, proceed as follows: 8 is contained in 48 6 times without a remainder. 8 is contained in 11 ounces once with 3 ounces remaining. 3 × 20 = 60; 60 + 6 = 66 pennyweights; 66 pennyweights ÷ 8 = 8 pennyweights and 2 pennyweights remaining; 2 × 24 grains = 48 grains; 48 grains ÷ 8 = 6 grains. Therefore, the entire quotient is 6 pounds 1 ounce 8 pennyweights 6 grains.

Example 2.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 spoons; what was the weight of each spoon?

Solution.—

	lb.	oz.	pwt.		
6)	2	8	10		
		5 oz.	8 pwt.	8 gr.	Ans.

Explanation.—Since we cannot divide 2 pounds by 6, we reduce it to ounces. 2 pounds = 24 ounces, and 24 ounces + 8 ounces = 32 ounces; 32 ounces ÷ 6 = 5 ounces and

2 ounces over. 2 ounces = 40 pennyweights. 40 pennyweights + 10 pennyweights = 50 pennyweights, and 50 pennyweights ÷ 6 = 8 pennyweights and 2 pennyweights over. 2 pennyweights = 48 grains, and 48 grains ÷ 6 = 8 grains. Hence, each spoon contains 5 ounces 8 pennyweights 8 grains.

62. Example.—Divide 820 rd. 4 yd. 2 ft. by 112.

Solution.—

```
              rd.  yd.  ft.  rd.  yd.  ft.  in.
       112)820    4    2 (7    1    2  5.143  Ans.
           784
           ---
            36 rd. rem.
            5.5
           ---
           180
          180
          ----
          198.0 yd.
            4   yd.
          ----
      112)202 yd. (1 yd.
          112
          ---
           90 yd. rem.
            3
          ---
          270 ft.
            2 ft.
          ---
      112)272 ft. (2 ft.
          224
          ---
           48 ft. rem.
           12
           --
           96
          48
          --
      112)576.0000 in. (5.1428+ in., or 5.143 in.
          560
          ---
           160
           112
           ---
            480
            448
            ---
             320
             224
             ---
              960
              896
              ---
               64
```

EXPLANATION.—The first quotient is 7 rods with 36 rods remaining. $5.5 \times 36 = 198$ yards; 198 yards + 4 yards = 202 yards; 202 yards ÷ 112 = 1 yard and 90 yards remaining. $90 \times 3 = 270$ feet; 270 feet + 2 feet = 272 feet; 272 feet ÷ 112 = 2 feet and 48 feet remaining; $48 \times 12 = 576$ inches; 576 inches ÷ 112 = 5.143 inches, nearly.

The preceding example is solved by long division, because the numbers are too large to deal with mentally. Instead of expressing the last result as a decimal, it might have been expressed as a common fraction. Thus, $576 \div 112 = 5\frac{16}{112} = 5\frac{1}{7}$ inches. The chief advantage of using a common fraction is that if the quotient be multiplied by the divisor, the result will always be the same as the original dividend.

63. **Rule.**—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination and add to it the number in the given dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

64. Divide:

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 T. 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans. (*a*) 17 mi. $41\frac{7}{11}$ rd.; (*b*) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.; (*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.; (*d*) 4 sq. yd. 4 sq. ft. $2\frac{5}{18}$ sq. in.; (*e*) 12 mi. 112 rd. 2 yd.; (*f*) 6 T. 14 cwt. 41 lb. $3\frac{14}{15}$ oz.; (*g*) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.; (*h*) 1 mi. $38\frac{22}{25}$ rd.

1. On Monday, 1 T. 3 cwt. of coal are burned under a boiler; on Tuesday, 1 T. 1 cwt. 54 lb.; on Wednesday, 1 T. 2 cwt. 16 lb.; on Thursday, 1 T. 2 cwt. 70 lb.; on Friday, 1 T. 3 cwt. 43 lb.; on Saturday, 15 cwt. 68 lb. How much coal was burned during the week?

Ans. 6 T. 8 cwt. 51 lb.

2. 504 gal. 2 qt. of water are drawn from a tank containing 30 hhd. 4 gal. 3 qt. of water. How much water remains?

Ans. 22 hhd. 4 gal. 1 qt.

3. A main line shaft is made up of 5 lengths, as follows: 16 ft. 3 in., 15 ft. 8 in., 15 ft. 2 in., 14 ft. 6 in., 12 ft. 10 in. If 10 hangers are used, one being at each end of the shaft, what is the distance between them, supposing them to be spaced equally?

Ans. 8 ft. $3\frac{2}{9}$ in.

4. If the distance around a flywheel is 47 ft. 3 in. and the belt extends $\frac{3}{5}$ of the way around, what is the distance covered by the belt?

Ans. 28 ft. $4\frac{1}{5}$ in.

5. A boiler shell is made up of three sheets, each 5 ft. $6\frac{11}{16}$ in. long. If the lap at each of the two middle seams is $2\frac{1}{2}$ in., what is the length of the shell?

Ans. 16 ft. $3\frac{1}{16}$ in.

6. In a return-tubular boiler the heating surface is divided as follows: outside of shell, 98 sq. ft. 9.8 sq. in.; heads, 5 sq. ft. $4\frac{1}{2}$ sq. in.; tubes, 683 sq. ft. 10.75 sq. in. What is the total area of the heating surface in square feet and square inches?

Ans. 786 sq. ft. 25.05 sq. in.

ARITHMETIC.

(PART 5.)

INVOLUTION.

1. If a product consists of equal factors, it is called a **power** of one of those equal factors, and one of the equal factors is called a **root** of the product. The power and the root are named according to the number of equal factors in the product. Thus, 3×3, or 9, is the *second power*, or **square**, of 3; $3 \times 3 \times 3$, or 27, is the *third power*, or **cube**, of 3; $3 \times 3 \times 3 \times 3$, or 81, is the **fourth power** of 3. Also, 3 is the **second root**, or **square root**, of 9; 3 is the **third root**, or **cube root**, of 27; 3 is the **fourth root** of 81.

2. For the sake of brevity,

3×3 is written 3^2, and read **three square**, or *three exponent two;*

$3 \times 3 \times 3$ is written 3^3, and read **three cube**, or *three exponent three;*

$3 \times 3 \times 3 \times 3$ is written 3^4, and read **three fourth**, or *three exponent four;*

and so on.

A number written above and to the right of another number, to show how often the latter number is used as a factor, is called an **exponent**. Thus, in 3^{12}, the number ^{12} is the exponent, and shows that 3 is to be used as a factor twelve times; so that 3^{12} is a contraction for

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3.$$

In an expression like 3^5, the exponent ^{5} shows how often 3 is used as a factor. Hence, if the exponent of a number is unity, the number is used once as a factor; thus, $3^1 = 3$, $4^1 = 4$, $5^1 = 5$.

3. If the side of a square contains 5 inches, the area of the square contains 5×5, or 5^2, square inches. If the edge of a cube contains 5 inches, the volume of the cube contains $5 \times 5 \times 5$, or 5^3, cubic inches. It is for this reason that 5^2 and 5^3 are called the square and cube of 5, respectively.

4. To find any power of a number:

EXAMPLE 1.—What is the third power, or cube, of 35?

SOLUTION.— $35 \times 35 \times 35$

$$
\begin{array}{rr}
\text{or} & 35 \\
 & 35 \\
\hline
 & 175 \\
 & 105 \\
\hline
 & 1225 \\
 & 35 \\
\hline
 & 6125 \\
 & 3675 \\
\hline
\textit{cube} = & 42875 \quad \text{Ans.}
\end{array}
$$

EXAMPLE 2.—What is the fourth power of 15?

SOLUTION.— $15 \times 15 \times 15 \times 15$

$$
\begin{array}{rr}
\text{or} & 15 \\
 & 15 \\
\hline
 & 75 \\
 & 15 \\
\hline
 & 225 \\
 & 15 \\
\hline
 & 1125 \\
 & 225 \\
\hline
 & 3375 \\
 & 15 \\
\hline
 & 16875 \\
 & 3375 \\
\hline
\textit{fourth power} = & 50625 \quad \text{Ans.}
\end{array}
$$

EXAMPLE 3.— 1.2^3 = what?

SOLUTION.— $1.2 \times 1.2 \times 1.2$

$$\begin{array}{lr} \text{or} & 1.2 \\ & 1.2 \\ \hline & 1.44 \\ & 1.2 \\ \hline & 288 \\ & 144 \\ \hline \textit{cube} = & 1.728 \end{array} \quad \text{Ans.}$$

EXAMPLE 4.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.— $\left(\frac{3}{8}\right)^3 = \frac{3^3}{8^3} = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

5. Rule.—I. *To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

Raise the following to the powers indicated:

(*a*) 85^2.
(*b*) $\left(\frac{12}{13}\right)^2$.
(*c*) 6.5^2.
(*d*) 14^4.
(*e*) $\left(\frac{3}{4}\right)^3$.
(*f*) $\left(\frac{5}{6}\right)^3$.
(*g*) $\left(\frac{7}{2}\right)^3$.
(*h*) 1.4^5.

Ans.
(*a*) 7,225.
(*b*) $\frac{144}{169}$.
(*c*) 42.25.
(*d*) 38,416.
(*e*) $\frac{27}{64}$.
(*f*) $\frac{125}{216}$.
(*g*) $\frac{343}{8}$.
(*h*) 5.37824.

EVOLUTION.

DEFINITIONS AND GENERAL REMARKS.

6. Evolution is the reverse of involution. It is the process of finding the root of a number that is considered as a power.

7. The **square root** of a number is that number which, when used twice as a factor, produces the number.

Thus, 2 is the square root of 4, since 2×2 (or 2^2) $= 4$.

8. The **cube root** of a number is that number which, when used three times as a factor, produces the number.

Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$ (or 3^3) $= 27$.

9. The **fourth root** of a number is that number which, when used four times as a factor, produces the number.

Thus, 9 is the fourth root of 6,561, since $9 \times 9 \times 9 \times 9$ (or 9^4) $= 6{,}561$.

10. The **fifth root** of a number is that number which, when used five times as a factor, produces the number.

Thus, 7 is the fifth root of 16,807, since $7 \times 7 \times 7 \times 7 \times 7$ (or 7^5) $= 16{,}807$.

11. The processes of finding squares and cubes and square roots and cube roots are very frequently employed in connection with the solution of problems pertaining to mensuration and engineering. The process of raising a number to some power, the exponent of the number being integral (*integral* is the adjective for integer; i. e., an integral number is one that does not contain a fraction or decimal) is very simple; but the reverse process, that of finding the roots, is very long and laborious, for which reason tables are generally employed. The tables so used are of two kinds—those giving the roots directly and logarithms. While

the roots of numbers can be found without the aid of a table, it is not customary to do this except in the case of square root, which is comparatively easy. At the same time it is well to know some general method of finding the roots of numbers, as it might be necessary to find a root when a table was not at hand. For purposes of this Course, a knowledge of how to use a table is all that is necessary.

12. Some idea of the importance of the processes of involution and evolution may be obtained from the following:

In finding the area of a square or a circle, it is necessary to square the length of a certain line; conversely, in finding the side of a square or the diameter of a circle that will have a given area it is necessary to find the square root. In finding the volume of a cube or a sphere it is necessary to cube the length of a certain line; conversely, in finding the length of one of the edges of a cube or the diameter of a sphere that will have a given volume, it is necessary to find the cube root. There are many other cases where it is required to extract square root and cube root, but enough has been stated so far to show the importance of the processes.

13. Cube root is not required as often as square root; fourth and fifth roots are required but very seldom, and not at all in connection with the work of this Course.

14. Having shown the necessity of some means of finding the roots of numbers, the manner of using the table following the Examination Questions in this Section will now be explained. But before studying the explanations, certain definitions and properties of numbers must be carefully considered.

15. The **radical sign** $\sqrt{}$, when placed before a number, indicates that some root of that number is to be found. The vinculum is almost always used in connection with the radical sign, as shown in Art. **16.**

16. The **index** of the root is a *small figure* placed *over* and to the *left* of the *radical sign*, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100,
$\sqrt[3]{125}$ denotes the *cube root* of 125.

17. When the square root is to be extracted, the index is generally omitted. Thus, $\sqrt{100}$ indicates the square root of 100. Also, $\sqrt{225}$ indicates the square root of 225.

18. In any number, the figures beginning with the first digit* at the left and ending with the last digit at the right, are called the **significant figures** of the number. Thus, the number 405,800 has the four significant figures 4, 0, 5, 8; and the number .000090067 has the five significant figures 9, 0, 0, 6, and 7.

19. The part of a number consisting of its significant figures is called the **significant part** of the number. Thus, in the number 28,070, the significant part is 2807; in the number .00812, the significant part is 812; and in the number 170.3, the significant part is 1703.

20. In speaking of the significant figures or of the significant part of a number, we consider the figures, in their proper order, from the first digit at the left to the last digit at the right, but we pay no attention to the position of the decimal point. Hence, *all numbers that differ only in the position of the decimal point have the same significant part.* For example, .002103, 21.03, 21,030, and 210,300 have the same significant figures, 2, 1, 0, and 3, and the same significant part 2103.

The **integral part** of a number is the part to the left of the decimal point or to the left of the fraction when the number consists of a whole number and a fraction.

21. The student will find the following principles of value, both in connection with the extraction of roots and in other arithmetical calculations:

a. In general, if any two numbers are multiplied together, no matter how many significant figures they contain, the first five significant figures of the product will be the same as the first five significant figures of the product obtained by multiplying the same two numbers when limited to five significant figures.

*A cipher is not a digit.

For example, the product of 4,562,357 and 6,421,849 is 29,298,767,738,093; limiting the numbers to five significant figures, the product of 45,624 and 64,218 is 2,929,882,032; and the value of both these products to five significant figures is 29,299. In other words, if only five significant figures are required in the product, it is not necessary to use more than five significant figures in the multiplier and multiplicand, the remaining figures, if any, being replaced by ciphers, and the fifth figures being increased by 1 if the sixth figure is 5 or a larger digit. In some cases, however, the fifth figure may be one unit too large or one unit too small; hence, if it is necessary that the fifth figure be absolutely exact, it is better to limit the multiplier and multiplicand to six figures instead of five.

For example, 4,562,347 × 6,421,849 = 29,298,703,519,603, or 29,299,000,000,000 to five significant figures; 4,562,300 × 6,421,800 = 29,298,178,140,000 = 29,298,000,000,000 to five significant figures, the fifth figure being 1 less than it should be; but 4,562,350 × 6,421,850 = 29,298,727,347,500 = 29,299,000,000,000 to five significant figures.

b. If the divisor and dividend are limited to six significant figures, the quotient will always be correct to five (usually to six) significant figures, regardless of how many significant figures there may have been in the dividend and divisor.

For example, 6,421,849 ÷ 4,562,357 = 1.407572+ = 1.4076 to five significant figures; also, 642,185 ÷ 456,236 = 1.407571+ = 1.4076 to five significant figures.

c. If the number whose root is to be extracted be limited to six significant figures, the root will be correct to five (usually to six) significant figures.

22. These principles may all be summed up in the following general statement: *In any series of arithmetical operations —addition, subtraction, multiplication, division, involution, and evolution—if it be desired to have the final result limited to a certain number of significant figures, it is unnecessary to use more significant figures in any of the numbers operated on than the desired number in the result plus 1.* For example, if only four significant figures are desired in the final result, all the

numbers used in the various operations may be limited to $4 + 1 = 5$ significant figures, the fifth figure being increased by 1 in all cases if the sixth figure is 5 or a greater digit.

From the foregoing, it follows that a table that will give five significant figures of the root correctly will be sufficiently extensive for all practical purposes. Such a table is here given, following the Examination Questions, and its use will now be explained.

23. The smallest number that can be written with one figure is 1, and the largest is 9. Their corresponding squares are 1 and 81, respectively. The smallest number that can be written with two figures is 10, and the largest is 99. Their corresponding squares are 100 and 9,801, respectively. Arrange the following numbers and their squares thus:

$$1^2 = 1 \qquad 9^2 = 81$$
$$10^2 = 100 \qquad 99^2 = 9{,}801$$
$$100^2 = 10{,}000 \qquad 999^2 = 998{,}001$$
$$1{,}000^2 = 1{,}000{,}000 \qquad 9{,}999^2 = 99{,}980{,}001$$

It is seen that the square of a number containing one figure is written with one or two figures; the square of a number containing two figures is written with three or four figures. Or, the square of a number is always written with twice as many figures as the given number, or twice as many less one.

24. In order to find the square root of a number, the first step is to point off the number into periods, or groups, of two figures each, beginning at the right if the number is integral, and at the decimal point if the number is decimal. The number of periods will be equal to the number of figures in the root if the number is a perfect square.

If the last period on the right of a decimal number contains but one figure, annex a cipher to complete the period.

Thus, the square root of 83,740,801 must contain four figures, since, pointing off the periods, we get 83′74′08′01, or four periods; consequently, there must be four figures in the root. In like manner, the square root of 50,625 must contain

three figures, since there are (5′06′25) three periods. The extreme left-hand period may contain either one or two figures, according to the size of the number squared.

25. The square of any number wholly decimal always contains twice as many figures as the number squared. For example, $.1^2 = .01$, $.13^2 = .0169$, $.751^2 = .564001$, etc.

The square of a number partly decimal contains twice as many decimal places as there are decimal places in the number. For example, $12.35^2 = 152.5225$.

26. It will also be noticed that the square of a decimal is always less than the decimal. Hence, the square root of a number wholly decimal is greater than the number itself. If it be required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to point off a decimal is to begin at the decimal point, and, going toward the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher to complete the period.

If the decimal point of a number is moved one or more places to the right (or left), the decimal point in the square will be moved twice as many places to the right (or left), thus:

$$3.567^2 = 12'.72'34'89.$$
$$356.7^2 = 12'72'34'.89.$$
$$.3567^2 = .12'72'34'89.$$

It will be observed that these squares differ only in the position of the decimal point, and when divided into periods, the corresponding group in each square contains the same figures.

Later it will be shown that numbers containing like periods have like figures in their roots.

27. There are comparatively few numbers that can be separated into exactly equal factors; these numbers are called **perfect powers,** and the factors are called *rational factors.* Numbers that cannot be separated into exactly equal factors are called **imperfect powers,** and the factors are called

irrational factors. In the numbers from 1 to 1,000, inclusive, there are only 48 perfect powers, not counting 1, and of these only 30 are perfect squares and 9 perfect cubes. These perfect powers are as follows: perfect squares, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961; perfect cubes, 8, 27, 64, 125, 216, 343, 512, 729, 1000; perfect fourth powers, 16, 81, 256, 625; perfect fifth powers, 32, 243; perfect sixth powers, 64, 729; perfect seventh power, 128. Of these numbers it will be noticed, that two of the perfect cubes, 64 and 729, the four perfect fourth powers, and the two perfect sixth powers are duplicated among the squares and cubes; hence, there are only 40 different numbers between 1 and 1000 that are perfect powers.

28. The root of any number that cannot be divided into as many equal factors as there are units in the index of the root contains an interminable decimal. For example, the number 20 lies between 16 ($= 4^2$) and 25 ($= 5^2$); hence, the square root of 20, or $\sqrt{20}$, is greater than 4 and less than 5, and is therefore equal to 4 plus an interminable decimal. In other words, no matter to how many figures the square root of 20 may be calculated, the root will never be found exactly.

29. Although the root of an imperfect power cannot be found exactly, as close an approximation may be obtained as is desired. In practice, five significant figures are all that are likely to be required, and four are generally sufficient. In the following examples, all roots will be calculated to five figures, unless the given number is a perfect power whose root contains less than five figures.

SQUARE ROOT.

30. The first step in finding the square root is to point off the number into periods of two figures each as previously described. The second step is to move the decimal point until it falls between the first (left-hand) period containing a digit and the next period to the right; in other words, the first step

is to make the first period the integral part of the number, if not already so. Call the result the **altered number.**

The second step is to search the table following the Examination Questions, and in the columns headed n^2 and find two consecutive numbers, one less and the other greater than the altered number. Opposite the smaller of the two numbers in the column headed n, will be found the first three figures of the square root. All the numbers in the columns headed n are printed in heavy-face type, and it will be noticed that there are two such columns on each page.

The third step is to find two more figures of the root and the fourth step is to locate the decimal point in the root; these two steps can be best illustrated by examples.

EXAMPLE 1.—What is the square root of 31,416?

SOLUTION.—Pointing off into periods, the result is 3′14′16. Moving the decimal point so that the first period becomes the integral part of the number, the altered number is 3.1416. Searching the table in the columns headed n^2, 3.1416 is found to lie between 3.1329 and 3.1684, on page 1. The number in the column headed n opposite 3.1329 is 1.77. The first three figures of the root are therefore 177. Find the difference between the two numbers between which the given number falls (call this the **first difference**), and the difference between the smaller number and the given number (call this the **second difference**); divide the second difference by the first difference, carrying the quotient to three decimal places and increasing the second figure by 1 if the third is 5 or a greater digit. The two figures of the quotient thus determined will be the fourth and fifth figures of the root. In the present example, dropping decimal points in the remainders, $3.1684 - 3.1329 = 355$, the first difference; $3.1416 - 3.1329 = 87$, the second difference; $87 \div 355 = .245+$, or .25. Hence, the first five figures of the root are 17,725. The decimal point is located in all cases by reference to the original number, after pointing off into periods.

There will be as many figures in the root preceding the decimal point as there are periods preceding the decimal point in the given number; if the number is entirely decimal, the root is entirely decimal, and there will be as many ciphers following the decimal point in the root as there are complete cipher periods following the decimal point in the given number.

Applying this principle, there are three periods preceding the decimal point in the given number; hence, there are three figures preceding the decimal point in the root, and $\sqrt{31,416} = 177.25$. Ans.

The operations may be arranged thus:

$$\begin{array}{rr@{\qquad}rr}
 & 1.78^2 = 3.1684 & \textit{altered number} = & 3.1416 \\
 & 1.77^2 = \underline{3.1329} & 1.77^2 = & \underline{3.1329} \\
\textit{first difference} = & 355 & \textit{second difference} = & 87
\end{array}$$

$$\begin{array}{l}
355\,)\,87.000\,(\,.245, \text{ or } .25 \\
\quad\;\; \underline{710} \\
\quad\;\; 1600 \\
\quad\;\; \underline{1420} \\
\quad\;\;\; 1800 \\
\quad\;\;\; \underline{1775}
\end{array}$$

First five significant figures of the root (the fifth figure being corrected) are 17725.

Locating decimal point, $\sqrt{31,416} = 177.25$. Ans.

NOTE.—Had the given number been 314.16, 3.1416, .031416, .00031416, etc. the significant figures of the root would have been the same as in the preceding case, since the altered number would have been 3.1416 in each instance; the decimal point would have been differently located, however. Thus, pointing off into periods, the given numbers are respectively 3'14'.16, 3'.14'16, .03'14'16, and .00'03'14'16, and the corresponding square roots are 17.725, 1.7725, .17725, and .017725.

Read very carefully Art. 34.

EXAMPLE 2.—What is the square root of .0031416?

SOLUTION.—Pointing off into periods, the result is .00'31'41'60; moving the decimal point, the altered number is 0031.4160 or 31.4160. Referring to the table in the columns headed n^2, 31.4160 is found to lie between 31.3600, opposite 5.60 and 31.4721, opposite 5.61, on page 5. The first three figures of the root are therefore 560. The first difference is 31.4721 − 31.3600 = 1121; the second difference is 31.4160 − 31.3600 = 560; 560 ÷ 1121 = .499+, or .50. Therefore, the first five figures of the root are 56050. Since there is one complete cipher period immediately following the decimal point in the given number, there is one cipher following the decimal point in the root, and $\sqrt{.0031416}$ = .056050, or .05605. Ans.

EXAMPLE 3.—What is the square root of 7,500?

SOLUTION.—Pointing off into periods, the result is 75'00. Moving the decimal point, the altered number is 75.00 or 75. Referring to the table, in the columns headed n^2, 75 is found to lie between 74.9956 and 75.1689, on page 8. The first difference is 75.1689 − 74.9956 = 1733; the second difference is 75 − 74.9956 = 44; 44 ÷ 1733 = .025+, or .03. The first three figures of the square root are 866, and the first five are 86603; there are two figures in the integral part of the root; hence, $\sqrt{7,500}$ = 86.603. Ans.

EXAMPLE 4.—What is the square root of 49,074,561,800?

SOLUTION.—Pointing off into periods, the result is 4'90'74'56'18'00. Moving the decimal point, the altered number is 4.90745618. Referring

to the table in the columns headed n^2, the altered number is found to lie between 4.8841 and 4.9284, on page 2. It is not necessary or advisable to retain more figures in the altered number than there are in the two numbers of the table between which it falls, in this case five figures; hence, throw off all figures after the fifth, increasing the fifth figure by 1 if the sixth is 5 or a greater digit. Doing so the altered number becomes 4.9075. The first difference is $4.9284 - 4.8841 = 443$; the second difference is $4.9075 - 4.8841 = 234$; $234 \div 443 = .528+$, or .53. The number opposite 4.8841 in the column headed n is 2.21; hence, the first five figures of the root are 22153. Since there are six periods on the left of the decimal point in the given number, there are six figures in the integral part of the root; as only five figures were determined, write a cipher for the sixth figure, obtaining 221,530. Therefore, to five significant figures, $\sqrt{49{,}074{,}561{,}800} = 221{,}530$. Ans.

EXAMPLES FOR PRACTICE.

Find the square root of:

(*a*) 5.		(*a*) 2.2361.
(*b*) .005.		(*b*) .070711.
(*c*) 149,263.		(*c*) 386.35.
(*d*) 792.06.	Ans.	(*d*) 28.144.
(*e*) 88.527.		(*e*) 9.4089.
(*f*) 1,000.		(*f*) 31.623.

CUBE ROOT.

31. An examination of the table will show that the columns headed n contain all the numbers between 1.00 and 9.99, inclusive, that is, all numbers that can be expressed by three figures, disregarding the decimal point. The columns headed n^2 contain the squares of all the numbers in the columns headed n. The columns headed n^3 contain the first six figures of the cubes of the numbers in the columns headed n. The preceding explanation for square root will suffice for the cube root, the only difference being in the first step—that of pointing off into periods. For cube root each period (except the first) must contain *three* figures. The process is the same; that is, begin at the decimal point and point off to the left and to the right periods of three figures each. If the right-hand period is not complete, annex ciphers until it contains three figures.

Then proceed exactly as before, using the columns headed n^3, and locate the decimal point by means of the principle given in the explanation to example 1, Art. **30.**

EXAMPLE 1.—The cube root of .0000062417 is what?

SOLUTION.—Pointing off into periods of three figures each, the result is .000'006'241'700. Moving the decimal point until it immediately follows the first period that contains a digit the altered number is 6.241700 or 6.24170, using but six figures, to correspond with the six figures of the table. Referring to the table, and looking in the columns headed n^3, the altered number is found to lie between 6.22950 and 6.33163, on page 1. The first difference is 6.33163 − 6.22950 = 10213; the second difference is 6.24170 − 6.22950 = 1220; 1220 ÷ 10213 = .119+, or .12. The number opposite 6.22950 in the column headed n is 1.84; hence, the first five significant figures of the cube root are 18412. Since there is one complete cipher period following the decimal point, there will be one cipher following the decimal point in the root; therefore, $\sqrt[3]{.0000062417} = .018412$. Ans.

Read very carefully Art. 34.

EXAMPLE 2.—The cube root of 50,932,676 is what?

SOLUTION.—Pointing off into periods of three figures each, the result is 50'932'676. Moving the decimal point, the altered number is 50.932676. Reducing to six figures and increasing the sixth figure by 1, since the seventh figure is 7, the altered number becomes 50.9327. Referring to the table in the columns headed n^3, 50.9327 is found to lie between 50.6530 and 51.0648, on page 3, the first three figures of the root being 370. The first difference is 51.0648 − 50.6530 = 4118; the second difference is 50.9327 − 50.6530 = 2797; 2797 ÷ 4118 = .679+, or .68; hence, the first five figures of the root are 37068. Since the integral part of the given number contains three periods there are three figures in the integral part of the root; therefore, $\sqrt[3]{50,932,676} = 370.68$. Ans.

EXAMPLE 3.—What is the cube root of .834?

SOLUTION.—There is but one period. Moving the decimal point, the altered number is 834, which falls between 833.238 and 835.897 in the columns headed n^3 on page 9 of the table. The first three figures of the root are 941. The first difference is 835.897 − 833.238 = 2659; the second difference is 834 − 833.238 = 762; 762 ÷ 2659 = .286+, or .29; hence, the first five figures of the root are 94129. Since the given number is wholly decimal, the root is wholly decimal; and since there is no complete cipher period between the decimal point and the first digit of the given number, there are no ciphers between the decimal point and the first digit of the root. Therefore, $\sqrt[3]{.834} = .94129$. Ans.

EXAMPLES FOR PRACTICE.

Find the cube root of:

(*a*)	78,347.809639.	Ans. (*a*)	42.79.
(*b*)	2.	(*b*)	1.2599.
(*c*)	4,180,769,192.462.	(*c*)	1,611.0.
(*d*)	.696.	(*d*)	.88621.
(*e*)	.375.	(*e*)	.72112.
(*f*)	513,229.783302144.	(*f*)	80.064.

---

32. The reason for pointing off the given number into periods of three figures each can be explained in the same manner as in the case of square root; viz.:

$1^3 = 1$	$3^3 = 27$	$9^3 = 729$
$10^3 = 1,000$	$30^3 = 27,000$	$99^3 = 970,299$
$100^3 = 1,000,000$	$300^3 = 27,000,000$	$999^3 = 997,002,999$

It will be noticed that the left-hand periods of these powers contain, respectively, for the first column, *one* figure; for the second column, *two* figures; and for the third column, *three* figures.

33. The following is a general rule for using the table to find the square root or cube root of any number.

Rule.—I. *Beginning at the decimal point, and going to the right and to the left, point off the given number whose root is to be found into periods having as many figures in each period as there are units in the index* (see Art. **16**) *of the root*

II. *If the decimal point does not immediately follow the right-hand figure of the first period containing a digit, move it from its position until it does follow the right-hand figure of the first period containing a digit. Call the result the* **altered number.** *If there are more than six figures in the altered number, drop all after the sixth figure, increasing the sixth figure by 1 if the seventh figure is 5 or a greater number. In the case of square root, retain only five figures, when the left-hand period contains but one significant figure*

III. *Refer to the table of powers that follows the Examination Questions, and looking in the columns having at the head "n"*

and an exponent of the same value as the index of the root, find between what two numbers in these columns the altered number falls. Subtract the smaller of these two numbers from the larger, and call the result the **first difference.** *Subtract the smaller of the two numbers from the altered number, and call the result the* **second difference.** *Divide the second difference by the first difference, and find the quotient to three figures, which reduce to two figures, increasing the second figure by one if the third figure is 5 or a greater number. The two figures thus found are the fourth and fifth figures of the root. The first three figures will be found in the column headed "n", opposite the smaller of the two numbers in the table between which the given number falls.*

IV. *Locate the decimal point by means of the principle that there will be as many figures in the root preceding the decimal point as there are periods preceding the decimal point in the given number; if the number is entirely decimal, the root is entirely decimal, and if there are any complete cipher periods immediately following the decimal point, there will be as many ciphers following the decimal point in the root as there are complete cipher periods following the decimal point in the given number.*

34. The student should study Arts. **1** to **34** very thoroughly, particularly the examples. Each example should be carefully considered by itself, as it illustrates some feature not present in the other examples. *The student should do the actual work on a separate sheet of paper performing each operation in the order given in the solution; he will find it advisable to work the Examples for Practice also. Any student who follows these instructions will have no difficulty in understanding the process.*

The student will notice that the reason for the second step, moving the decimal point, is to get the decimal point in the same relative position that it occupies in the corresponding numbers of the table.

ROOTS OF FRACTIONS.

35. If the given number is in the form of a fraction, and it is required to find some root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the required root of the decimal. If, however, the numerator and denominator of the fraction are perfect powers, extract the required root of each separately, and write the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

EXAMPLE 1.—What is the square root of $\frac{9}{64}$?

SOLUTION.— $\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}$. Ans.

EXAMPLE 2.—What is the square root of $\frac{5}{8}$?

SOLUTION.— $\sqrt{\frac{5}{8}} = \sqrt{.625} = .79057$, since $\frac{5}{8} = .625$. Ans.

EXAMPLE 3.—What is the cube root of $\frac{27}{125}$?

SOLUTION.— $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$. Ans.

EXAMPLE 4.—What is the cube root of $\frac{1}{4}$?

SOLUTION.—Since $\frac{1}{4} = .25$, $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{.25} = .62996$. Ans.

36. Rule.—*Extract the required root of the numerator and denominator separately; or, reduce the fraction to a decimal, and extract the root of the decimal.*

EXAMPLES FOR PRACTICE.

(*a*) $\sqrt{\frac{9}{16}} = ?$

(*b*) $\sqrt[3]{\frac{27}{1728}} = ?$

(*c*) $\sqrt[3]{\frac{27}{375}} = ?$

(*d*) $\sqrt[3]{\frac{560}{128}} = ?$

Ans. (*a*) $\frac{3}{4}$. (*b*) $\frac{1}{4}$. (*c*) .41602. (*d*) 1.6355.

---

37. On page 10 of the table are given the squares and cubes of the numbers expressed by the first nine digits. By aid of this little table, the page on which the first three figures of the required root are to be found is instantly located. Thus, after moving the decimal point (if necessary) in the given number find, in the column in which n has an exponent equal to the index of the root sought, between what two

numbers the altered number falls; the number in the left-hand column opposite the smaller of these two numbers is the page of the table sought.

For example, on what pages will the first three figures of the (*a*) square root and (*b*) cube root of .00432176 be found?

Pointing off and moving the decimal point the altered numbers, reduced to six figures, become, respectively, (*a*) 43.2176, (*b*) 4.32176. Referring to page 10 of the table, in the column headed n^2, 43.2176 falls between 36, opposite 6 in the column headed n, and 49 opposite 7 in the column headed n; hence, the first three figures of the square root will be found on page 6. 4.32176 falls between 1 and 8 in the column headed n^3; hence, the first three figures of the cube root will be found on page 1.

38. In the following articles will be described how more than five figures of the root can be found, and exact methods for extracting any root, the index being an integer, to any number of figures. *The student may omit everything from this point to Art.* ***52****, if he so desires.*

39. If a root has been found to five significant figures and it is desired to obtain more figures, perhaps the easiest way is to proceed as follows: Raise the number indicated by the root to the power indicated by an exponent equal to the index of the root; if the result so obtained is less than the given number, add 1 to the right-hand figure of the root and raise the new number to the same power; but if the result so obtained is greater than the given number, subtract 1 from the right-hand figure of the root and raise the new number to same power. The result of these operations is to obtain powers of two consecutive numbers having *five* significant figures each, one of the powers being a little greater and the other a little less than the given number. Then proceeding exactly as previously described, divide the second difference by the first difference and obtain four more figures of the root.

Consider example 1, Art. **30.** The square root of 31,416 to five significant figures is 177.25. $177.25^2 = 31{,}417.5625$,

which is a little greater than 31,416; hence, subtracting 1 from 177.25, $177.24^2 = 31,414.0176$, which is a little less than 31,416. The first difference is $31,417.5625 - 31,414.0176 = 3.5449$. The second difference is $31,416 - 31,414.0176 = 1.9824$; $1.9824 \div 3.5449 = .55922$, or .5592 to four figures. Therefore, $\sqrt{31,416} = 177.245592$ to nine significant figures.

Suppose it has been found that the cube root of 37,267 is 33.402 to five significant figures, and it is desired to obtain more figures. Proceed exactly as before. $33.402^3 = 37,266.397760808$, which is a little less than 37,267; hence, adding 1 to 33.402, $33.403^3 = 37,269.744941827$. The first difference is $37,269.744941827 - 37,266.397760808 = 3.347181019$; the second difference is $37,267 - 37,266.397760808 = .602239192$; $.602239192 \div 3.347181019 = .17992+$, or .1799 to four figures. Therefore, $\sqrt[3]{37,267} = 33.4021799$ to nine significant figures.

40. As before stated, it is customary to use some kind of a table instead of extracting the roots directly, still the square root is so frequently required that it is well to learn how to extract square root directly. There are several good methods, and none are much harder than long division. The method given here for square root and cube root is the simplest and easiest to remember and apply of any.

SQUARE ROOT—EXACT METHOD.

41. The method is best explained by giving several examples with full explanations of each step. To make the work clearer to the student and easier to follow, the figures in the root and the successive numbers resulting from their use are printed in light-face and italic type alternately.

Example 1.—Find the square root of 31,505,769.

Solution.—

```
                                              root
                           3 1'5 0'5 7'6 9 ( 5 6 1 3   Ans.
(a)     5          (b)  2 5
        5               ---
       ---         (c)    6 5 0
(d)  1 0 0                6 3 6
         6                -----
     -----         (e)      1 4 5 7
     1 0 6                  1 1 2 1
         6                  -------
     -----                    3 3 6 6 9
     1 1 2 0                  3 3 6 6 9
           1                  ---------
     -------
     1 1 2 1
           1
     -------
     1 1 2 2 0
             3
     ---------
     1 1 2 2 3
```

Explanation.—First point off into periods of two figures each. Now, find the largest single number whose square is less than or equal to 31, the first period. This is evidently 5, since $6^2 = 36$, which is greater than 31. Write it to the right, as in long division, and also to the left, as shown at (*a*). This is the first figure of the root. Now, multiply the 5 at (*a*) by the 5 in the root, and write the result under the first period, as shown at (*b*). Subtract and obtain 6 as a remainder.

Add the root already found to the 5 at (*a*), getting 10, and annex a cipher to this 10, thus making it 100, as shown at (*d*), which call the **first trial divisor.** Bring down the next period, 50, and annex it to the remainder 6, as shown at (*c*), which call the **first dividend.** Divide the first dividend (*c*) by the first trial divisor (*d*) and obtain 6, which is *probably* the next figure of the root. Write 6 in the root, as shown, and also add it to 100, the trial divisor, making it 106. This is called the **first complete divisor.**

Multiply the first complete divisor, 106, by 6, the second figure in the root, and subtract the result from the first dividend (*c*); the remainder is 14. Add the second figure of the root to the complete divisor, 106, and annex a cipher, thus getting 1120, which call the **second trial divisor.** Annex the next period, to the remainder in the second column

making it 1457, as shown at (*e*), which call the **second dividend.** Dividing 1457 by 1120, we get 1 as the next figure of the root. Adding this last figure of the root to 1120, the result is 1121, the **second complete divisor.** Multiplying the second complete divisor by the third figure of the root and subtracting from the second dividend, 1457, the remainder is 336.

Now, adding the last figure of the root to 1121 and annexing a cipher as before, the result is 11220, the **third trial divisor.** Annexing the next and last period, 69, to the remainder in the second column the result is 33669, the **third dividend.** Dividing 33669 by 11220, the result is 3, the fourth figure of the root. Adding the fourth figure of the root to 11220, the result is 11223, the **third complete divisor.** Multiplying the third complete divisor by the fourth figure of the root, the result is 33669. Subtracting the product from the third dividend, there is no remainder; hence, $\sqrt{31,505,769} = 5,613$.

Read very carefully that part of Art. 34 which is printed in Italics.

EXAMPLE 2.—What is the square root of .000576?

SOLUTION.—

	.0 0′0 5′7 6 (.0 2 4 Ans (root)
2	4
2	1 7 6
4 0	1 7 6
4	
4 4	

EXPLANATION.—Beginning at the decimal point and pointing off the number into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure of the root must be a cipher. The remaining portion of the solution should be perfectly clear from what has preceded.

42. If the number is not a perfect power, the root will consist of an interminable number of decimal places. The result may be carried to any required number of decimal places by annexing periods of two ciphers each to the number.

EXAMPLE 1.—What is the square root of 3? Find the result to five decimal places.

SOLUTION.—

		root
	3.0 0′0 0′0 0′0 0′0 0 (	1.7 3 2 0 5+ Ans.
1	1	
1	2 0 0	
2 0	1 8 9.	
7	1 1 0 0	
2 7	1 0 2 9	
7	7 1 0 0	
3 4 0	6 9 2 4	
3	1 7 6 0 0 0 0	
3 4 3	1 7 3 2 0 2 5	
3	2 7 9 7 5	
3 4 6 0		
2		
3 4 6 2		
2		
3 4 6 4 0 0		
5		
3 4 6 4 0 5		

EXPLANATION.—Annex five periods of two ciphers each to the right of the decimal point. The first figure of the root is found to be 1. To get the second figure, we find that, on dividing 200 by 20, it is 10. This is evidently too large.

Trying 9, we add 9 to 20 and multiply 29 by 9; the result is 261, a result which is considerably larger than 200; hence, 9 is too large. In the same way, it is found that 8 is also too large. Trying 7, 7 times 27 is 189, a result smaller than 200; therefore, 7 is the second figure of the root. The next two figures, 3 and 2, are easily found. The fifth figure in the root is a cipher, since the trial divisor, 34640, is greater than the new dividend, 17600. In a case of this kind we annex another cipher to 34640, thereby making it 346400, and bring down the next period, making the 17600, 1760000. Dividing the fourth dividend, 1760000, by the fourth trial divisor, 346400, the result is 5.0+. Hence, the next figure of the root is 5, and, as we now have five decimal places, we stop.

The square root of 3 is, then, 1.73205+.

If the second figure of the quotient last obtained, 5.0+, had been 5 or a greater digit, the figure in the fifth decimal place would have been increased by 1.

EXAMPLE 2.—What is the square root of .3 to five decimal places?

SOLUTION.—

```
                                          root
                 .3 0'0 0'0 0'0 0'0 0 ( .5 4 7 7 2+   Ans.
     5            2 5
     5            ---
   ---              5 0 0
   1 0 0            4 1 6
       4            -----
   -----              8 4 0 0
   1 0 4              7 6 0 9
       4              -------
   -------              7 9 1 0 0
   1 0 8 0              7 6 6 2 9
         7              ---------
   -------                2 4 7 1 0 0
   1 0 8 7                2 1 9 0 8 4
         7                -----------
   ---------                2 8 0 1 6
   1 0 9 4 0
           7
   ---------
   1 0 9 4 7
           7
   -----------
   1 0 9 5 4 0
             2
   -----------
   1 0 9 5 4 2
```

EXPLANATION.—In the above example we annex a cipher to .3, making the first period .30, since every period of a decimal, as was mentioned before, must have two figures in it. The remainder of the work should be perfectly clear.

43. If it is required to find the square root of a mixed number, begin at the decimal point and point off the periods both to the right and to the left. The manner of finding the root will then be exactly the same as in the previous cases.

EXAMPLE.—What is the square root of 258.2449?

SOLUTION.—

```
                              root
              2'5 8.2 4'4 9 ( 1 6.0 7   Ans.
   1          1
   1          -
   ---        1 5 8
   2 0        1 5 6
     6        -----
   ---            2 2 4 4 9
   2 6            2 2 4 4 9
     6            ---------
   -------
   3 2 0 0
         7
   -------
   3 2 0 7
```

Explanation.—In the above example, since 320 is greater than 224, we place a cipher for the third figure of the root and annex a cipher to 320, making it 3200. Then, bringing down the next period, 49, 7 is found to be the fourth figure of the root. Since there is no remainder, the square root of 258.2449 is 16.07.

44. Proof.—To prove square root, square the result obtained. If the number is an exact power, the square of the root will equal it; if it is not an exact power, the square of the root will very nearly equal it.

45. **Rule.—I.** *Begin at units place and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if there be any.*

II. *Find the greatest number whose square is contained in the first, or left-hand, period. Write this number as the first figure in the root; also, write it at the left of the given number.*

Multiply this number at the left by the first figure of the root, and subtract the result from the first period.

III. *Add the first figure of the root to the number in the first column on the left and annex a cipher to the result; this is the first trial divisor. Annex the second period to the remainder in the second column; this is the first dividend. Divide the dividend by the trial divisor for the second figure in the root and add this figure to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure in the root and subtract this result from the dividend. (If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.) Add the second figure of the root to the complete divisor. Annex a cipher for a new trial divisor, and bring down the third period and annex it to the last remainder for the second dividend.*

IV. *Continue in this manner to the last period, after which, if any additional places in the root are required, bring down cipher periods and continue the operation.*

V. *If at any time the trial divisor is larger than the dividend, place a cipher in the root, annex a cipher to the trial divisor, and bring down another period.*

VI. *If the root contains an interminable decimal and it is desired to terminate the operation at some point, say, the fourth decimal place, carry the operation one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign +.*

46. Short Method.—If the number whose root is to be extracted is not an exact square, the root will be an interminable decimal. It is then usual to extract the root to a certain number of significant figures. In such cases the work may be greatly shortened as follows: Determine to how many significant figures the work is to be carried, say seven, for example; divide this number by 2 and take the next higher number. In the above case, we have $7 \div 2 = 3\frac{1}{2}$; hence, we take 4, the next higher number. Now extract the root in the usual manner until four significant figures have been obtained. Then form the trial divisor in the usual manner, but omitting to annex the cipher; divide the last remainder by the trial divisor, as in long division, obtaining as many figures of the quotient as there are remaining figures of the root, in this case $7 - 4 = 3$. The quotient so obtained is the remaining figures of the root.

Consider example 2, Art. **42.** Here there are five figures in the root. We therefore extract the root to three figures in the usual manner, obtaining .547 for the first three root figures. The next trial divisor is 1094 (with the cipher omitted) and the last remainder is 791. Then, $791 \div 1094 = .723$, and the next two figures of the root are 72, the whole root being .54772+. Always carry the division one place farther than desired, and if the last figure is 5 or a greater digit, increase the preceding figure by 1. This method should not be used unless the root is to contain five or more figures.

NOTE.—If the last figure of the root found in the regular manner is a cipher, carry the process one place farther before dividing as described above.

EXAMPLES FOR PRACTICE.

Find the square root of:

(*a*) 186,624.
(*b*) 2,050,624.
(*c*) 29,855,296.
(*d*) .0116964.
(*e*) 198.1369.
(*f*) 994,009.
(*g*) 2.375 to four decimal places.
(*h*) 1.625 to three decimal places.
(*i*) .3025.
(*j*) .571428.
(*k*) .78125.

Ans. { (*a*) 432.
(*b*) 1,432.
(*c*) 5,464.
(*d*) .10815 –
(*e*) 14.0761.
(*f*) 997.
(*g*) 1.5411.
(*h*) 1.275.
(*i*) .55.
(*j*) .75593 – .
(*k*) .88388 + .

CUBE ROOT—EXACT METHOD.

47. The process of extracting cube root is very similar to that just described for square root, the work being arranged in three columns instead of two. An example will best illustrate the method.

EXAMPLE.—What is the cube root of 375,741,853,696?

SOLUTION.—

(1)	(2)	(3)
		3 7 5′7 4 1′8 5 3′6 9 6 (7 2 1 6 Ans. (root)
7	4 9	3 4 3
7	9 8	
		3 2 7 4 1
1 4	1 4 7 0 0	3 0 2 4 8
7	4 2 4	
		2 4 9 3 8 5 3
2 1 0	1 5 1 2 4	1 5 5 7 3 6 1
2	4 2 8	
		9 3 6 4 9 2 6 9 6
2 1 2	1 5 5 5 2 0 0	9 3 6 4 9 2 6 9 6
2	2 1 6 1	
2 1 4	1 5 5 7 3 6 1	
2	2 1 6 2	
2 1 6 0	1 5 5 9 5 2 3 0 0	
1	1 2 9 8 1 6	
2 1 6 1	1 5 6 0 8 2 1 1 6	
1		
2 1 6 2		
1		
2 1 6 3 0		
6		
2 1 6 3 6		

Explanation.—Write the work in three columns as follows: On the right place the number whose cube root is to be extracted, and point it off into periods of *three* figures each. Call this column (3). Find the largest number whose cube is less than or equal to the first period, in this case 7. Write the 7 on the right, as shown, for the first figure of the root, and also on the extreme left at the head of column (1). Multiply the 7 in column (1) by the first figure of the root 7, and write the product 49 at the head of column (2). Multiply the number in column (2) by the first figure of the root 7, and write the product 343 under the figures in the first period. Subtract, obtaining 32 for the remainder. Add the first figure of the root to the number in column (1), obtaining 14. Multiply the last number in column (1) by the first figure of the root, add the product to the number in column (2), and obtain 147. Add the first figure of the root to the last number in column (1), and obtain 21. Annex *one* cipher to the number in column (1), and obtain 210. Also, annex *two* ciphers to the number in column (2), and obtain 14,700 for the first trial divisor. Bring down the next period, annexing it to the remainder in column (3), and obtain 32,741 for the first dividend. Dividing the first dividend by the first trial divisor, we obtain $\frac{32741}{14700} = 2+$, and write the 2 as the second figure of the root. Add the 2 to the number in column (1), and obtain 212, which, multiplied by the second figure of the root, and added to the trial divisor, gives 15,124, the first complete divisor. This last result, multiplied by the second figure of the root and subtracted from the first dividend, gives a remainder of 2,493. Adding the second figure of the root to the number in column (1), we get 214; this, multiplied by the second figure of the root and added to complete divisor, gives 15,552. Adding the second figure of the root to the number in column (1) gives 216. Annexing one cipher to the number in column (1) gives 2,160. Annexing two ciphers to the number in column (2) gives 1,555,200, the second trial divisor. Annexing the third period to the remainder in

column (3), we obtain 2,493,853 for the second dividend. Dividing the second dividend by the second trial divisor, we obtain $\frac{2493853}{1555200} = 1+$, and write 1 as the third figure of the root. The remainder of the work is continued in the same manner and should be perfectly clear from what has preceded.

48. In extracting the cube root of a decimal, proceed as above, taking care that each period contains *three* figures. Begin the pointing off at the decimal point, going toward the right. If the last period does not contain three figures, annex ciphers until it does.

EXAMPLE 1.—What is the cube root of .009129329?

SOLUTION.—

		root
		.0 0 9'1 2 9'3 2 9 (.2 *0* 9 Ans.
2	4	8
2	8	1 1 2 9 3 2 9
4	1 2 *0 0* 0 0	1 1 2 9 3 2 9
2	5 4 8 1	
6 *0* 0	1 2 5 4 8 1	
9		
6 0 9		

EXPLANATION.—Beginning at the decimal point, and pointing off as shown, the largest number whose cube is less than 9 is seen to be 2; hence, 2 is the first figure of the root. When finding the second figure, it is seen that the first trial divisor 1,200 is greater than the dividend; hence, write a cipher for the second figure of the root; bring down the next period to form the second dividend; annex two ciphers to the trial divisor to form the second trial divisor; also, annex one cipher to the 60 in column (1). Dividing the second dividend by the second trial divisor, we get $\frac{1129329}{120000} = 9+$, and write 9 as the third figure of the root. Complete the work as before.

EXAMPLE 2.—What is the cube root of 78,347.809639?

SOLUTION.—

		root
		7 8'3 4 7.8 0 9'6 3 9 (4 2.7 9
4	1 6	6 4
4	3 2	
		1 4 3 4 7
8	4 8 0 0	1 0 0 8 8
4	2 4 4	
		4 2 5 9 8 0 9
1 2 0	5 0 4 4	3 7 6 6 4 8 3
2	2 4 8	
		4 9 3 3 2 6 6 3 9
1 2 2	5 2 9 2 0 0	4 9 3 3 2 6 6 3 9
2	8 8 6 9	
1 2 4	5 3 8 0 6 9	
2	8 9 1 8	
1 2 6 0	5 4 6 9 8 7 0 0	
7	1 1 5 3 7 1	
1 2 6 7	5 4 8 1 4 0 7 1	
7		
1 2 7 4		
7		
1 2 8 1 0		
9		
1 2 8 1 9		

EXPLANATION.—Since we have a mixed number, begin at the decimal point and point off periods of three figures each, in both directions. The first period contains but two figures, and the largest number whose cube is less than 78 is 4; consequently, 4 is the first figure of the root. The remainder of the work should be perfectly clear. When dividing the second dividend by the second trial divisor for the third figure of the root, the quotient was 8+; but, on trying it, it was found that 8 was too large, the complete divisor being considerably larger than the trial divisor. Therefore, 7 was used instead of 8.

EXAMPLE 3.—What is the cube root of 5 to five decimal places?

root

SOLUTION.— 5.000′000′000′000′000(1.709975+

1	1	1
1	2	4000
2	300	3913
1	259	87000000
30	559	78443829
7	308	8556171000
37	8670000	7889992299
7	45981	666178701000
44	8715981	614014317973
7	46062	52164383027
5100	876204300	
9	461511	
5109	876665811	
9	461592	
5118	877127740300	
9	3590839	
51270	877163311139	
9		
51279		
9		
51288		
9		
512970		
7		
512977		

Root correct to five decimal places is 1.70998−. Ans.

EXPLANATION.—In the preceding example we annex five periods of ciphers, of three ciphers each, to the 5 for the decimal part of the root, placing the decimal point between the 5 and the first cipher. In practice it is not necessary to write these cipher periods after the given number; when one cipher is added to the number in column (1) and two to the number in column (2), three ciphers would be added to the number in column (3). Since the quotient obtained by

dividing the fourth dividend by the fourth trial divisor is 666178701000 ÷ 87712740300 = 7.5+ the root correct to five decimal places is 1.70998−.

EXAMPLE 4.—What is the cube root of .5 to four decimal places?

SOLUTION.—

		root
		.500'000'000'000(.7937+ Ans.
7	49	343
7	98	157000
14	14700	150039
7	1971	6961000
210	16671	5638257
9	2052	1322743000
219	1872300	1321748953
9	7119	994047
228	1879419	
9	7128	
2370	188654700	
3	166579	
2373	188821279	
3		
2376		
3		
23790		
7		
23797		

EXPLANATION.—In the above example we annex two ciphers to the .5 to complete the first period, and three periods of three ciphers each. The largest number whose cube is less than 500 is 7; this we write as the first figure of the root. The remainder of the work should be perfectly plain from the explanations of the preceding examples. Since the quotient obtained by dividing the third dividend by the third trial divisor is 1322743000 ÷ 188654700 = 7.0+, the root is correct as found to the fourth decimal place.

EXAMPLE 5.—What is the cube root of .05 to four decimal places?

SOLUTION.—

		root
		.0 5 0'0 0 0'0 0 0'0 0 0 (.3 6 8 4+
3	9	27
3	18	23000
6	2700	19656
3	576	3344000
90	3276	3180032
6	612	163968000
96	388800	162685504
6	8704	1282496
102	397504	
6	8768	
1080	40627200	
8	44176	
1088	40671376	
8		
1096		
8		
11040		
4		
11044		

49. PROOF.—To prove cube root, cube the result obtained. If the given number is an exact power, the cube of the root will equal it; if not an exact power, the cube of the root will very nearly equal it.

50. **Rule.—I.** *Arrange the work in three columns, placing the number whose cube root is to be extracted in the third or right-hand column. Begin at units place, and separate the number into periods of three figures each, proceeding from the decimal point toward the right with the decimal part, if there is any.*

II. *Find the greatest number whose cube is not greater than the number expressed by the first period that contains a digit. Write this number as the first figure of the root; also, write it at the head of the first column. Multiply the number in the first column by the first figure in the root, and write the result in the second column. Multiply the number in the second column by the first figure of the root, and subtract the product*

from the first period. Add the first figure of the root to the number in the first column. Multiply the last number in the first column by the first figure of the root, and add the product to the number in the second column. Add the first figure of the root to the number in the first column. Annex one cipher to the last number in the first column, two ciphers to the last number in the second column, and annex the second period to the remainder in the third column. The last numbers in the second and third columns are, respectively, the first trial divisor and the first dividend.

III. *Divide the first dividend by the first trial divisor to find the second figure of the root. Add the second figure of the root to the number in the first column, multiply the sum by the second figure of the root, and add the result to the first trial divisor to form the first complete divisor. Multiply the first complete divisor by the second figure of the root, and subtract the result from the first dividend in the third column. Add the second figure of the root to the number in the first column; multiply the sum by the second figure of the root, and add the product to the complete divisor. Add the second figure of the root to the number in the first column. Annex one cipher to the number in the first column, and two ciphers to the last number in the second column to form the second trial divisor. Annex the third period to the remainder in the third column for the second dividend.*

IV. *If there are more periods to be brought down, proceed as before. If there is a remainder after the root of the last period has been found, annex cipher periods, and proceed as before. The figures of the root thus obtained will be decimals.*

V. *If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say the fifth significant figure, carry the operation one place farther, and if the sixth figure is 5 or greater, increase the fifth figure by 1 and omit the sign +.*

51. The method of Art. **46** can be applied to cube root (or any other root) as well as to square root. Thus, in

example 3, Art. **48**, there are $5+1=6$ figures in the root. Extracting the root in the usual manner to $6 \div 2 = 3$, say 4 figures, we get for the first four figures 1,709. The last remainder is 8,556,171, and the next trial divisor, with the ciphers omitted, is 8,762,043. Hence, the next two figures of the root are $8{,}556{,}171 \div 8{,}762{,}043 = .976$, say .98. Therefore, the root is 1.70998.

EXAMPLES FOR PRACTICE.

Find the cube root of:

		Ans.
(*a*)	$\frac{27}{512}$.	(*a*) $\frac{3}{8}$.
(*b*)	2 to five decimal places.	(*b*) 1.25992+.
(*c*)	4,180,769,192.462 to five decimal places.	(*c*) 1,610.96238.
(*d*)	$\frac{87}{125}$.	(*d*) .8862+.
(*e*)	$\frac{3}{8}$.	(*e*) .7211+.
(*f*)	513,229.783302144 to three decimal places.	(*f*) 80.064.

RATIO.

52. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$, or .2, of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

53. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4 and express this comparison as a ratio, it may be done as follows: $20 \div 4$; $20 : 4$, or $\frac{20}{4}$.

All three are read *the ratio of 20 to 4.* The ratio of 4 to 20 would be expressed thus: $4 \div 20$; $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one oftenest met with, while the third form, called the fractional form, possesses great advantages to students of algebra and of higher mathematical subjects. The second form is better adapted to arithmetical subjects and is one we shall ordinarily adopt.

54. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together, they are called a **couplet;** when considered separately, the first term is called the **antecedent** and the second term the **consequent.** Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent and 4 the consequent.

55. A ratio may be **direct** or **inverse.** The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal ratio.** The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i. e., the reciprocal of a fraction is the fraction inverted. Hence, the inverse ratio of 20 to 4 may be expressed as 4 : 20, or as $\frac{1}{20} : \frac{1}{4}$. Both have equal values; for, $4 \div 20 = \frac{1}{5}$, and $\frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$.

56. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that the ratio between the first two numbers is the same as the ratio between the other two numbers.

57. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20 : 4 is 5—it is the quotient obtained by dividing the antecedent by the consequent. The value of a ratio is always an abstract number, regardless of whether the terms are abstract or concrete numbers.

58. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or if expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then, transposing the terms, as 4 : 20; or as $\frac{20}{4}$, and then inverting, as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

59. Instead of expressing the value of a ratio by a single number, as above, it is convenient to express it by means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30,* the consequent, we obtain $\frac{1\frac{1}{2}}{1}$, or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

*This evidently does not alter the value of the ratio, since by the laws of fractions, both numerator and denominator may be divided by the same number without changing the value of the fraction.

PROPORTION.

60. **Proportion** is an equality of ratios, the equality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8 : 4 and 6 : 3, which both have the same value, 2, we may employ one of the three following forms:

$$8 : 4 :: 6 : 3 \qquad (1)$$
$$8 : 4 = 6 : 3 \qquad (2)$$
$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

61. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and in time will probably entirely supersede the first form. In this arithmetic we shall adopt the second form, unless some statement can be made clearer by using the third form.

62. A proportion may be *read* in two ways. The old way to read the above proportion was: *8 is to 4 as 6 is to 3;* the new way is: *the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

63. Each ratio of a proportion is termed a **couplet.** In the above proportion, 8 : 4 is a couplet, and so is 6 : 3.

64. The numbers forming the proportion are called **terms**; and they are numbered consecutively from left to right, thus:

first second third fourth
8 : 4 = 6 : 3

Hence, in any proportion, the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

65. The first and fourth terms of a proportion are called the **extremes,** and the second and third terms the **means.** Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

66. A **direct proportion** is one in which both couplets are direct ratios.

67. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6 must be written $8 : 4 = 6 : 3$; i. e., the second ratio (couplet) must be inverted.

68. Proportion forms one of the most useful sections of arithmetic. In our grandfathers' arithmetics, it was called "The rule of three."

69. **Rule.**—*In any proportion, the product of the extremes equals the product of the means.*

Thus, in the proportion,

$$17 : 51 = 14 : 42$$

$17 \times 42 = 51 \times 14$, since both products equal 714.

70. **Rule.**—*The product of the extremes divided by either mean gives the other mean.*

EXAMPLE.—What is the third term of the proportion $17 : 51 = \quad : 42$?

SOLUTION.—Applying the rule, $17 \times 42 = 714$, and $714 \div 51 = 14$. Ans.

71. **Rule.**—*The product of the means divided by either extreme gives the other extreme.*

EXAMPLE.—What is the first term of the proportion $\quad : 51 = 14 : 42$?

SOLUTION.—Applying the rule, $51 \times 14 = 714$, and $714 \div 42 = 17$. Ans.

72. When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, the last example would be written

$$x : 51 = 14 : 42,$$

and for the value of x we have $x = \frac{51 \times 14}{42} = 17$.

73. The principle of all calculations in proportion is this: *Three of the terms are always given and the remaining one is to be found.*

74. Example.—If 4 men can earn \$25 in one week, how much can 12 men earn in the same time?

Solution.—The required term must bear the same relation to the given term of the same kind as one of the remaining terms bears to the other remaining term. We can then form a proportion by which the required term may be found.

The first question the student must ask himself in every calculation by proportion is: "What is it I want to find?" In this case it is dollars. We have two sets of men, one set earning \$25, and we want to know how many dollars the other set earns. It is evident that the *amount* 12 men earn bears the same relation to the *amount* that 4 men earn as 12 men bears to 4 men. Hence, we have the proportion, the amount 12 men earn is to \$25 as 12 men is to 4 men; or, since either extreme equals the product of the means divided by the other extreme, we have

$$\text{The amount 12 men earn} : \$25 = 12 \text{ men} : 4 \text{ men},$$

or the amount 12 men earn $= \frac{\$25 \times 12}{4} = \$75$. Ans.

Since it matters not which place x, or the required term, occupies, the problem could be stated in any of the following forms, the value of x being the same in each:

(*a*) \$25 : the amount 12 men earn = 4 men : 12 men; or the amount 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

(*b*) 4 men : 12 men = \$25 : the amount 12 men earn; or the amount that 12 earn $= \frac{\$25 \times 12}{4}$, or \$75, since either extreme equals the product of the means divided by the other extreme.

(*c*) 12 men : 4 men = the amount 12 men earn : \$25; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

75. If the proportion is an inverse one, first form it as though it were a direct proportion and then invert one of the couplets.

EXAMPLES FOR PRACTICE.

76. Find the value of x in each of the following:

$$
\begin{array}{ll}
(a) & \$16 : \$64 = x : \$4. \\
(b) & x : 85 = 10 : 17. \\
(c) & 24 : x = 15 : 40. \\
(d) & 18 : 94 = 2 : x. \\
(e) & \$75 : \$100 = x : 100. \\
(f) & 15 \text{ pwt.} : x = 21 : 10. \\
(g) & x : 75 \text{ yd.} = \$15 : \$5.
\end{array}
\qquad \text{Ans.} \left\{
\begin{array}{ll}
(a) & x = \$1. \\
(b) & x = 50. \\
(c) & x = 64. \\
(d) & x = 10\tfrac{4}{9}. \\
(e) & x = 75. \\
(f) & x = 7\tfrac{1}{7} \text{ pwt.} \\
(g) & x = 225 \text{ yd.}
\end{array}
\right.
$$

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what will be the circumference of a circle 31 inches in diameter? Ans. $97\frac{3}{7}$ in.

INVERSE PROPORTION.

77. In Art. **67**, an inverse proportion was defined as one which required one of the couplets to be expressed as an inverse ratio. Sometimes the word *inverse* occurs in the statement of the example; in such cases the proportion can be written directly, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, *if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term.* Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example and

ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct; otherwise, it is inverse, and one of the couplets must be inverted.

78. Example.—A's *rate* of doing work is to B's as 5 : 7; if A does a piece of work in 42 days, in what time will B do it?

Solution.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence, it will take B a *less* number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated

$$5 : 7 = x : 42,$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires, as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5} = 58.8$ days.

EXAMPLES FOR PRACTICE.

79. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. $6\frac{2}{3}$ hr.

2. The circular seam of a boiler requires 50 rivets when the pitch is $2\frac{1}{2}$ in.; how many would be required if the pitch were $3\frac{1}{8}$ in.? Ans. 40.

3. The spring hangers on a certain locomotive are $2\frac{1}{2}$ in. wide and $\frac{3}{4}$ in. thick; those on another engine are of same sectional area, but are 3 in. wide; how thick are they? Ans. $\frac{5}{8}$ in.

4. A locomotive with driving wheels 16 ft. in circumference runs a certain distance in 5,000 revolutions; how many revolutions would it make in going the same distance if the wheels were 22 ft. in circumference (no allowance for slip being made in either case)? Ans. $3,636\frac{4}{11}$ rev.

UNIT METHOD.

80. In the older books on arithmetic, a large number of problems were solved by proportion; but these problems can be solved much more easily by the **unit method**, which we now proceed to explain by means of examples.

EXAMPLE 1.—If a pump discharging 4 gallons of water per minute can fill a tank in 20 hours, how long will it take a pump discharging 12 gallons per minute to fill the tank?

SOLUTION.—A pump discharging 4 gallons per minute fills the tank in 20 hours. Therefore, a pump discharging 1 gallon per minute fills it in 4×20 hours. Hence, a pump discharging 12 gallons per minute fills it in $\frac{4 \times 20 \text{ hours}}{12} = \frac{20 \text{ hours}}{3} = 6\frac{2}{3}$ hr. Ans.

EXAMPLE 2.—If 4 men earn \$65.80 in 7 days, how much can 14 men, paid at the same rate, earn in 12 days?

SOLUTION.— 4 men in 7 days earn \$65.80.

Therefore, 1 man in 7 days earns $\frac{\$65.80}{4}$.

Therefore, 1 man in 1 day earns $\frac{\$65.80}{4 \times 7}$.

Therefore, 1 man in 12 days earns $\frac{\$65.80 \times 12}{4 \times 7}$.

Therefore, 14 men in 12 days earn $\frac{\$65.80 \times 12 \times 14}{4 \times 7}$.

Canceling,

14 men in 12 days earn $\$65.80 \times 3 \times 2 = \$65.80 \times 6 = \$394.80$. Ans.

81. The student will notice that in the solution of these examples, in the successive steps, the operations of multiplication and division were merely indicated, and no multiplication or division was performed until the very last, and then the answer was obtained easily by cancelation. In arithmetical calculations, the student should make it an invariable habit to indicate the multiplications and divisions that occur in the successive steps of a solution, and not to perform these operations until the very last. Then, he will probably be able to use the principle of cancelation.

EXAMPLE.—If a block of granite 8 feet long, 5 feet wide, and 3 feet thick weighs 7,200 pounds, what is the weight of a block of granite 12 feet long, 8 feet wide, and 5 feet thick?

SOLUTION.—If a block 8 feet long, 5 feet wide, and three feet thick weighs 7,200 pounds, a block 1 foot long, 5 feet wide, and 3 feet thick weighs $\frac{7,200}{8}$ pounds; a block 1 foot long, 1 foot wide, and 3 feet thick weighs $\frac{7,200}{8 \times 5}$; and a block 1 foot long, 1 foot wide, and 1 foot thick weighs $\frac{7,200}{8 \times 5 \times 3}$ pounds. Therefore, by the same reasoning, a block 12 feet long, 8 feet wide, and 5 feet thick weighs

$$\frac{7,200 \times 12 \times 8 \times 5}{8 \times 5 \times 3} \text{ pounds} = \frac{7,200 \times \overset{4}{\cancel{12}} \times \cancel{8} \times \cancel{5}}{\cancel{8} \times \cancel{5} \times \cancel{3}} = 28,800 \text{ lb. Ans.}$$

EXAMPLES FOR PRACTICE.

1. If a pump discharges 90,000 gallons of water in 20 hours, in what time will it discharge 144,000 gallons? Ans. 32 hr.

2. When the barometer stands at 30 inches, the pressure of the atmosphere is 14.7 pounds per square inch. What is the atmospheric pressure per square inch when the barometer stands at 29.5 inches? Give answer correct to three figures. Ans. 14.5.

MENSURATION AND USE OF LETTERS IN FORMULAS.

FORMULAS.

1. The term **formula,** as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule. Any formula can be expressed in words, and when so expressed it becomes a rule.

Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note book, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred.

As the term "quantity" is a very convenient one to use, we will define it. In mathematics, the word **quantity** is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say "two or more numbers," or "two or more quantities"; the word quantity is more general in its meaning than the word number.

2. The signs used in formulas are the ordinary signs indicative of operations and the signs of aggregation. All

these signs are explained in arithmetic, but some of them will here be explained in order to refresh the student's memory.

3. The signs indicative of operations are six in number, viz.: $+$, $-$, $\times$, $\div$, $|$, $\sqrt{}$.

Division is indicated by the sign $\div$, or by placing a straight line between the two quantities. Thus, $25 \mid 17$, $25 / 17$, and $\frac{25}{17}$ all indicate that 25 is to be divided by 17. When both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\sqrt{}$) indicates that some root of the quantity on the right is to be taken; it is called the **radical sign.** To indicate what root is to be taken, a small figure, called the **index,** is placed within the sign, this being always omitted when the square root is to be indicated. Thus, $\sqrt{25}$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken; etc.

4. The signs of aggregation are four in number; viz., $\overline{\quad}$, ( ), [ ], and { }, respectively called the **vinculum,** the **parenthesis,** the **brackets,** and the **brace;** they are used when it is desired to indicate that all the quantities included by them are to be subjected to the same operation. Thus, if we desire to indicate that the sum of 5 and 8 is to be multiplied by 7, and we do not wish to actually add 5 and 8 before indicating the multiplication, we may employ any one of the four signs of aggregation as here shown: $\overline{5+8} \times 7$, $(5+8) \times 7$, $[5+8] \times 7$, $\{5+8\} \times 7$. The vinculum is placed above those quantities which are to be treated as one quantity and subjected to the same operations.

5. While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{}(5+8)$, $\sqrt[3]{}[5+8]$, or $\sqrt[3]{}\{5+8\}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where but one sign of aggregation is needed (except, of course, when a root is to be indicated), the parenthesis is always used. Hence, $(5 + 8) \times 7$ would be the usual way of expressing the product of 5 plus 8, and 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20 - 5) \div 3] \times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20 - 5) \div 3] \times 9 - 21\} \div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9, and that after 21 has been subtracted from the product thus obtained, the result is to be divided by 8.

Should it be necessary to use all four of the signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(\overline{20 - 5} \div 3) \times 9 - 21] \div 8\} \times 12$.

6. As stated in *Arithmetic*, when several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication must always be performed first. Thus, $2 + 3 \times 4$ equals 14, 3 being multiplied by 4 before adding to 2. Similarly, $10 \div 2 \times 5$ equals 1, since 2×5 equals 10, and $10 \div 10$ equals 1. Hence, in the above case, if the brace were omitted, the result would be $\frac{1}{4}$, whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in order of importance. For example, $5 - 9 \div 3$ equals 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

7. There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be performed; it is (=), and is called the sign of **equality**; it means that all on one side of it is exactly equal to all on the other side. For example, $2 = 2$, $5 - 3 = 2$, $5 \times (14 - 9) = 25$.

8. Having called particular attention to certain signs used in formulas, the formulas themselves will now be explained. First, consider the well-known rule for finding the horsepower of a steam engine, which may be stated as follows:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (viz., the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \begin{matrix}\text{mean effective pressure}\\ \text{(in pounds per square inch)}\end{matrix} \times \begin{matrix}\text{stroke}\\ \text{(in feet)}\end{matrix}$$

$$\times \begin{matrix}\text{area of piston}\\ \text{(in square inches)}\end{matrix} \times \begin{matrix}\text{number of strokes}\\ \text{(per minute)}\end{matrix} \div 33{,}000.$$

This expression could be shortened by representing each quantity by a single letter; thus, representing horsepower by the letter "H," the mean effective pressure in pounds per square inch by "P," the length of stroke in feet by "L," the area of the piston in square inches by "A," the number of strokes per minute by "N," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H = \frac{P \times L \times A \times N}{33{,}000},$$

a much simpler and shorter expression. This last expression is called a *formula.*

9. The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that, when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{PLAN}{33,000}.$$

10. The sign of multiplication, evidently, cannot be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers which are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus, $2 \cdot 3$ means the same as 2×3; $542 \cdot 749 \cdot 1,006$ indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a \times \sqrt{b+c}$, $3 \times (b+c)$, $(b+c) \times a$, etc., writing them $a\sqrt{b+c}$, $3(b+c)$, $(b+c)a$, etc. The sign is not omitted when several quantities are included by a vinculum and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3 \times \overline{b+c}$, $\overline{b+c} \times a$, $\sqrt{b+c} \times a$, etc. are always written as here printed.

11. Before proceeding further, we will explain one other device that is used by formula makers and which is apt to puzzle one who encounters it for the first time—it is the use of what mathematicians call *primes* and *subs.*, and

what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial letters of the names of the quantities. For example, they represent volume by v, pressure by p, height by h, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity which it represents. Some authors carry the practice a little further and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned when two quantities have the same name but represent different things. Thus, consider the word *pressure* as applied to steam at different stages between the boiler and the condenser. First, there is *absolute* pressure, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by p, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter and the gauge pressure by a small (lower-case) letter. Doing so, P represents absolute pressure and p, gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways and still retain the letter p or P to represent the word pressure: First, by the use of the prime mark; thus, p' or P' (read *p prime* and *P major prime*) may be considered to represent the initial gauge pressure or the initial absolute pressure. Second, by the use of sub. figures; thus, p_1 or P_1 (read *p sub. one* and *P major sub. one*). Third, by the use of sub. letters; thus, p_i or P_i (read *p sub. i* and *P major sub. i*). In the same manner p'' (read *p second*), p_2, or p_r might be used to represent the gauge pressure at release, etc. The sub. letters have the advantage of still further identifying the quantity represented; in many instances,

however, it is not convenient to use them, in which case primes and subs. are used instead. The prime notation may be continued as follows: p''', p^{iv}, p^{v}, etc.; it is inadvisable to use superior figures, for example, p^1, p^2, p^3, p^a, etc., as they are liable to be mistaken for exponents.

12. The main thing to be remembered by the student is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1, w_2, and w_3 represent the weights of three different bodies; s_1, s_2, and s_3, their specific heats; and t_1, t_2, and t_3, their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that those letters having the *same* subs. refer to the same bodies. Thus, w_1, s_1, and t_1 all refer to one of the three bodies; w_2, s_2, t_2, to another body, etc.

It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters and then perform the indicated operations. Thus, suppose that the values of w_1, s_1, and t_1 are, respectively, 2 pounds, .0951, and 80°; of w_2, s_2, and t_2, 7.8 pounds, 1, and 80°; and of w_3, s_3, and t_3, $3\frac{1}{4}$ pounds, .1138, and 780°; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 1 \times 80 + 3\frac{1}{4} \times .1138 \times 780}{2 \times .0951 + 7.8 \times 1 + 3\frac{1}{4} \times .1138}$$

$$= \frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97°.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

13. The student should now be able to apply any formula involving only algebraic expressions that he may meet with, and which does not require the use of logarithms for its solution. We will, however, call his attention to one or two other facts that he may have forgotten.

Expressions similar to $\dfrac{160}{\frac{660}{25}}$ sometimes occur, the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light, it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$, or whether $\frac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\dfrac{\frac{160}{660}}{25}$. In every case the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\dfrac{160}{7+\frac{660}{25}}$, the heavy line is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7+\frac{660}{25}=\frac{175+660}{25}$, if we substitute $\frac{175+660}{25}$ for $7+\frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7+\frac{660}{25}}=\frac{160}{\frac{175+660}{25}}=\frac{160}{\frac{835}{25}}.$$

14. Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc. of some quantity, as 37, by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the

denominator is *always* the index of the root. Hence, instead of writing $\sqrt[3]{37^2}$ for the cube root of the square of 37, it may be written $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2} = 37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1 + a^2 b)^3}$ may also be written $(1 + a^2 b)^{\frac{3}{5}}$, a much simpler expression.

15. We will now give several examples showing how to apply some of the more difficult formulas that the student may encounter.

1. The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h),$$

in which A = area of segment;
π = 3.1416;
r = radius;
E = angle obtained by drawing lines from the center to the extremities of arc of segment;
c = chord of segment;
h = height of segment.

Example.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is 83.46°, radius is 7.5 inches, and height of segment is 1.91 inches?

Solution.—Applying the formula just given,

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h) = \frac{3.1416 \times 7.5^2 \times 83.46}{360} - \frac{10}{2}(7.5 - 1.91)$$
$$= 40.968 - 27.95 = 13.018 \text{ sq. in., nearly. Ans.}$$

2. The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}.$$

Example.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long?

Solution.—In order to apply the formula, suppose we let a represent the side that is 21 feet long; b, the side that is 50 feet long; and c, the side that is 46 feet long. Then, substituting in the formula

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2\,b}\right)^2} = \frac{50}{2}\sqrt{21^2 - \left(\frac{21^2 + 50^2 - 46^2}{2 \times 50}\right)^2}$$

$$= \frac{50}{2}\sqrt{441 - \left(\frac{441 + 2{,}500 - 2{,}116}{100}\right)^2} = 25\sqrt{441 - \left(\frac{825}{100}\right)^2}$$

$$= 25\sqrt{441 - 8.25^2} = 25\sqrt{441 - 68.0625} = 25\sqrt{372.9375}$$

$$= 25 \times 19.312 = 482.8 \text{ sq. ft., nearly. Ans.}$$

The operations in the above examples have been extended much farther than was necessary; it was done in order to show the student every step of the process. The last formula is perfectly general, and the same answer would have been obtained had the 50-foot side been represented by a, the 46-foot side by b, and the 21-foot side by c.

3. The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P = \frac{S\,A}{1 + q\,\dfrac{l^2}{G^2}},$$

in which $P =$ load (pressure) in pounds;

$S =$ ultimate strength (in pounds per square inch) of the material composing the column;

$A =$ area of cross-section of column in square inches;

$q =$ a factor (multiplier) whose value depends upon the shape of the ends of the column and on the material composing the column;

$l =$ length of column in inches;

$G =$ least radius of gyration of cross-section of column.

The values of S, q, and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load that will break a hollow steel column whose outside diameter is 14 inches; inside diameter, 11 inches; length, 20 feet, and whose ends are flat?

SOLUTION.—For steel, $S = 150{,}000$, and $q = \frac{1}{25{,}000}$ for flat-ended steel columns; A, the area of the cross-section, $= .7854\,(d_1^2 - d_2^2) = .7854\,(14^2 - 11^2)$, d_1 and d_2 being the outside and inside diameters,

respectively; $l = 20 \times 12 = 240$ inches; and $G^2 = \frac{d_1^2 + d_2^2}{16} = \frac{14^2 + 11^2}{16}$. Substituting these values in the formula

$$P = \frac{S A}{1 + q \frac{l^2}{G^2}} = \frac{150{,}000 \times .7854\,(14^2 - 11^2)}{1 + \frac{1}{25{,}000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}}$$

$$= \frac{150{,}000 \times 58.905}{1 + .1163} = \frac{8{,}835{,}750}{1.1163} = 7{,}915{,}211 \text{ lb.} \quad \text{Ans.}$$

4. EXAMPLE.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following?

(*a*) $E = \sqrt[3]{\frac{B\,C\,D}{A\left(2 + \frac{D^2}{C^2}\right)}}$; (*b*) $E = \frac{A - \frac{3}{4} D + \frac{4\,B^2}{A + C}}{A - \sqrt{\frac{2\,B^2}{A + 22}}}$.

SOLUTION.—(*a*) Substituting,

$$E = \sqrt[3]{\frac{8 \times 5 \times 4}{10\left(2 + \frac{4^2}{5^2}\right)}}.$$

To simplify the denominator, square the 4 and 5, add the resulting fraction to 2, and multiply by 10. Simplifying, we have

$$E = \sqrt[3]{\frac{160}{10\left(2 + \frac{16}{25}\right)}} = \sqrt[3]{\frac{160}{10 \times \frac{66}{25}}} = \sqrt[3]{\frac{160}{\frac{660}{25}}} = \sqrt[3]{\frac{200}{33}}.$$

Reducing the fraction to a decimal before extracting the cube root,

$$E = \sqrt[3]{6.0606} = 1.823. \quad \text{Ans.}$$

(*b*) Substituting,

$$E = \frac{10 - \frac{3}{4} \times 4 + \frac{4 \times 8^2}{10 + 5}}{10 - \sqrt{\frac{2 \times 8^2}{10 + 22}}} = \frac{10 - 3 + \frac{4 \times 64}{15}}{10 - \sqrt{\frac{2 \times 64}{32}}}$$

$$= \frac{7 + 17.066+}{10 - \sqrt{4}} = \frac{24.066+}{8} = 3.008+. \quad \text{Ans.}$$

16. In the preceding pages, the unknown quantity has always been represented by the single letter at the left of the sign of equality, while the letters at the right have represented known values from which the required values could be found. It is possible, however, to find the value of the quantity represented by any letter in a formula, if the values represented by all the others are known. For example, let it be required to find how many strokes per minute an

engine having a piston area of 78.54 square inches must make in order to develop 60 horsepower, if the mean effective pressure is 40 pounds per square inch and the length of stroke is $1\frac{1}{4}$ ft. By substituting the given values in the formula $H = \frac{P\,L\,A\,N}{33{,}000}$, we have

$$60 = \frac{40 \times 1\frac{1}{4} \times 78.54 \times N}{33{,}000},$$

in which N, the number of strokes, is to be found.

But it is evident that the expression on the right of the sign of equality is equal to $\frac{40 \times 1\frac{1}{4} \times 78.54}{33{,}000} \times N$, a fraction whose numerator is composed of three factors. Reducing the numerator to a single number by performing the indicated multiplications, we obtain, after canceling,

$$60 = \frac{119}{1{,}000} \times N = .119\,N.$$

If 60 equals $.119\,N$, then N equals 60 divided by .119; hence,

$$N = \frac{60}{.119} = 504.2 \text{ strokes per minute.}$$

The method of procedure is essentially the same when the unknown quantity occurs in the denominator of a formula. Thus, in the formula $f = \frac{m\,v^2}{r}$, suppose that $f = 375$, $m = 1.25$, and $v = 60$. Then, substituting,

$$375 = \frac{1.25 \times 60^2}{r} = \frac{4{,}500}{r}.$$

But, if 375 equals 4,500 divided by r, then $375 \times r = 4{,}500$; hence, r must equal 4,500 divided by 375, or $r = \frac{4{,}500}{375} = 12$.

EXAMPLES FOR PRACTICE.

Find the numerical values of x in the following formulas, when $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

1. $x = \frac{d + c^2}{d^2 - 40}$. Ans. $x = \frac{7}{30}$.

2. $x = \frac{\frac{2}{3}(A + e)}{c\,e}$. Ans. $x = 1\frac{1}{3}$.

3. $x = \sqrt{\frac{d^2}{2c}} + \sqrt{A B^2}$. Ans. $x = 29$.

4. $x = \frac{A e}{\sqrt{16 B c}} + \frac{5}{16}$. Ans. $x = 2$.

5. $x = (c + 2e)\left(\sqrt[3]{B} - \frac{1}{c}\right) + \frac{e^2 - c^2}{e^2 + c^2}$. Ans. $x = 12\frac{5}{18}$.

6. $x = \sqrt{\frac{B c d}{A\left(2 + \frac{d^3}{e^2}\right)}}$. Ans. $x = .396+$.

MENSURATION.

17. Mensuration treats of the measurement of lines, angles, surfaces, and solids.

LINES AND ANGLES.

18. A **straight line** is one that does not change its direction throughout its whole length. To distinguish one straight line from another, two of its points are designated by letters. The line shown in Fig. 1 would be called the line $A B$.

Fig. 1.

19. A **curved line** changes its direction at every point. Curved lines are designated by three or more letters, as the curved line $A B C$, Fig. 2.

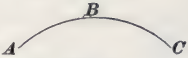


Fig. 2.

20. Parallel lines (Fig. 3) are those which are equally distant from each other at all points.


Fig. 3.

21. A line is **perpendicular** to another (see Fig. 4) when it meets that line so as not to incline towards it on either side.


Fig. 4.

22. A **vertical line** is one that points towards the center of the earth, and is also known as a *plumb* line.

23. A **horizontal line** (see Fig. 5) is one that makes a right angle with any vertical line.

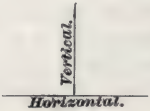


Fig. 5.

24. An **angle** is the opening between two lines which intersect or meet; the point of meeting is called the **vertex** of the angle. Angles are distinguished by naming the vertex and a point on each line. Thus, in Fig. 6, the angle formed by the lines *A B* and *C B* is called the angle *A B C*, or the angle *C B A*; the letter at the vertex is always placed in the middle. When an angle stands alone so that it cannot be mistaken for any other angle, only the vertex letter need be used. Thus, the angle referred to might be designated simply as the angle *B*.

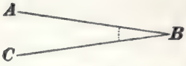


FIG. 6.

25. If one straight line meets another straight line at a point between its ends, as in Fig. 7, two angles, *A B C* and *A B D*, are formed, which are called **adjacent angles.**

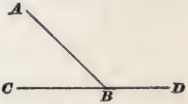


FIG. 7.

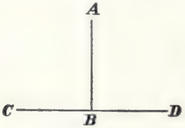


FIG. 8.

26. When these adjacent angles, *A B C* and *A B D*, are equal, as in Fig. 8, they are called **right angles.**

27. An **acute angle** is less than a right angle. *A B C*, Fig. 9, is an acute angle.

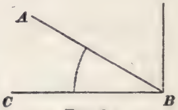


FIG 9.

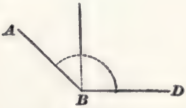


FIG. 10.

28. An **obtuse angle** is greater than a right angle. *A B D* (Fig. 10) is an obtuse angle.

29. A **circle** (see Fig. 11) is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

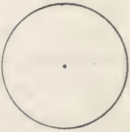


FIG. 11.

30. An **arc** of a circle is any part of its circumference; thus *a e b*, Fig. 12, is an arc of the circle.

31. The circumference of every circle is considered to be divided into 360 equal parts, or arcs, called **degrees;** every degree is subdivided into 60 equal parts, called **minutes,** and every minute is again divided into 60 equal parts, called **seconds.**

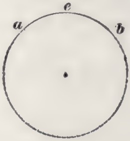


FIG. 12.

Since 1 degree is $\frac{1}{360}$ of any circumference, it follows that the length of a degree will be different in circles of different sizes, but the proportion of the length of an arc of 1 degree to the whole circumference will always be the same, viz., $\frac{1}{360}$ of the circumference.

Degrees, minutes, and seconds are denoted by the symbols °, ′, ″. Thus, the expression 37° 14′ 44″ is read 37 degrees 14 minutes 44 seconds.

32. The arcs of circles are used to measure angles. An angle having its vertex at the center of a circle is measured by the arc included between its sides; thus, in Fig. 13, the arc *F B* measures the angle *F O B*. If the arc *F B* contains 20°, or $\frac{20}{360}$ of the circumference, the angle *F O B* would be an angle of 20°; if it contained 20° 14′ 18″, it would be an angle of 20° 14′ 18″, etc.

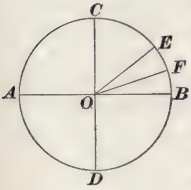


FIG. 13.

In the figure, if the line *C D* be drawn perpendicular to *A B*, the adjacent angles will be equal, and the circle will be divided into four equal angles, each of which will be a right angle. A right angle, therefore, is an angle of $\frac{360°}{4}$, or 90°; two right angles are measured by 180°, or half the circumference, and four right angles by the whole circumference, or 360°. One-half of a right angle, as *E O B*, is an angle of 45°. An *acute* angle may now be defined as an angle of *less* than 90°, and an *obtuse* angle as one of *more*

than 90°. These values are important and should be remembered.

33. From the foregoing it will be evident that if a number of straight lines on the same side of a given straight line meet at the same point, the sum of all the angles formed is equal to two right angles, or 180°. Thus, in Fig. 14, angles $C\,O\,B + D\,O\,C + E\,O\,D + F\,O\,E + A\,O\,F = 2$ right angles, or 180°.

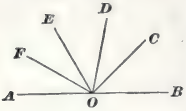


FIG. 14.

34. Also, if through a given point any number of straight lines be drawn, the sum of all the angles formed about the points of intersection equals four right angles, or 360°. Thus, in Fig. 15, angles $H\,O\,F + F\,O\,C + C\,O\,A + A\,O\,G + G\,O\,E + E\,O\,D + D\,O\,B + B\,O\,H = 4$ right angles, or 360°.

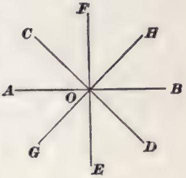


FIG. 15.

EXAMPLE.—In a flywheel with 12 arms, how many degrees are there in the angle included between the center lines of any two arms, the arms being spaced equally ?

SOLUTION.—Since there are 12 arms, there are 12 angles, which together equal 360°. Hence, one angle equals $\frac{1}{12}$ of 360°, or $\frac{360°}{12} = 30°$. Ans.

EXAMPLES FOR PRACTICE.

1. How many seconds are in 32° 14′ 6″ ? Ans. 116,046 sec.

2. How many degrees, minutes, and seconds do 38,582 seconds amount to ? Ans. 10° 43′ 2″.

3. How many right angles are there in an angle of 170° ? Ans. $1\frac{8}{9}$ right angles.

4. In a pulley with five arms, what part of a right angle is included between the center lines of any two arms ? Ans. $\frac{4}{5}$ of a right angle.

5. If one straight line meets another so as to form an angle of 20° 10′, what does its adjacent angle equal ? Ans. 159° 50′.

6. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle ? Ans. 30°.

QUADRILATERALS.

35. A **plane figure** is any part of a plane or flat surface bounded by straight or curved lines.

36. A **quadrilateral** is a plane figure bounded by four straight lines.

37. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **square,** the **rectangle,** the **rhombus,** and the **rhomboid.**

38. A **rectangle** (Fig. 16) is a parallelogram whose angles are all right angles.

FIG. 16.


FIG. 17.

39. A **square** (Fig. 17) is a rectangle whose sides are all of the same length.

40. A **rhomboid** (Fig. 18) is a parallelogram whose opposite sides are equal and parallel, and whose angles are not right angles.


FIG. 18.

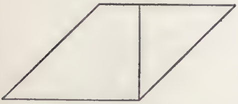


FIG. 19.

41. A **rhombus** (Fig. 19) is a parallelogram having equal sides, and whose angles are not right angles.

42. A **trapezoid** (Fig. 20) is a quadrilateral which has only two of its sides parallel.


FIG. 20.

43. The **altitude** of a parallelogram or a trapezoid is the perpendicular distance between the parallel lines, as shown by the vertical lines in Figs. 18, 19, and 20.

44. The **base** of *any* plane figure is the side on which it is supposed to stand.

45. The **area** of a surface is expressed by the number of unit squares it will contain.

46. A **unit square** is the square having a unit for its side. For example, if the unit is 1 inch, the unit square is the square each of whose sides measures 1 inch in length, and the area of a surface would be expressed by the number of square inches it would contain. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area of the given surface would be the number of square feet it would contain, etc.

The square that measures 1 inch on a side is called a **square inch,** and the one that measures 1 foot on a side is called a **square foot.** Square inch and square foot are abbreviated to sq. in. and sq. ft.

47. To find the area of any parallelogram:

Rule 1.—*Multiply the base by the altitude.*

NOTE.—Before multiplying, the base and altitude must be reduced to the same kind of units; that is, if the base should be given in feet and the altitude in inches, they could not be multiplied together until either the altitude had been reduced to feet or the base to inches. This principle holds throughout the subject of mensuration.

EXAMPLE 1.—The sides of a square piece of sheet iron are each $10\frac{1}{4}$ inches long. How many square inches does it contain?

SOLUTION.— $10\frac{1}{4}$ inches $= 10.25$ inches when reduced to a decimal. The base and altitude are each 10.25 inches. Multiplying them together, $10.25 \times 10.25 = 105.06+$ sq. in. Ans.

EXAMPLE 2.—What is the area in square rods of a piece of land in the shape of a rhomboid, one side of which is 8 rods long and whose length, measured on a line perpendicular to this side, is 200 feet?

SOLUTION.—The base is 8 rods and the altitude 200 feet. As the answer is to be in rods, the 200 feet should be reduced to rods. Reducing $200 \div 16\frac{1}{2} = 200 \div \frac{33}{2} = 12.12$ rods. Hence, area $= 8 \times 12.12 = 96.96$ sq. rd. Ans.

48. To find the area of a trapezoid:

Rule 2.—*Multiply one-half the sum of the parallel sides by the altitude.*

EXAMPLE.—A board 14 feet long is 20 inches wide at one end and 16 inches wide at the other. If the ends are parallel, how many square feet does the board contain?

SOLUTION.—One-half the sum of the parallel sides $= \frac{20 + 16}{2}$ $= 18$ inches $= 1\frac{1}{2}$ feet. The length of the board corresponds to the altitude of a trapezoid. Hence, $14 \times 1\frac{1}{2} = 21$ sq. ft. Ans.

49. Having given the area of a parallelogram and one dimension, to find the other dimension:

Rule 3.—*Divide the area by the given dimension.*

EXAMPLE.—What is the width of a parallelogram whose area is 212 square feet and whose length is $26\frac{1}{2}$ feet?

SOLUTION.— $212 \div 26\frac{1}{2} = 212 \div \frac{53}{2} = 8$ ft. Ans.

The following examples illustrate a few special cases:

EXAMPLE 1.—An engine room is 22 feet by 32 feet. The engine bed occupies a space of 3 feet by 12 feet; the flywheel pit, a space of 2 feet by 6 feet, and the outer bearing a space of 2 feet by 4 feet. How many square feet of flooring will be required for the room?

SOLUTION.—Area of engine bed $= 3 \times 12 = 36$ sq. ft.
Area of flywheel pit $= 2 \times 6 = 12$ sq. ft.
Area of outer bearing $= 2 \times 4 = 8$ sq. ft.
Total, 56 sq. ft.
Area of engine room $= 22 \times 32 = 704$ sq. ft.

$704 - 56 = 648$ sq. ft. of flooring required. Ans.

EXAMPLE 2.—How many square yards of plaster will it take to cover the sides and ceiling of a room 16×20 feet and 11 feet high, having four windows, each 7×4 feet, and three doors, each 9×4 feet over all, the baseboard coming 6 inches above the floor?

SOLUTION.—

Area of ceiling $= 16 \times 20 = 320$ sq. ft.
Area of end walls $= 2(16 \times 11) = 352$ sq. ft.
Area of side walls $= 2(20 \times 11) = 440$ sq. ft.
Total area $= 1{,}112$ sq. ft.

From the above must be deducted:

Windows $= 4(7 \times 4) = 112$ sq. ft.
Doors $= 3(9 \times 4) = 108$ sq. ft.

Baseboard less the width of three doors $= (72 - 12) \times \frac{6}{12} = 30$ sq. ft.

Total number of feet to be deducted $= 112 + 108 + 30 = 250$ sq. ft.

Hence, number of square feet to be plastered $= 1{,}112 - 250$ $= 862$ sq. ft., or $95\frac{7}{9}$ sq. yd. Ans.

EXAMPLE 3.—How many acres are contained in a rectangular tract of land 800 rods long and 520 rods wide?

SOLUTION.— $800 \times 520 = 416{,}000$ sq. rd. Since there are 160 square rods in 1 acre, the number of acres $= 416{,}000 \div 160 = 2{,}600$ acres. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area in square feet of a rhombus whose base is 84 inches and whose altitude is 3 feet? Ans. 21 sq. ft.

2. A flat roof, 46 feet by 80 feet in size, is covered by tin roofing weighing one-half pound per square foot; what is the total weight of the roofing? Ans. 1,840 lb.

3. One side of a room measures 16 feet. If the floor contains 240 square feet, what is the length of the other side? Ans. 15 ft.

4. How many square feet in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end? Ans. 15 sq. ft.

5. How much would it cost to lay a sidewalk a mile long and 8 feet 6 inches wide, at the rate of 20 cents per square foot? How much at the rate of $1.80 per square yard? Ans. $8,976 in each case.

6. How many square yards of plastering will be required for the ceiling and walls of a room 10 ft. × 15 ft. and 9 feet high; the room contains one door $3\frac{1}{2}$ ft. × 7 ft., three windows $3\frac{1}{2}$ ft. × 6 ft., and a baseboard 8 inches high? Ans. 53.5 sq. yd.

THE TRIANGLE.

50. A **triangle** is a plane figure having three sides.

FIG. 21.

51. An **isosceles** triangle is one having two of its sides equal; see Fig. 21.

52. An **equilateral** triangle (Fig. 22) is one having all of its sides of the same length.

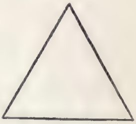


FIG. 22.

FIG. 23.

53. A **scalene** triangle (Fig. 23) is one having no two of its sides equal.

54. A **right-angled** triangle (Fig. 24) is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse.** A right-angled triangle is now usually called a **right triangle.**

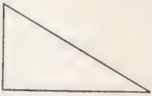


FIG. 24.

55. In any triangle the sum of the three angles equals two right angles, or 180°. Thus, in Fig. 25, the sum of the angles A, B, and C equals two right angles, or 180°. Hence, if any two angles of a triangle are given and it is required to find the third angle:

FIG. 25.

Rule 4.—*Add the two given angles and subtract their sum from 180°; the remainder will be the third angle.*

EXAMPLE.—If two angles of a triangle are 48° 16′ and 47° 50′, what does the third angle equal?

SOLUTION.—First reduce 48° 16′ and 47° 50′ to minutes, for convenience in adding and subtracting the angles. $48° = 48 \times 60' = 2,880'$; $2,880' + 16' = 2,896'$; hence, $48°\ 16' = 2,896'$. In like manner, $47°\ 50' = 47 \times 60' + 50' = 2,820' + 50' = 2,870'$. Adding the two angles and subtracting the sum from 180° reduced to minutes, $2,896' + 2,870' = 5,766'$; $180° = 180 \times 60' = 10,800'$; $10,800 - 5,766 = 5,034'$. Reducing this last number to degrees and minutes, $\frac{5,034}{60} = 83\frac{54}{60}° = 83°\ 54'$. Hence, the third angle in the triangle $= 83°\ 54'$. Ans.

56. In any right triangle there can be but one right angle, and since the sum of all the angles is two right angles, it is evident that the sum of the two acute angles must equal one right angle, or 90°. Therefore, if in any right triangle one acute angle is known, to find the other acute angle:

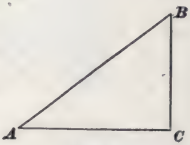


FIG. 26.

Rule 5.—*Subtract the known acute angle from 90°; the result will be the other acute angle.*

EXAMPLE.—If one acute angle, as A, of the right triangle $A\,B\,C$, Fig. 26, equals 30°, what does the angle B equal?

SOLUTION.— $90° - 30° = 60°$. Ans.

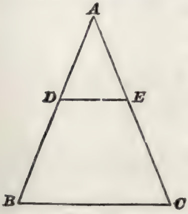


FIG. 27.

57. If a straight line be drawn through two sides of a triangle, parallel to the third side, a second triangle will be formed whose sides will be proportional to the corresponding sides of the first triangle. Thus, in the triangle $A\,B\,C$, Fig. 27, if the line $D\,E$ be drawn parallel to the side $B\,C$, the triangle $A\,D\,E$ will be formed and we shall have

(1) Side $A\,D$: side $D\,E$ = side $A\,B$: side $B\,C$; and,

(2) Side $A\,E$: side $D\,E$ = side $A\,C$: side $B\,C$; also,

(3) Side $A\,D$: side $A\,E$ = side $A\,B$: side $A\,C$.

EXAMPLE.—In Fig. 27, if $A\,B = 24$, $B\,C = 18$, and $D\,E = 8$, what does $A\,D$ equal?

SOLUTION.—Writing these values for the sides in (1),

$$A\,D : 8 = 24 : 18; \text{ whence, } A\,D = \frac{24 \times 8}{18} = 10\tfrac{2}{3}. \quad \text{Ans.}$$

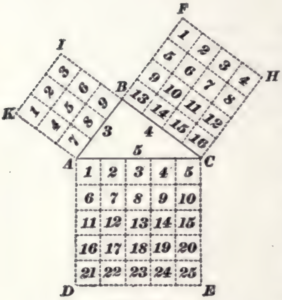


FIG. 28.

58. In any right triangle, the square described on the hypotenuse is equal to the sum of the squares described upon the other two sides. If $A\,B\,C$, Fig. 28, is a right triangle right-angled at B, then the square described upon the hypotenuse $A\,C$ is equal to the sum of the squares described upon the sides $A\,B$ and $B\,C$. Hence, having given the two sides forming the right angle in a right triangle, to find the hypotenuse:

Rule 6.—*Square each of the sides forming the right angle; add the squares together and take the square root of the sum.*

EXAMPLE.—If $A\,B = 3$ inches and $B\,C = 4$ inches, what is the length of the hypotenuse $A\,C$?

SOLUTION.—Squaring each of the given sides, $3^2 = 9$ and $4^2 = 16$. Taking the square root of the sum of 9 and 16, the hypotenuse $= \sqrt{9 + 16} = \sqrt{25} = 5$ in. Ans.

59. If the hypotenuse and one side are given, the other side can be found as follows:

Rule 7.—*Subtract the square of the given side from the square of the hypotenuse, and extract the square root of the remainder.*

EXAMPLE 1.—The side given is 3 inches, the hypotenuse is 5 inches; what is the length of the other side?

SOLUTION.— $3^2 = 9$; $5^2 = 25$. $25 - 9 = 16$, and $\sqrt{16} = 4$ in. Ans.

EXAMPLE 2.—If from a church steeple which is 150 feet high a rope is to be attached to the top and to a stake in the ground, which is 85 feet from the center of the base (the ground being supposed to be level), what must be the length of the rope?

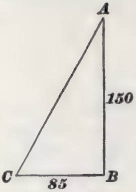


FIG. 29.

SOLUTION.—In Fig. 29, $A\ B$ represents the steeple, 150 feet high; C a stake 85 feet from the foot of the steeple, and $A\ C$ the rope. Here we have a right triangle right-angled at B, and $A\ C$ is the hypotenuse. The square of $A\ B = 150^2 = 22{,}500$; of $C\ B$, $85^2 = 7{,}225$. $22{,}500 + 7{,}225 = 29{,}725$; $\sqrt{29{,}725} = 172.4$ ft., nearly. Ans.

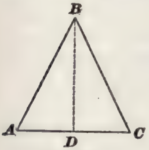


FIG. 30.

60. The **altitude** of any triangle is a line, as $B\ D$, drawn from the vertex B of the angle opposite the base $A\ C$, perpendicular to the base, as in Fig. 30, or to the base extended, as in Fig. 31.

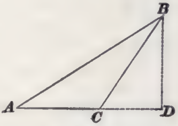


FIG. 31.

61. If in any parallelogram a straight line, called the **diagonal,** be drawn, connecting two opposite corners, it will divide the parallelogram into two equal triangles, as $A\ D\ B$ and $D\ B\ C$ in Fig. 32. The area of each triangle will equal one-half the area of the parallelogram, i. e., one-half the product of the base and the altitude. Hence, to find the area of any triangle:

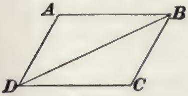


FIG. 32.

Rule 8.—*Multiply the base by the altitude and divide the product by 2.*

EXAMPLE.—What is the area in square feet of a triangle whose base is 18 feet and whose altitude is 7 feet 9 inches?

SOLUTION.— 7 ft. 9 in. $= 7\frac{3}{4}$ ft. $= \frac{31}{4}$ ft. $18 \times \frac{31}{4} = 139\frac{1}{2}$, and one-half of $139\frac{1}{2} = 69\frac{3}{4}$ sq. ft. Ans.

To find the altitude or base of a triangle, having given the area and the base or altitude:

Rule 9.—*Multiply the area by 2 and divide by the given dimension.*

Example.—What must be the height of a triangular piece of sheet metal to contain 100 square inches, if the base is 10 inches long?

Solution.— $100 \times 2 = 200$; $200 \div 10 = 20$ in. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area of a triangle whose base is 18 feet long and whose altitude is 10 feet 6 inches? Ans. 94.5 sq. ft.

2. Two angles of a scalene triangle together equal 100° 4′. What is the size of the third angle? Ans. 79° 56′.

3. One angle of a right triangle equals 20° 10′ 5″. What is the size of the other acute angle? Ans. 69° 49′ 55″.

4. A ladder 65 feet long reaches to the top of a wall when its foot is 25 feet from the wall. How high is the wall? Ans. 60 ft.

5. Draw a triangle, and through two of its sides draw a line parallel to the base. Letter the different lines, and then, without referring to the text, write out the proportions existing between the sides of the two triangles.

6. A triangular piece of sheet metal weighs 24 pounds. If the base of the triangle is 4 feet and its height 6 feet, how much does the metal weigh per square foot? Ans. 2 lb.

7. The area of a triangle is 16 square inches. If the altitude is 4 inches, what does the base measure? Ans. 8 in.

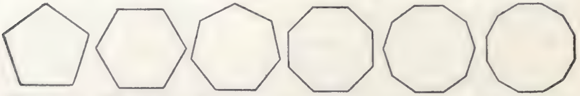
8. Two sides of a right triangle are 92 feet and 69 feet long. How long is the hypotenuse? Ans. 115 ft.

POLYGONS.

62. A **polygon** is a plane figure bounded by straight lines. The term is usually applied to a figure having more than four sides. The bounding lines are called the **sides,** and the sum of the lengths of all the sides is called the **perimeter** of the polygon.

63. A **regular polygon** is one in which all the sides and all the angles are equal.

64. A polygon of five sides is called a **pentagon**; one of six sides, a **hexagon**; one of seven sides, a **heptagon,**



Pentagon. Hexagon. Heptagon. Octagon. Decagon. Dodecagon.

Fig. 33.

etc. Regular polygons having from five to twelve sides are

shown in Fig. 33. In any polygon, the sum of all the interior angles, as $A+B+C+D+E$, Fig. 34, equals 180° multiplied by a number which is two less than the number of sides in the polygon. Hence, to find the size of any one of the interior angles of a *regular* polygon:

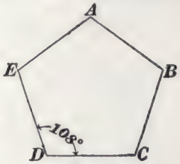


FIG. 34.

Rule 10.—*Multiply 180° by the number of sides less two and divide the result by the number of sides; the quotient will be the number of degrees in each interior angle.*

EXAMPLE 1.—If Fig. 34 is a regular pentagon, how many degrees are there in each interior angle?

SOLUTION.—In a pentagon there are five sides; hence, $5-2=3$ and $180 \times 3 = 540$; $540 \div 5 = 108°$ in each angle. Ans.

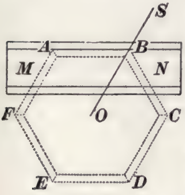


FIG. 35.

EXAMPLE 2.—It is desired to make a miter-box in which to cut a strip of molding to fit around a column having the shape of a regular hexagon. At what angle should the saw run across the miter-box?

SOLUTION.—In Fig. 35, let $A\,B$, $B\,C$, $C\,D$, etc. represent the pieces of molding as they will fit around the column. First find the size of one of the equal angles of the polygon by the above rule. Number of sides $=6$; $6-2=4$; hence, $180 \times 4 = 720$, and $720 \div 6 = 120°$ in each angle. Now, let $M\,N$ represent the miter-box and $O\,S$ the direction in which the saw should run; then, $A\,B\,O$ is the angle made by the saw with the side of the miter-box; but as the polygon is a regular one, this angle is one-half the interior angle $A\,B\,C$, which we have found to be 120°. Hence, the saw should run at an angle of $\frac{120}{2} = 60°$ with the side of the miter-box. Ans.

65. The area of any regular polygon may be found by drawing lines from the center to each angle and computing the area of each triangle thus formed. Hence, to find the area of any regular polygon:

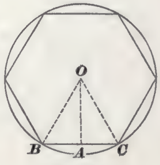


FIG. 36.

Rule 11.—*Multiply the length of a side by half the distance from the side to the center, and that product by the number of sides. The last product will be the area of the figure.*

EXAMPLE.—In Fig. 36 the side $B C$ of the regular hexagon is 12 inches and the distance $A O$ is 10.4 inches; required the area of the polygon.

SOLUTION.— $10.4 \div 2 = 5.2$; $12 \times 5.2 \times 6 = 374.4$ sq. in. Ans.

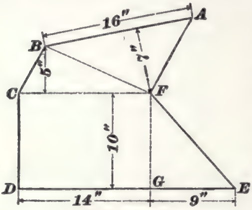


FIG. 37.

66. To obtain the area of any irregular polygon, draw diagonals dividing the polygon into triangles and quadrilaterals, and compute the areas of these separately; their sum will be the area of the figure.

EXAMPLE.—It is required to find the area of the polygon $A B C D E F$, Fig. 37.

SOLUTION.—Draw the diagonals $B F$ and $C F$ and the line $F G$ perpendicular to $D E$, dividing the figure into the triangles $A B F$, $B C F$, and $F G E$ and the rectangle $F C D G$. Let it be supposed that the altitudes of the figures and the lengths of the sides $A B$, $D G$, and $G E$ are as indicated in the polygon above. Then,

$$\text{Area } A B F = \frac{16 \times 7}{2} = 56 \text{ sq. in.}$$

$$\text{Area } B C F = \frac{14 \times 5}{2} = 35 \text{ sq. in.}$$

$$\text{Area } F C D G = 14 \times 10 = 140 \text{ sq. in.}$$

$$\text{Area } F G E = \frac{9 \times 10}{2} = 45 \text{ sq. in.}$$

Total area $= 56 + 35 + 140 + 45 = 276$ sq. in. Ans.

EXAMPLES FOR PRACTICE.

1. How many degrees are there in one of the angles of a regular octagon? Ans. 135°.

2. Find the area of the polygon $A B C D E F$ (see Fig. 37), supposing each of the given dimensions to be increased to $1\frac{1}{2}$ times the length given in the figure. Ans. 621 sq. in.

3. What is the area of a regular heptagon whose sides are 4 inches long, the distance from one side to the center being 4.15 inches? Ans. 58.1 sq. in.

4. At what angle should the saw run in a miter-box to cut strips to fit around the edge of a table top made in the shape of a regular pentagon? Ans. 54°.

THE CIRCLE.

67. A **circle** (Fig. 38) is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

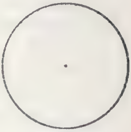


FIG. 38.

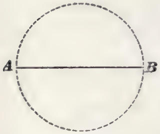


FIG. 39.

68. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference; thus, $A\ B$ (Fig. 39) is a diameter of the circle.

69. The **radius** of a circle, $A\ O$ (Fig. 40), is a straight line drawn from the center O to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii,** and all radii of a circle are equal.

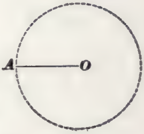


FIG. 40.

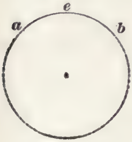


FIG. 41.

70. An **arc** of a circle (see $a\ e\ b$, Fig. 41) is any part of its circumference.

71. A **chord** is a straight line joining any two points in a circumference; or it is a straight line joining the extremities of an arc; thus, the straight line $A\,B$, Fig. 42, is a chord of the circle whose corresponding arc is $A\,E\,B$.

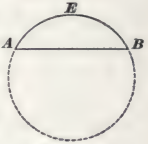


FIG. 42.

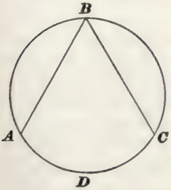


FIG. 43.

72. An **inscribed angle** is one whose vertex lies on the circumference of a circle and whose sides are chords. It is measured by one-half the intercepted arc. Thus, in Fig. 43, $A\ B\ C$ is an inscribed angle, and it is measured by one-half the arc $A\ D\ C$.

EXAMPLE.—If in Fig. 43, the arc $A\,D\,C = \frac{2}{5}$ of the circumference, what is the measurement of the inscribed angle $A\,B\,C$?

SOLUTION.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ of the circumference. The whole circumference $= 360°$; hence, $360° \times \frac{1}{5} = 72°$; therefore, angle $A\,B\,C$ is an angle of 72°.

73. If a circle is divided into halves, each half is called a **semicircle,** and each half circumference is called a **semi-circumference.**

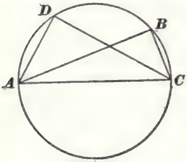


FIG. 44.

Any angle inscribed in a semicircle is a right angle, since it is measured by one-half a semi-circumference, or $180° \div 2 = 90°$. Thus, the angles $A\,D\,C$ and $A\,B\,C$, Fig. 44, are right angles, since they are inscribed in a semicircle.

74. An **inscribed** polygon is one whose vertexes lie on the circumference of a circle and whose sides are chords, as $A\,B\,C\,D\,E$, Fig. 45.

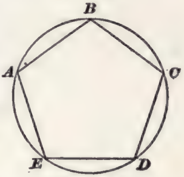


FIG. 45.

The sides of an inscribed regular hexagon have the same length as the radius of the circle.

If, in any circle, a radius be drawn perpendicular to any chord, it bisects (cuts in halves) the chord. Thus, if the radius $O\,C$, Fig. 46, is perpendicular to the chord $A\,B$, $A\,D = D\,B$.

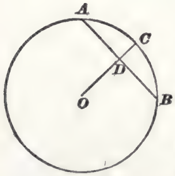


FIG. 46.

EXAMPLE.—If a regular pentagon is inscribed in a circle and a radius is drawn perpendicular to one of the sides, what are the lengths of the two parts of the side, the perimeter of the pentagon being 27 inches?

SOLUTION.—A pentagon has five sides, and since it is a regular pentagon, all the sides are of equal lengths; the perimeter of the pentagon, which equals the distance around it, or equals the sum of all the sides, is 27 inches. Therefore, the length of one side $= 27 \div 5 = 5\frac{2}{5}$ inches. Since the pentagon is an inscribed pentagon, its sides are chords, and as a radius perpendicular to a chord bisects it, we have $5\frac{2}{5} \div 2 = 2\frac{7}{10}$ inches, which equals the length of each of the parts of the side cut by a radius perpendicular to it. Ans.

75. If, from any point on the circumference of a circle, a perpendicular is let fall upon a diameter, it will divide the diameter into two parts, one of which will be in the same ratio to the perpendicular as the perpendicular is to the other part. That is, the perpendicular will be a *mean proportional* between the two parts of the diameter.

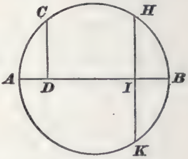


FIG. 47.

If $A\,B$, Fig. 47, is the given diameter and C any point on the circumference, then $A\,D : C\,D = C\,D : D\,B$, $C\,D$ being a mean proportional between $A\,D$ and $D\,B$.

EXAMPLE.—If $H\,K = 30$ feet and $I\,B = 8$ feet, what is the diameter of the circle, $H\,K$ being perpendicular to $A\,B$?

SOLUTION.— 30 feet ÷ 2 feet = 15 feet = $I\,H$. And $B\,I : I\,H = I\,H : I\,A$, or $8 : 15 = 15 : I\,A$.

Therefore, $I\,A = \frac{15^2}{8} = \frac{225}{8} = 28\frac{1}{8}$ feet and $I\,A + I\,B = 28\frac{1}{8} + 8 = 36\frac{1}{8}$ feet $= A\,B$, the diameter of the circle. Ans.

76. When the diameter of a circle and the lengths of the two parts into which it is divided are given, the length of the perpendicular may be found by multiplying the lengths of the two parts together and extracting the square root of the product.

EXAMPLE.—In Fig. 47, the diameter of the circle $A\,B$ is $36\frac{1}{8}$ feet and the distance $B\,I$ is 8 feet; what is the length of the line $H\,K$?

SOLUTION.—As the diameter of the circle is $36\frac{1}{8}$ feet and as $B\,I$ is 8 feet, $I\,A$ is equal to $36\frac{1}{8} - 8 = 28\frac{1}{8}$ feet. The two parts, therefore, are 8 and $28\frac{1}{8}$ feet, and their product $= 8 \times 28\frac{1}{8} = 8 \times \frac{225}{8} = 225$; the square root of their product $= \sqrt{225} = 15$ feet, and as $H\,K = I\,H + I\,K$, or $2\,I\,H$, $H\,K = 15 \times 2 = 30$ ft. Ans.

77. To find the circumference of a circle, the diameter being given:

Rule 12.—*Multiply the diameter by 3.1416.*

EXAMPLE.—What is the circumference of a circle whose diameter is 15 inches?

SOLUTION.— $15 \times 3.1416 = 47.124$ in. Ans.

78. To find the diameter of a circle, the circumference being given:

Rule 13.—*Divide the circumference by 3.1416.*

EXAMPLE.—What is the diameter of a circle whose circumference is 65.973 inches?

SOLUTION.— $65.973 \div 3.1416 = 21$ in. Ans.

79. To find the length of an arc of a circle:

Rule 14.—*Multiply the length of the circumference of the circle of which the arc is a part by the number of degrees in the arc and divide by 360.*

EXAMPLE.—What is the length of an arc of 24°, the radius of the arc being 18 inches?

SOLUTION.— $18 \times 2 = 36$ in. = the diameter of the circle. $36 \times 3.1416 = 113.1$ in., the circumference of the circle of which the arc is a part.

$113.1 \times \frac{24}{360} = 7.54$ in., or the length of the arc. Ans.

80. To find the area of a circle:

Rule 15.—*Square the diameter and multiply by .7854.*

EXAMPLE.—What is the area of a circle whose diameter is 15 inches?

SOLUTION.— $15^2 = 225$; and $225 \times .7854 = 176.72$ sq. in. Ans.

81. Given the area of a circle, to find its diameter:

Rule 16.—*Divide the area by .7854 and extract the square root of the quotient.*

EXAMPLE 1.—The area of a circle = 17,671.5 square inches. What is its diameter in feet?

SOLUTION.— $\sqrt{\frac{17,671.5}{.7854}} = 150$ inches.

$\frac{150}{12} = 12\frac{1}{2}$ feet, or the diameter. Ans.

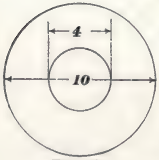


FIG. 48.

EXAMPLE 2.—What is the area of a flat circular ring, Fig. 48, whose outside diameter is 10 inches and inside diameter is 4 inches?

SOLUTION.—The area of the large circle = $10^2 \times .7854 = 78.54$ sq. in.; the area of the small circle = $4^2 \times .7854 = 12.57$ sq. in. The area of the ring is the difference between these areas, or $78.54 - 12.57 = 65.97$ sq. in. Ans.

82. To find the area of a sector (a **sector** of a circle is the area included between two radii and the circumference, as, for example, the area *B A C O*, Fig. 36):

Rule 17.—*Divide the number of degrees in the arc of the sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

Example.—The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 75°. The diameter of the circle is 12 inches; what is the area of the sector?

Solution.— $\frac{75}{360} = \frac{5}{24}$; and $12^2 \times .7854 = 113.1$ sq. in.

$113.1 \times \frac{5}{24} = 23.56$ sq. in., the area. Ans.

83. To find the area of a segment of a circle (a **segment** of a circle is the area included between a chord and its arc; for example, the area *A B C*, Fig. 49) when its chord and height are given. There is no exact method, except by applying principles of trigonometry. The following rule gives results that are exact enough for practical purposes.

Rule 18.—*Divide the diameter by the height of the segment; subtract .608 from the quotient and extract the square root of the remainder. This result multiplied by 4 times the square of the height of the segment and then divided by 3 will give the area, very nearly.*

The rule, expressed as a formula, is as follows, where D = the diameter of the circle and h = the height of the segment (see Fig. 49):

$$\text{Area of } A\,B\,C\,A = \frac{4\,h^2}{3}\sqrt{\frac{D}{h} - .608}.$$

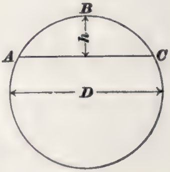


Fig. 49.

Example.—What is the area of the segment of a circle whose diameter is 54 inches, the height of the segment being 20 inches?

Solution.—Substituting in the formula,

$$\text{Area} = \frac{4 \times 20^2}{3}\sqrt{\frac{54}{20} - .608}. \quad \frac{4 \times 20^2}{3} = \frac{4 \times 400}{3}$$

$$= \frac{1,600}{3};\ \sqrt{\frac{54}{20} - .608} = \sqrt{2.092} = 1.446;\ \frac{1,600}{3} \times 1.446 = 771.2 \text{ sq. in.}$$

Ans.

Note.—Had the chord *A C*, Fig. 49, been given instead of the diameter, the diameter would have been found as explained in Art. **75.**

EXAMPLES FOR PRACTICE.

1. An angle inscribed in a circle intercepts one-third of the circumference. How many degrees are there in the angle? Ans. 60°.

2. Suppose that in Fig. 47, the diameter $AB = 15$ feet and the distance $BI = 3$ feet. What is the length of the line HK? Ans. 12 ft.

3. The diameter of a flywheel is 18 feet. What is the distance around it to the nearest 16th of an inch? Ans. 56 ft. $6\frac{9}{16}$ in.

4. A carriage wheel was observed to make $71\frac{5}{8}$ turns while going 300 yards. What was its diameter? Ans. 4 ft., nearly.

5. What is the length of an arc of 64°, the radius of the arc being 30 inches? Ans. 33.51 in.

6. Find the area of a circle 2 feet 3 inches in diameter. Ans. 3.976 sq. ft.

7. What must be the diameter of a circle to contain 100 square inches? Ans. 11.28 in.

8. Compute the area of a segment whose height is 11 inches and the radius of whose arc is 21 inches. Ans. 289.06 sq. in.

9. Find the area of a flat circular ring whose outside diameter is 12 inches and whose inside diameter is 6 inches. Ans. 84.82 sq. in.

THE PRISM AND CYLINDER.

84. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides which enclose it are called the **faces,** and their lines of intersection are called the **edges.**

85. A **prism** is a solid whose ends are equal and parallel polygons and whose sides are parallelograms. Prisms take their names from the form of their bases. Thus, a triangular prism is one having a triangle for its base; a hexagonal prism is one having a hexagon for its base, etc.

86. A **cylinder** is a body of uniform diameter whose ends are equal parallel circles.

FIG. 50.

87. A **parallelopipedon** (Fig. 50) is a prism whose bases (ends) are parallelograms.

88. A **cube** (Fig. 51) is a prism whose faces and ends are squares. All the faces of a cube are equal.

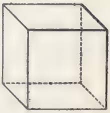


FIG. 51.

In the case of plane figures, we are concerned with perimeters and areas. In the case of solids, we are

concerned with the areas of their outside surfaces and with their contents or volumes.

89. The **entire surface** of any solid is the area of the whole outside of the solid, including the ends.

The **convex surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

90. A **unit of volume** is a cube each of whose edges is equal in length to the unit. The **volume** is expressed by the number of times it will contain a *unit of volume*.

Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges each measure 1 inch, this cube being 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

Instead of the word *volume*, the expression **cubical contents** is sometimes used.

91. To find the area of the convex surface of a prism or cylinder:

Rule 19.—*Multiply the perimeter of the base by the altitude.*

Example 1.—A block of marble is 24 inches long and its ends are 9 inches square. What is the area of its convex surface?

Solution.— $9 \times 4 = 36$ = the perimeter of the base; $36 \times 24 = 864$ sq. in., the convex area. Ans.

To find the entire area of the outside surface, add the areas of the two ends to the convex area. Thus, the area of the two ends $= 9 \times 9 \times 2 = 162$ sq. in.; $864 + 162 = 1{,}026$ sq. in. Ans.

Example 2.—How many square feet of sheet iron will be required for a pipe $1\frac{1}{2}$ feet in diameter and 10 feet long, neglecting the amount necessary for lapping?

Solution.—The problem is to find the convex surface of a cylinder $1\frac{1}{2}$ feet in diameter and 10 feet long. The perimeter, or circumference, of the base $= 1\frac{1}{2} \times 3.1416 = 1.5 \times 3.1416 = 4.712$ ft. The convex surface $= 4.712 \times 10 = 47.12$ sq. ft. of metal. Ans.

92. To find the volume of a prism or a cylinder:

Rule 20.—*Multiply the area of the base by the altitude.*

Example 1.—What is the weight of a length of wrought-iron shafting 16 feet long and 2 inches in diameter? Wrought iron weighs .28 pound per cubic inch.

Solution.—The shaft is a cylinder 16 ft. long. The area of one end, or the base, $= 2^2 \times .7854 = 3.1416$ sq. in. Since the weight of the iron is given per cubic inch, the contents of the shaft must be found in cubic inches. The length, 16 ft., reduced to inches $= 16 \times 12 = 192$ in.; $3.1416 \times 192 = 603.19$ cu. in. = the volume. The weight $= 603.19 \times .28 = 168.89$ lb. Ans.

Example 2.—Find the cubical contents of a hexagonal prism, Fig. 52, 12 inches long, each edge of the base being 1 inch long.

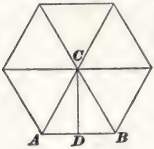


Fig. 52.

Solution.—In order to obtain the area of one end, the distance $C\,D$ from the center C to one side must be found.

In the right triangle $C\,D\,A$, side $A\,D = \frac{1}{2}\,A\,B$, or $\frac{1}{2}$ inch, and since the polygon is a hexagon, side $C\,A$ = distance $A\,B$, or 1 inch (Art. **74**). Hence, $C\,A$ being the hypotenuse, the length of side $C\,D = \sqrt{1^2 - (\frac{1}{2})^2} = \sqrt{1^2 - .5^2} = \sqrt{.75}$, or .866 inch. Area of triangle $A\,C\,B = \frac{1 \times .866}{2} = .433$ sq. in.; area of the whole polygon $= .433 \times 6 = 2.598$ sq. in. Hence, the contents of the prism $= 2.598 \times 12 = 31.176$ cu. in. Ans.

Example 3.—It is required to find the number of cubic feet of steam space in the boiler shown in Fig. 53. The boiler is 16 feet long between heads, 54 inches in diameter, and the mean water-line $M\,N$ is at a distance of 16 inches from the top of the boiler. The volume of the steam outlet casting may be neglected.

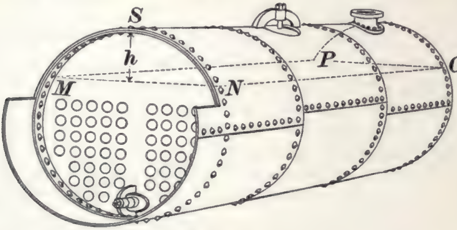


Fig. 53.

Solution.—The volume of the steam space, which is that space within the boiler above the surface $M\,N\,O\,P$ of the water, is found by the rule for finding the volume of a prism or cylinder, the area $M\,N\,S$ being the base and the length $N\,O$ the altitude. First obtain the area of the segment $M\,N\,S$, whose height h is 16 inches, in square feet; then multiply the result by 16, the length of the boiler.

By the formula given in Art. **83**, the area of the segment =

$$\frac{4\,h^2}{3}\sqrt{\frac{D}{h} - .608} = \frac{4 \times 16^2}{3}\sqrt{\frac{54}{16} - .608.}$$

$$\frac{4 \times 16^2}{3} = 341.33;\ \sqrt{\frac{54}{16} - .608} = \sqrt{2.767} = 1.663.$$

Hence, the area $= 341.33 \times 1.663 = 567.63$ sq. in. This reduced to sq. ft. $= 567.63 \div 144 = 3.942$ sq. ft., and the volume therefore $= 3.942 \times 16 = 63.07$ cu. ft. Ans.

In the above solution, the space occupied by the stays is not considered, for sake of simplicity. They are not shown in the figure.

EXAMPLE 4.—In the above boiler there are 60 tubes, $3\frac{1}{4}$ inches outside diameter. How many gallons of water will it take to fill the boiler up to the mean water level, there being 231 cubic inches in a gallon?

SOLUTION.—Find the volume in cubic inches of that part of the boiler below the surface of the water $M\,N\,O\,P$, since the contents of a gallon is given in cubic inches, and from it subtract the volume of the tubes in cubic inches.

This may be done by first finding the *total* area of one end of the boiler in square inches, from it subtracting the area of the segment $M\,N\,S$, and the areas of the ends of the tubes in square inches, and then by multiplying the result by the length of the boiler *in inches*.

Total area of one end $= 54^2 \times .7854 = 2{,}290.23$ sq. in.

Area of segment $M\,N\,S$, as found in last example, $= 567.63$ sq. in.

Area of the end of one tube $= 3.25^2 \times .7854 = 8.2958$ sq. in.

Area of the ends of the 60 tubes $= 8.2958 \times 60 = 497.75$ sq. in.

Hence, the area to be subtracted $= 567.63 + 497.75 = 1{,}065.38$ sq. in. Subtracting, $2{,}290.23 - 1{,}065.38 = 1{,}224.85$ sq. in. $=$ net area.

The cubical contents $= 1{,}224.85 \times 16 \times 12 = 235{,}171.2$ cu. in. This divided by 231 will give the number of gallons; whence, $235{,}171.2 \div 231 = 1{,}018.06$ gal. of water. Ans.

EXAMPLES FOR PRACTICE.

1. Find the area in square inches of the convex surface of a bar of iron $4\frac{1}{4}$ inches in diameter and 8 feet 5 inches long. Ans. 1,348.53 sq. in.

2. Find the area of the entire surface of the above bar.
Ans. 1,376.9 sq. in.

3. What is the area of the entire surface of the hexagonal prism whose base is shown in Fig. 52? Ans. 77.196 sq. in.

4. A multitubular boiler has the following dimensions: diameter, 50 inches; length between heads, 15 feet; number of tubes, 56; outside diameter of tubes, 3 inches; distance of mean water-line from top of boiler, 16 inches. (*a*) Compute the steam space in cubic feet. (*b*) Find the number of gallons of water required to fill the boiler up to the mean water-line.

Ans. $\begin{cases} (a) & 56.4 \text{ cu. ft.} \\ (b) & 800 \text{ gal.} \end{cases}$

THE PYRAMID AND CONE.

93. A **pyramid** (Fig. 54) is a solid whose base is a polygon and whose sides are triangles uniting at a common point, called the **vertex.**

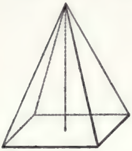


FIG. 54.

94. A **cone** (Fig. 55) is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex.**

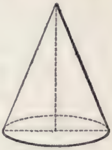


FIG. 55.

95. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

96. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

97. To find the convex area of a pyramid or cone:

Rule 21.—*Multiply the perimeter of the base by one-half the slant height.*

EXAMPLE 1.—What is the convex area of a pentagonal pyramid if one side of the base measures 6 inches and the slant height is 14 inches?

SOLUTION.—The base of a pentagonal pyramid is a pentagon, and, consequently, has fives sides.

$6 \times 5 = 30$ inches, or the perimeter of the base. $30 \times \frac{14}{2} = 210$ sq. in., or the convex area. Ans.

EXAMPLE 2.—What is the entire area of a right cone whose slant height is 17 inches and whose base is 8 inches in diameter?

SOLUTION.—The perimeter of the base $= 8 \times 3.1416 = 25.1328$ in.

$$\text{Convex area} = 25.1328 \times \frac{17}{2} = 213.63 \text{ sq. in.}$$

$$\text{Area of base} = 8^2 \times .7854 = \underline{50.27} \text{ sq. in.}$$

$$\text{Entire area} = 263.90 \text{ sq. in. Ans.}$$

98. To find the volume of a pyramid or cone:

Rule 22.—*Multiply the area of the base by one-third of the altitude.*

EXAMPLE 1.—What is the volume of a triangular pyramid, each edge of whose base measures 6 inches and whose altitude is 8 inches?

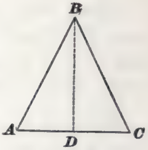


FIG. 56.

SOLUTION.—Draw the base as shown in Fig. 56; it will be an equilateral triangle, all of whose sides are 6 inches long.

Draw a perpendicular $B\,D$ from the vertex to the base; it will divide the base into two equal parts, since an equilateral triangle is also isosceles, and will be the altitude of the triangle. In order to obtain the area of the base, this altitude must be determined.

In the right triangle $B\,D\,A$, the hypotenuse $B\,A = 6$ inches and side $A\,D = 3$ inches, to find the other side,

$$B\,D = \sqrt{6^2 - 3^2} = 5.2 \text{ in., nearly.}$$

Area of the base, or $B\,A\,C$, $= \dfrac{6 \times 5.2}{2} = 15.6$ sq. in. Hence, the volume $= 15.6 \times \dfrac{8}{3} = 41.6$ cu. in. Ans.

EXAMPLE 2.—What is the volume of a cone whose altitude is 18 inches and whose base is 14 inches in diameter?

SOLUTION.—Area of the base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, the volume $= 153.94 \times \dfrac{18}{3} = 923.64$ cu. in. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of a square pyramid whose slant height is 28 inches and one edge of whose base is $7\frac{1}{2}$ inches long.

Ans. 420 sq. in.

2. What is the volume of a triangular pyramid, one edge of whose base measures 3 inches and whose altitude is 4 inches? Ans. 5.2 cu. in.

3. Find the volume of a cone whose altitude is 12 inches and the circumference of whose base is 31.416 inches. Ans. 314.16 cu. in.

NOTE.—Find the diameter of the base and then its area.

THE FRUSTUM OF A PYRAMID OR CONE.

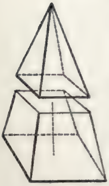


FIG. 57.

99. If a pyramid be cut by a plane, parallel to the base, so as to form two parts, as in Fig. 57, the lower part is called the **frustum** of the pyramid.

If a cone be cut in a similar manner, as in Fig. 58, the lower part is called the **frustum** of the cone.

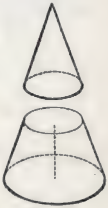


FIG. 58.

100. The upper end of the frustum of a pyramid or cone is called the **upper base,** and the lower end the **lower base.** The **altitude** of a frustum is the perpendicular distance between the bases.

101. To find the convex surface of a frustum of a pyramid or cone:

Rule 23.—*Multiply one-half the sum of the perimeters of the two bases by the slant height of the frustum.*

EXAMPLE 1.—Given, the frustum of a triangular pyramid, in which one side of the lower base measures 10 inches, one side of the upper base measures 6 inches, and whose slant height is 9 inches; find the area of the convex surface.

SOLUTION.— 10 in. $\times$ 3 = 30 in., the perimeter of the lower base. 6 in. $\times$ 3 = 18 in., the perimeter of the upper base.

$\frac{30 + 18}{2} = 24$ in., or one-half the sum of the perimeters of the two bases. $24 \times 9 = 216$ sq. in., the convex area. Ans.

EXAMPLE 2.—If the diameters of the two bases of a frustum of a cone are 12 inches and 8 inches, respectively, and the slant height is 12 inches, what is the entire area of the frustum?

SOLUTION.— $\frac{(12 \times 3.1416) + (8 \times 3.1416)}{2} \times 12 = 376.99$ sq. in., the area of the convex surface.

Area of the upper base = $8^2 \times .7854 = 50.27$ sq. in.
Area of the lower base = $12^2 \times .7854 = 113.1$ sq. in.
The entire area of the frustum = $376.99 + 50.27 + 113.1 = 540.36$ sq. in. Ans.

102. To find the volume of the frustum of a pyramid or cone:

Rule 24.—*Add together the areas of the upper and lower bases and the square root of the product of the two areas; multiply the sum by one-third of the altitude.*

EXAMPLE 1.—Given, a frustum of a square pyramid (one whose base is a square); each edge of the lower base is 12 inches, each edge of the upper base is 5 inches, and its altitude is 16 inches; what is its volume?

SOLUTION.—Area of upper base = $5 \times 5 = 25$ sq. in.; area of lower base = $12 \times 12 = 144$ sq. in.; the square root of the product of the two areas = $\sqrt{25 \times 144} = 60$. Adding these three results, and multiplying by one-third the altitude, $25 + 144 + 60 = 229$; $229 \times \frac{16}{3} = 1{,}221\frac{1}{3}$ cu. in. = the volume. Ans.

EXAMPLE 2.—How many gallons of water will a round tank hold which is 4 feet in diameter at the top, 5 feet in diameter at the bottom, and 8 feet deep?

SOLUTION.—There are 231 cubic inches in a gallon, and the volume of the tank should be found in cubic inches. The tank is in the shape of the frustum of a cone. The upper diameter $= 4 \times 12 = 48$ inches; the lower diameter $= 5 \times 12 = 60$ inches, and the depth $= 8 \times 12 = 96$ inches. Area of upper base $= 48^2 \times .7854 = 1{,}809.56$ sq. in.; area of lower base $= 60^2 \times .7854 = 2{,}827.44$ sq. in.; $\sqrt{1{,}809.56 \times 2{,}827.44} = 2{,}261.95$.

Whence, $1{,}809.56 + 2{,}827.44 + 2{,}261.95 = 6{,}898.95$; $6{,}898.95 \times \frac{96}{3} = 220{,}766.4$ cu. in. = contents. Now, since there are 231 cu. in. in 1 gallon, the tank will hold $220{,}766.4 \div 231 = 955.7$ gal., nearly. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of the frustum of a square pyramid, one edge of whose lower base is 15 inches long, one edge of whose upper base is 14 inches long, and whose slant height is 1 inch. Ans. 58 sq. in.
2. Find the volume of the above frustum, supposing its altitude to be 3 inches. Ans. 631 cu. in.
3. Find the volume of the frustum of a cone whose altitude is 12 feet and the diameters of whose upper and lower bases are 8 and 10 feet, respectively. Ans. 766.55 cu. ft.
4. If a tank had the dimensions of example 3, how many gallons would it hold? Ans. 5,734.2 gal., nearly.

THE SPHERE AND CYLINDRICAL RING.

103. A **sphere** (Fig. 59) is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center.

The word **ball,** or **globe,** is generally used instead of sphere.

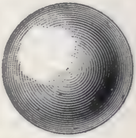


FIG. 59.

104. To find the area of the surface of a sphere:

Rule 25.—*Square the diameter and multiply the result by 3.1416.*

EXAMPLE.—What is the area of the surface of a sphere whose diameter is 14 inches?

SOLUTION.—Diameter squared $\times 3.1416 = 14^2 \times 3.1416 = 14 \times 14 \times 3.1416 = 615.75$ sq. in. Ans.

From this it will be seen that the surface of a sphere equals the circumference of a great circle multiplied by the diameter, a rule often used; a *great circle* of a sphere is the intersection of its surface with a plane passing through its center; for instance, the *great circle* of a sphere 6 inches diameter is a circle of 6 inches diameter. Any number of *great circles* could be described on a given sphere.

105. To find the volume of a sphere:

Rule 26.—*Cube the diameter and multiply the result by .5236.*

EXAMPLE.—What is the weight of a lead ball 12 inches in diameter, a cubic inch of lead weighing .41 pound?

SOLUTION.—Diameter cubed $\times$ .5236 $= 12 \times 12 \times 12 \times .5236 = 904.78$ cu. in., or the volume of the ball. The weight, therefore, $= 904.78 \times .41 = 370.96$ lb. Ans.

106. To find the convex area of a cylindrical ring:

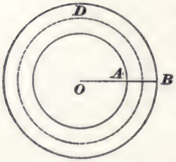


FIG. 60.

A **cylindrical ring** (Fig. 60) is a cylinder bent to a circle. The **altitude** of the cylinder before bending is the same as the length of the dotted center line *D*. The **base** will correspond to a cross-section on the line *A B* drawn from the center *O*. Hence, to find the convex area:

Rule 27.—*Multiply the circumference of an imaginary cross-section on the line A B by the length of the center line D.*

EXAMPLE.—If the outside diameter of the ring is 12 inches and the inside diameter is 8 inches, what is its convex area?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12 + 8}{2} = 10$, and $10 \times 3.1416 = 31.416$ in., the length of the center line.

The radius of the inner circle is 4 inches; of the outside circle, 6 inches; therefore, the diameter of the cross-section on the line *A B* is 2 inches. Then, $2 \times 3.1416 = 6.2832$ in., and $6.2832 \times 31.416 = 197.4$ sq. in., the convex area. Ans.

107. To find the volume of a cylindrical ring:

Rule 28.—*The volume will be the same as that of a cylinder whose altitude equals the length of the dotted center line D* (Fig. 61) *and whose base is the same as a cross-section of the ring on the line A B drawn from the center O. Hence, to find the volume of a cylindrical ring, multiply the area of an imaginary cross-section on the line A B by the length of the center line D.*

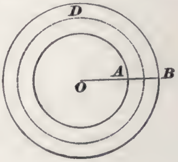


FIG. 61.

EXAMPLE.—What is the volume of a cylindrical ring whose outside diameter is 12 inches and whose inside diameter is 8 inches?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12+8}{2} = 10$.

$10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the outside circle = 6 inches; of the inside circle = 4 inches; therefore, the diameter of the cross-section on the line $A\ B$ = 2 inches.

Then, $2^2 \times .7854 = 3.1416$ sq. in., the area of the imaginary cross-section.

And $3.1416 \times 31.416 = 98.7$ cu. in., the volume. Ans.

EXAMPLES FOR PRACTICE.

1. What is the volume of a sphere 30 inches in diameter?
Ans. 14,137.2 cu. in.

2. How many square inches in the surface of the above sphere?
Ans. 2,827.44 sq. in.

3. Required the area of the convex surface of a circular ring, the outside diameter of the ring being 10 inches and the inside diameter $7\frac{1}{2}$ inches. Ans. 107.95 sq. in.

4. Find the cubical contents of the ring in the last example.
Ans. 33.73 cu. in.

5. The surface of a sphere contains 314.16 square inches. What is the volume of the sphere? Ans. 523.6 cu. in.

PRINCIPLES OF MECHANICS.

MATTER AND ITS PROPERTIES.

DEFINITION OF MECHANICS.

1. Mechanics is that science which treats of the action of forces on bodies and the effects that they produce; it treats of the laws that govern the movement and equilibrium of bodies and shows how they may be applied.

MATTER.

2. Matter is anything that occupies space. It is the substance of which all bodies consist. Matter is composed of *molecules* and *atoms.*

3. A **molecule** is the smallest portion of matter that can exist without changing its nature.

4. An **atom** is an indivisible portion of matter.

Atoms unite to form molecules, and a collection of molecules forms a mass or body.

A drop of water may be divided and subdivided until each particle is so small that it can only be seen by the most powerful microscope, but each particle will still be water.

Now, imagine the division to be carried on still further, until a limit is reached beyond which it is impossible to go without changing the nature of the particle. The particle of water is now so small that, if it be divided again, it will

cease to be water, and will be something else; we call this particle a *molecule.*

If a molecule of water be divided, it will yield 2 atoms of hydrogen gas and 1 of oxygen gas. If a molecule of sulphuric acid be divided, it will yield 2 atoms of hydrogen, 1 of sulphur, and 4 of oxygen.

5. **Bodies** are composed of collections of molecules. Matter exists in three conditions or forms: *solid*, *liquid*, and *gaseous.*

6. A **solid body** is one whose molecules change their relative positions with great difficulty; as iron, wood, stone, etc.

7. A **liquid body** is one whose molecules tend to change their relative positions easily. Liquids readily adapt themselves to the shape of vessels that contain them, and their upper surface always tends to become perfectly level. Water, mercury, molasses, etc., are liquids.

8. A **gaseous body,** or gas, is one whose molecules tend to separate from one another; as air, oxygen, hydrogen, etc.

Gaseous bodies are sometimes called **aeriform** (air-like) **bodies.** They are divided into two classes: the so-called "*permanent*" *gases* and *vapors.*

9. A **permanent gas** is one that remains a gas at ordinary temperatures and pressures.

10. A **vapor** is a body that at ordinary temperatures is a liquid or solid, but when heat is applied, becomes a gas, as steam.

One body may, under different conditions, exist in all three states; as, for example, mercury, which at ordinary temperatures is a liquid, becomes a solid (freezes) at 40° below zero, and a vapor (gas) at 600° above zero. By means of great cold, all gases, even hydrogen, have been liquefied, and many solidified.

By means of heat, all solids have been liquefied, and a great many vaporized. It is probable that, if we had the means of producing sufficiently great extremes of heat and

cold, all solids might be converted into gases, and all gases into solids.

11. Every portion of matter possesses certain qualities called *properties*. Properties of matter are divided into two classes: *general* and *special*.

12. **General properties of matter** are those that are common to all bodies. They are as follows: *Extension*, *impenetrability*, *weight*, *indestructibility*, *inertia*, *mobility*, *divisibility*, *porosity*, *compressibility*, *expansibility*, and *elasticity*.

13. **Extension** is the property of occupying space. Since all bodies must occupy space, it follows that extension is a general property.

14. By **impenetrability** we mean that no two bodies can occupy exactly the same space at the same time.

15. **Weight** is the measure of the earth's attraction upon a body. All bodies have weight. In former times it was supposed that gases had no weight, since, if unconfined, they tend to move away from the earth, but, nevertheless, they will finally reach a point beyond which they cannot go, being held in suspension by the earth's attraction. Weight is measured by comparison with a standard. The standard is a bar of platinum weighing 1 pound, owned and kept by the Government.

16. **Inertia** means that a body cannot put itself in motion nor bring itself to rest. To do either it must be acted upon by some force.

17. **Mobility** means that a body can be changed in position by some force acting upon it.

18. **Divisibility** is that property of matter by virtue of which a body may be separated into parts.

19. **Porosity** is the term used to denote the fact that there is space between the molecules of a body. The molecules of a body are supposed to be spherical, and, hence, there is space between them, as there would be between

peaches in a basket. The molecules of water are larger than those of salt; so that when salt is dissolved in water, its molecules wedge themselves between the molecules of the water, and, unless too much salt is added, the water will occupy no more space than it did before. This does not prove that water is penetrable, for the molecules of salt occupy the space that the molecules of water did not.

Water has been forced through iron by pressure, thus proving that iron is porous.

20. Compressibility is a natural consequence of porosity. Since there is space between the molecules, it is evident that by means of force (pressure) they can be brought closer together, and thus the body be made to occupy a smaller space.

21. Expansibility is the term used to denote the fact that the molecules of a body will, under certain conditions (when heated, for example), move farther apart, and so cause the body to *expand*, or occupy a greater space.

22. Elasticity is that property of matter which enables a body when distorted within certain limits to resume its original form when the distorting force is removed. Glass, ivory, and steel are very elastic, clay and putty in their natural state being very slightly so.

23. Indestructibility is the term used to denote the fact that we cannot *destroy* matter. A body may undergo thousands of changes, be resolved into its molecules, and its molecules into atoms, which may unite with other atoms to form other molecules and bodies entirely different in appearance and properties from the original body, but the same number of atoms remain. The whole number of atoms in the universe is exactly the same now as it was millions of years ago, and will always be the same. *Matter is indestructible.*

24. Special properties are those that are not possessed by all bodies. Some of the most important are as

follows: *hardness*, *tenacity*, *brittleness*, *malleability*, and *ductility*.

25. Hardness.—A piece of copper will scratch a piece of wood, steel will scratch copper, and tempered steel will scratch steel in its ordinary state. We express all this by saying that steel is *harder* than copper, and so on. Emery and corundum are extremely hard, and the diamond is the hardest of all known substances. It can only be polished with its own powder.

26. Tenacity is the term applied to the power with which some bodies resist a force tending to pull them apart. Steel is very tenacious.

27. Brittleness. — Some bodies possess considerable power to resist either a pull or a pressure, but they are easily broken when subjected to shocks or jars; for example, good glass will bear a *greater* compressive force than most woods, but may be easily broken when dropped upon a hard floor; this property is called *brittleness*.

28. Malleability is that property which permits of some bodies being hammered or rolled into sheets. Gold is the most malleable of all substances.

29. Ductility is that property which enables some bodies to be drawn into wire. Platinum is the most ductile of all substances.

MOTION AND VELOCITY.

DEFINITIONS.

30. Motion is the opposite of rest and indicates a changing of position in relation to some object which is for that purpose regarded as being fixed. If a large stone is rolled down hill, it is in motion in relation to the hill.

If a person is on a railway train and walks in the opposite direction from that in which the train is moving, and with the same speed, he will be in motion as regards the

train, but at rest with respect to the earth, since, until he gets to the end of the train, he will be directly over the spot at which he was when he started to walk.

31. The **path of a body in motion** is the line described by a certain point in the body called its center of gravity. No matter how irregular the shape of the body may be, nor how many turns and twists it may make, the line that indicates the direction of this point for every instant that it is in motion is the path of the body.

32. Velocity is rate of motion. It is measured by a unit of space passed over in a unit of time. When equal spaces are passed over in equal times, the velocity is said to be **uniform.**

If the flywheel of an engine keeps up a constant speed of a certain number of revolutions per minute, the velocity of any point is uniform. A railway train having a constant speed of 40 miles per hour moves 40 miles every hour, or $\frac{40}{60} = \frac{2}{3}$ mile every minute, and since equal spaces are passed over in equal times, the velocity is uniform.

33. Variable Velocity.—When a body moves in such a way that the spaces passed over in equal periods of time are unequal, its velocity is said to be **variable.**

34. The rate of motion of a body with a variable velocity may increase or decrease at a uniform rate. When the velocity varies in either of these ways, the body is said to have a **uniformly varying velocity.**

The most familiar example of uniformly varying velocity is a falling weight. Suppose a stone is dropped from a high bridge. It starts from a state of rest with no velocity, but its velocity constantly increases until it strikes. Its increase in velocity during any equal units of time is nearly the same. Thus, at the end of the first second its velocity will have increased from 0 to a rate of 32.16 feet per second, nearly; during the next second the velocity will have increased by the same amount, making the velocity at the end of the second second 64.32 feet. At the end of the third

second a like increase will have taken place, and the velocity will then be $3 \times 32.16 = 96.48$ feet per second.

35. The change in the velocity of a body during a period of time is called its **acceleration** for that period. Thus, in the case of the falling weight just considered, the change of 32.16 feet per second that takes place in its velocity during each second is its acceleration in feet per second for each second considered. If the period of time considered had been 2 seconds, the acceleration would have been the increase in velocity during this time, that is, 64.32 feet per second for each 2 seconds considered.

36. The change in velocity may be from a higher to a lower rate. Thus, when a stone is thrown upwards, it leaves the hand with a given velocity; its upward motion is constantly resisted by the force of gravity and the resistance of the air, and in consequence of these resistances, it moves slower and slower until it finally stops and begins to return to the earth. A change in velocity of this kind is sometimes called **retardation.**

37. The **mean** or **average velocity** of a body moving with a variable velocity can only be given for a stated period of time, and is numerically equal to the uniform velocity that will take the body over the same distance in the same time.

RULES FOR VELOCITY PROBLEMS.

38. Uniform and Average Velocity.—

Let $d =$ distance;
$t =$ time;
$v =$ velocity.

Rule 1.—*To find the uniform or the average velocity that a body must have to pass over a certain distance or space in a given time, divide the distance by the time.*

Or, $$v = \frac{d}{t}.$$

EXAMPLE 1.—The piston of a steam engine travels 3,000 feet in 5 minutes; what is its average velocity in feet per minute?

SOLUTION.—Here 3,000 feet is the distance, and 5 minutes is the time. Applying the rule, $3{,}000 \div 5 = 600$ ft. per min. Ans.

CAUTION.—Before applying the above or any of the succeeding rules, care must be taken to reduce the values given to the denominations required in the answer. Thus, in the above example, if the velocity is required in feet per second instead of in feet per minute, the 5 minutes must be reduced to seconds before dividing. The operation will then be: 5 minutes $= 5 \times 60 = 300$ seconds. Applying the rule, $3{,}000 \div 300 = 10$ ft. per sec. Ans.

If the velocity is required in inches per second, it is necessary to reduce the 3,000 feet to inches and the 5 minutes to seconds, before dividing. Thus, 3,000 feet $\times 12 = 36{,}000$ inches. 5 minutes $\times 60 = 300$ seconds. Now applying the rule, $36{,}000 \div 300 = 120$ in. per sec. Ans.

EXAMPLE 2.—A railroad train travels 50 miles in $1\frac{1}{2}$ hours; what is its average velocity in feet per second?

SOLUTION.—Reducing the miles to feet and the hours to seconds, 50 miles $\times 5{,}280 = 264{,}000$ feet. $1\frac{1}{2}$ hours $\times 60 \times 60 = 5{,}400$ seconds. Applying the rule, $264{,}000 \div 5{,}400 = 48\frac{8}{9}$ ft. per sec. Ans.

39. If the uniform velocity (or the average velocity) and the time are given, and it is required to find the distance that a body having the given velocity will travel in the given time:

Rule 2.—*Multiply the velocity by the time.*

Or, $$d = v\,t.$$

EXAMPLE 1.—The velocity of sound in still air is 1,092 feet per second; how many miles will it travel in 16 seconds?

SOLUTION.—Reducing the 1,092 feet to miles, $1{,}092 \div 5{,}280 = \frac{1092}{5280}$. Applying the rule, $\frac{1092}{5280} \times 16 = 3.31$ mi., nearly. Ans.

EXAMPLE 2.—The piston speed of an engine is 11 feet per second, how many miles does the piston travel in 1 hour and 15 minutes?

SOLUTION.— 1 hour and 15 minutes reduced to seconds $= 4{,}500$ seconds $=$ the time. 11 feet reduced to miles $= \frac{11}{5280}$ mile $=$ velocity in miles per second. Applying the rule, $\frac{11}{5280} \times 4{,}500 = 9.375$ mi. Ans.

40. If the distance through which a body moves is given, and also its average or uniform velocity, and it is desired to know how long it takes the body to move through the given distance:

Rule 3.—*Divide the distance, or space passed over, by the velocity.*

Or, $$t = \frac{d}{v}.$$

EXAMPLE 1.—Suppose that the radius of the crank of a steam engine is 15 inches and that the shaft makes 120 revolutions per minute; how long will it take the crankpin to travel 18,849.6 feet?

SOLUTION.—Since the radius, or distance from the center of the shaft to the center of the crankpin, is 15 inches, the diameter of the circle it moves in is 15 inches $\times$ 2 = 30 inches = 2.5 feet. The circumference of this circle is 2.5 $\times$ 3.1416 = 7.854 feet. 7.854 $\times$ 120 = 942.48 feet = distance that the crankpin travels in 1 minute = velocity in feet per minute. Applying the rule, 18,849.6 $\div$ 942.48 = 20 min. Ans.

EXAMPLE 2.—A point on the rim of an engine flywheel travels at the rate of 150 feet per second; how long will it take it to travel 45,000 feet?

SOLUTION.—Applying the rule,

45,000 $\div$ 150 = 300 sec. = 5 min. Ans.

EXAMPLES FOR PRACTICE.

1. A locomotive has drivers 80 inches in diameter. If they make 293 revolutions per minute, what is the velocity of the train in (*a*) feet per second? (*b*) miles per hour?

Ans. $\begin{cases} (a) & 102.277 \text{ ft. per sec.} \\ (b) & 69.734 \text{ mi. per hr.} \end{cases}$

2. Assuming the velocity of steam as it enters the cylinder to be 900 feet per second, how far can it travel, if unobstructed, during the time the flywheel of an engine revolves 7 times, if the number of revolutions per minute are 120? Ans. 3,150 ft.

3. The average speed of the piston of an engine being 528 feet per minute, how long will it take the piston to travel 4 miles?

Ans. 40 min.

4. A speed of 40 miles per hour equals how many feet per second?

Ans. $58\frac{2}{3}$ ft.

5. The earth turns around once in 24 hours. If the diameter be taken as 8,000 miles, what is the velocity of a point on the equator in miles per minute? Ans. $17.45\frac{1}{3}$ mi. per min.

6. The stroke of an engine is 28 inches. If the engine makes 11,400 strokes per hour, (*a*) what is its speed in feet per minute? (*b*) how far will this piston travel in 11 minutes?

Ans. $\begin{cases} (a) & 443\frac{1}{3} \text{ ft. per min.} \\ (b) & 4{,}876 \text{ ft. } 8 \text{ in.} \end{cases}$

FORCE.

GENERAL PRINCIPLES.

41. Force is known only by the effects it produces on matter. Forces cannot, therefore, be compared in the same way that quantities of matter or distances are compared; the only way in which forces can be examined in reference to one another is by noting their relative effects in the production of motion or a change of state in matter.

The most familiar conception of force is that of a push or pull that tends to produce or destroy motion. If the force is great enough, its effect is seen in a change in the state of motion of the body on which it acts; that is, it either produces motion in the body or destroys some or all of the motion already existing. If, however, the body acted on by a force is so situated that the force applied to it is opposed by a resisting force of equal magnitude, no change in motion is produced. In this case the force is not perceivable, unless some other force is introduced whose effects will reveal the existence of the first force.

42. Forces are called by various names according to the ways in which they manifest themselves. Manifestations of force are: *attraction*, *repulsion*, *cohesion*, *adhesion*, *acceleration*, *retardation*, *resistance*, etc., and the forces producing these manifestations are called *attractive*, *repulsive*, *cohesive*, *adhesive*, *accelerating*, *retarding*, *resisting*, etc. forces.

43. Comparison of Forces.—In considering the effects of a force on a body, some standard of comparison must be used. The standard most commonly adopted in English-speaking countries is the **pound,** which is the force required to raise a standard mass of matter from the ground under certain specified conditions.

In practice, force is always regarded as a pressure; that is, a force is considered the equivalent of the pressure exerted by a weight. For example, the effect of a force of

20 pounds acting upon a body is the same as the pressure of 20 pounds exerted by a weight of 20 pounds.

44. In order that we may compare the effect of a force on a body with that of another force on another body, it is necessary that the following *three conditions* be fulfilled in regard to both bodies:

1. The point of application, or point at which the force acts upon the body, must be known.
2. The direction of the force, or, what is the same thing, the straight line along which the force tends to move the point of application, must be known.
3. The magnitude or value of the force in comparison with the given standard must be known.

45. Reduction of Forces.—When a number of forces act upon a body and produce a certain effect, it is often necessary to find a *single* force that, when substituted for the given forces, will produce the *same* effect. In order to find the point of application, the direction, and the magnitude of this single force, it is necessary to know the above three conditions of every one of the given forces.

NEWTON'S LAWS OF MOTION.

46. The fundamental principles of the relations between force and motion were first stated by Sir Isaac Newton, and are called *Newton's Three Laws of Motion.* They are as follows:

1. *All bodies continue in a state of rest, or of uniform motion in a straight line, unless acted upon by some external force that compels a change.*
2. *Every motion or change of motion is proportional to the acting force, and takes place in the direction of the straight line along which the force acts.*
3. *To every action there is always opposed an equal reaction.*

These laws in their accepted form, as just given, have been more or less indirectly derived from experiment, but

they are so comprehensive as to defy complete experimental verification.

47. Exemplification of the First Law.—In the first law of motion, it is stated that a body once set in motion by any force, no matter how small, will move forever in a straight line, and always with the same velocity, unless acted upon by some other force that compels a change. It is not possible to actually verify this law, on account of the earth's attraction for all bodies, but, from astronomical observations, we are certain that the law is true. This law is often called the **law of inertia.**

48. The word **inertia** is so abused that a full understanding of its meaning is important. Inertia is not a force, although it is often so called. If a force acts upon a body and puts it in motion, the effect of the force is stored in the body, and a second body, in stopping the first, will receive a blow equal in every respect to the original force, assuming that there has been no resistance of any kind to the motion of the first body.

It is dangerous for a person to jump from a fast-moving train, for the reason that, since his body has the same velocity as the train, it has the same force stored in it that would cause a body of the same weight to take the same velocity as the train, and the effect of a sudden stoppage is the same as the effect of a blow necessary to give the person that velocity.

By "bracing" himself and jumping in the same direction that the train is moving, and running, he brings himself gradually to rest, and thus reduces the danger. If a body is at rest, it must be acted upon by a force in order to be put in motion, and, no matter how great the force may be, it cannot be *instantly* put in motion.

The resistance thus offered to being put in motion is commonly, but erroneously, called the *resistance of inertia.* It should be called the *resistance due to inertia.*

49. Exemplification of the Second Law.—From the second law, we see that if two or more forces act upon a

body, their final effect on the body will be in proportion to their magnitudes and to the directions in which they act.

Thus, if the wind is blowing due west with a velocity of 50 miles per hour, and a ball is thrown due north with the same velocity, or 50 miles per hour, the wind will carry the ball west while the force of the throw is carrying it north, and the combined effect will be to cause it to move northwest.

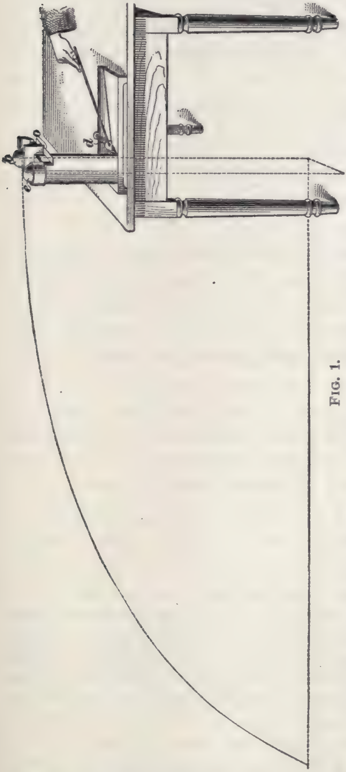


FIG. 1.

The amount of departure from due north will be proportional to the force of the wind and independent of the velocity due to the force of the throw.

50. In Fig. 1 a ball *e* is supported in a cup, the bottom of which is attached to the lever *o* in such a manner that *o* will swing the bottom horizontally and allow the ball to drop. Another ball *b* rests in a horizontal groove that is provided with a slit in the bottom. A swinging arm is actuated by the spring *d* in such a manner that, when drawn back, as shown, and then released, it will strike the lever *o* and the ball *b* at the same time. This gives *b*

an impulse in a horizontal direction, and swings *o* so as to allow *e* to fall.

On trying the experiment, it is found that *b* follows a path shown by the curved dotted line, and reaches the floor at the same instant as *e*, which drops vertically. This shows that the force that gave the first ball its horizontal movement had no effect on the vertical force that compelled both balls to fall to the floor; the vertical force produces the same effect as if the horizontal force had not acted. The second law may also be stated as follows: *A force has the same effect in producing motion, whether it acts upon a body at rest or in motion, and whether it acts alone or with other forces.*

51. Exemplification of the Third Law.—The third law states that action and reaction are equal. A man cannot lift himself by his boot straps for the reason that he presses downwards with the same force that he pulls upwards; the downward reaction equals the upward action, and is opposite to it.

In springing from a boat, we must exercise caution or the reaction will drive the boat from the shore. When we jump from the ground, we tend to push the earth from us, while the earth reacts and pushes us from it.

EXAMPLE.—Two men pull on a rope in opposite directions, each exerting a force of 100 pounds; what is the force that the rope resists?

SOLUTION.—Imagine the rope to be fastened to a tree, and that one man pulls with a force of 100 pounds. The rope evidently resists 100 pounds. According to Newton's third law, the reaction of the tree is also 100 pounds. Now, suppose the rope to be slacked, but that one end is still fastened to the tree; the second man then takes hold of the rope near the tree, and pulls with a force of 100 pounds, the first man pulling as before. The resistance of the rope is 100 pounds, as before, since the second man merely takes the place of the tree. He is obliged to exert a force of 100 pounds to keep the rope from slipping through his fingers. If the rope is passed around the tree, and each man pulls an end with a force of 100 pounds in the same and parallel directions, the stress in the rope is 100 pounds, as before, but the tree must resist the pull of both men, or 200 pounds.

52. Dynamics, also called **kinetics,** is that branch of mechanics that treats of forces and their effects when they *produce a change in motion* in the bodies on which they act.

53. Statics is that branch of mechanics that treats of forces and their effects when they do *not produce a change in motion* in the bodies on which they act.

GRAVITATION AND WEIGHT.

54. Every body in the universe exerts a certain attractive force on every other body, which tends to draw the two together. To scientists, this attractive force is known as **gravitation.**

If a body is held in the hand, a downward pull is felt, and if the hold is loosened, the body will fall to the ground. This pull, which we commonly call **weight,** is the attraction between the earth and the body.

55. The attraction between the earth and bodies at or near its surface is denoted by the term **force of gravity.** This attraction is generally considered as acting along the line joining the center of gravity of the body and the center of the earth. By **center of gravity** is meant that point of a body at which its whole weight may be assumed to be concentrated.

56. The weight of a body is directly proportional to the force of gravity. From this it follows that the weight of a body can only be uniform everywhere if the force of gravity is uniform. As a matter of fact, the force of gravity varies in different locations; consequently, the weight of the body is not the same at all points on the surface of the earth. This has been conclusively shown by sensitive spring balances.

ACCELERATING AND RETARDING FORCES.

57. According to the first law of motion, if a body is set in motion by a force and the force then ceases to act, the body will continue to move at the rate it had at the

instant the action of the force was discontinued, unless acted upon by some other force.

If, however, a force acts upon a body for a given period of time, say 1 second, and imparts to it a certain amount of motion, and then, instead of ceasing to act, acts with the same intensity during the next second, it will impart to the body an increase in velocity equal to the velocity imparted during the first second. Then, the velocity at the end of the second second will be twice that at the end of the first. During the third second a like increase in velocity will be produced, making the velocity at the end of the third second three times as great as at the end of the first. This uniform increase in velocity will continue as long as the constant force continues to act on the body. A constant force when producing a constant acceleration is called a **constant accelerating force.**

58. If a constant force is applied to a body in motion in such a manner that it opposes the motion, its effect will be to reduce the motion by a certain amount, which will be the same for each second during which it acts. In this case, the force is called a **constant retarding force.**

We thus see that the effect of a constant force acting upon a body in motion is to produce a uniform acceleration or retardation in the velocity of motion of the body, it being assumed that the motion of the body is not opposed by varying resisting forces.

59. Acceleration Due to the Force of Gravity.—If a body falls freely under the action of the force of gravity, its velocity will increase at a uniform rate; in other words, it will be accelerated. Since the force of gravity varies in different localities, it follows that the acceleration produced by it is not everywhere the same. The greatest range in the acceleration due to the force of gravity in the United States is from a minimum of about 32.089 feet per second up to a maximum of about 32.186 feet per second for each second. In the latitude of Scranton, Pa., and at the level of the sea, the acceleration is nearly 32.16 feet per second;

this value will be used in all calculations in this Course that involve the use of acceleration due to the force of gravity. In accordance with the practice of most scientific writers, we will denote the acceleration due to the force of gravity by the letter g.

60. Mass.—If the weight of a body at any place, as determined by a spring balance, is divided by the acceleration due to the force of gravity at that place, a numerical value will be obtained that, for the same body, will be the same wherever it may be weighed. This quotient is called the **mass** of the body, and is generally designated by the letter m.

Rule 4.—*To find the mass of a body, divide its weight by the acceleration due to the force of gravity.*

Let W = the weight of a body;
g = acceleration due to gravity;
m = mass of the body.

Then,
$$m = \frac{W}{g}.$$

EXAMPLE.—What is the mass of a body weighing 96.48 pounds?

SOLUTION.—Applying the rule, $m = \frac{96.48}{32.16} = 3$. Ans.

61. Law of Gravitation.—The attractive force by which one body tends to draw another body toward it is directly proportional to its mass and inversely proportional to the square of the distance between their centers of gravity.

62. Laws of Weight.—

1. *Bodies weigh most at the surface of the earth. Below the surface, the weight decreases directly as the distance to the center of the earth decreases.*

2. *Above the surface, the weight decreases inversely as the square of the distance.*

63. Change in Motion of a Body.—A change in the motion of a body cannot take place without the action of an accelerating or retarding force. The force required to

produce a given acceleration or retardation in a body is given by the following rule, where

f = force in pounds;
a = acceleration or retardation in feet per second.

Rule 5.—*Multiply the mass of the body by the acceleration, or retardation, in feet per second.*

Or, $$f = m a.$$

Since $m = \frac{W}{g}$ (see rule 4), this may also be written $f = \frac{W}{g} a$.

EXAMPLE.—What force will be required to give a body weighing 90 pounds an acceleration of 5 feet per second?

SOLUTION.—Applying rule **5**, we get

$$f = \frac{90}{32.16} \times 5 = 13.99+ \text{ lb., say 14 lb.} \quad \text{Ans.}$$

64. According to the first law of motion, a body in motion not acted upon by any external force will continue its motion without any further application of a force. In practice, however, the motion of a body is always opposed by some resisting force or forces. According to the third law of motion, the force required to overcome the resistance is equal to the resistance.

The opposing forces are usually constant, or nearly so. Taking the opposing forces into account, the actual force required to accelerate a body meeting with resistance will be the sum of the accelerating force and the opposing forces.

ILLUSTRATION.—Imagine a weight of 321.6 pounds to be lying on a smooth plane surface. Assume that it has been determined experimentally that a force of 100 pounds is required to be exerted continually to overcome the friction between the weight and the surface. What force will be required to produce an acceleration of 2 feet per second?

By rule **5**, the accelerating force is $\frac{321.6}{32.16} \times 2 = 20$ pounds. As a force of 100 pounds must be exerted continually to overcome the resistance due to friction, a force of $100 + 20 = 120$ pounds will be required to produce the required acceleration.

65. The question is often asked, what force is required to start a flywheel and keep it going at a stated number of

revolutions per minute? This question, or similar questions, cannot be answered without knowing the time in which the flywheel is to attain the given speed. An accelerating force depending on the mass of the wheel, its diameter, the distribution of the material of which it is made, the time of acceleration, and the constant resistances, is required to bring the wheel up to its speed. When this speed has been attained, the force required to keep it going will be that required to overcome the frictional and air resistances.

MOMENTUM.

66. Experience teaches us that the same force acting upon bodies of different weights produces different effects. For example, if a given force imparts a velocity of 10 feet per second in a certain time to a body weighing 1 pound, we know from experience and observation that it cannot impart the same velocity in the same time when acting upon a body weighing 1,000 pounds.

Scientists have shown that the velocity imparted to a body in a given time by a force *varies directly as the force* and *inversely as the mass* of the body. Hence, forces may be compared with one another by comparing their effects in imparting velocities to bodies whose masses are known.

67. The product obtained by multiplying the mass of a body by its velocity in feet per second is called the **momentum** of the body; it represents the magnitude of the force that will produce the given velocity in the body in 1 second. Hence, we may call momentum the **time effect** of a force.

68. According to the third law of motion, action and reaction are equal to each other. Consequently, if the force required to produce a stated momentum in a given time is known, it is likewise known what force is required to *destroy* this momentum, or to bring the body to rest, in an equal period of time.

When a liquid body is flowing in a stream, as from a nozzle, the weight to be considered in problems involving

momentum is the weight of the liquid discharged in 1 second. For example, let it be required to estimate the force with which a man must hold the nozzle of a fire hose to prevent its slipping through his hands when a stream of water issues from it with a velocity of 20 feet per second, the area of the opening in the nozzle being 1 square inch and the weight of a cubic inch of water .0361 pound. The volume of water discharged in 1 second is $1 \times 12 \times 20 = 240$ cubic inches, and since the weight of 1 cubic inch is .0361 pound, the weight discharged per second is $240 \times .0361 = 8.664$ pounds. The momentum of the stream is $\frac{8.664}{32.16} \times 20 = 5.38$ pounds. This is the constant force required to give a body of water weighing 8.664 pounds a velocity of 20 feet per second, in 1 second; it also represents the magnitude of the reaction on the nozzle, and the man must hold the nozzle with a force equal to the reaction, or 5.38 pounds, in order to prevent its slipping through his hands.

WORK, POWER, AND ENERGY.

WORK.

69. Work is the overcoming of resistance continually occurring along the path of motion.

Motion in itself is not work; a force must overcome a resistance in order that work may be done.

70. Unit of Work.—The unit by which the work done by a force is measured is the work done in overcoming a resistance of 1 pound through a space of 1 foot; this unit is called a **foot-pound.** According to the definition, it may be considered as the force required to raise 1 pound 1 foot vertically. All work is measured by this standard.

A horse going up a hill does an amount of work equal to its own weight, plus the weight of the wagon and its contents, plus the frictional resistances reduced to an equivalent

weight, multiplied by the vertical height of the hill. Thus, if the horse weighs 1,200 pounds, the wagon and contents 1,200 pounds, and the frictional resistances equal 400 pounds, then if the vertical height of the hill is 100 feet, the work done is equal to $(1{,}200 + 1{,}200 + 400) \times 100 = 280{,}000$ foot-pounds.

Rule 6.—*In all cases the force (or resistance) multiplied by the distance through which it acts equals the work. If a weight is raised, the weight multiplied by the vertical height of the lift equals the work.*

71. The total amount of work done in overcoming a given resistance through a given distance is independent of time; that is, it is immaterial whether it takes 1 minute or 1 year in which to do it; but in order to compare the *rate* at which work is done by different machines with a common standard, time must be considered. If one machine does a certain amount of work in 10 minutes and another machine does exactly the same amount of work in 5 minutes, the second machine can do twice as much work as the first in an equal period of time.

POWER.

72. **Power** is a term used to denote the rate at which work is done.

73. The common unit used for expressing the rate at which work is done is the **horsepower.**

One horsepower is 33,000 foot-pounds of work per minute; in other words, it is 33,000 pounds raised vertically 1 foot in 1 minute, or 1 pound raised vertically 33,000 feet in 1 minute, or any combination that will, when multiplied together, give 33,000 foot-pounds in 1 minute. Thus, the work done in raising 110 pounds vertically 5 feet in 1 second is a horsepower; for, since in 1 minute there are 60 seconds, $110 \times 5 \times 60 = 33{,}000$ foot-pounds in 1 minute.

EXAMPLE.—In a steam engine the force impelling the piston forwards and backwards is 10,000 pounds. This force overcomes the

resistance due to the load at the rate of 600 feet per minute; that is, it moves the piston back and forth at that rate. What is the horsepower of the engine?

SOLUTION.—According to rule **6**, the work done is $10,000 \times 600 = 6,000,000$ foot-pounds per minute. Then, as 33,000 foot-pounds per minute represent a horsepower, the horsepower of the engine is $6,000,000 \div 33,000 = 181.818$. Ans.

ENERGY.

74. Energy is a term used to express the ability of an agent to do work.

75. Kinetic Energy.—If we have a body at rest, a certain amount of force must be exerted and a certain amount of work must be done to set it in motion. A part of this force will be required to overcome those resistances outside of the body, such as friction and the resistance of the air, that oppose the motion of all bodies with which we have to do; another part acts to overcome the inertia of the body, to start it from its state of rest, and give it motion (see Newton's first law). The force that overcomes the resistance due to the inertia of the body does work, and the work so performed is stored in the body; in being brought to rest, the body is capable of overcoming a resistance and of doing an amount of work exactly equal to the work expended in giving it motion. The ability that the moving body has to do work while being brought to rest is called the **kinetic energy** of the body.

76. Rule for the Energy of a Moving Body.—

Let w = weight of body in pounds;
v = its velocity in feet per second;
E = kinetic energy in foot-pounds.

Then, the kinetic energy of a moving body may be found as follows:

Rule 7.—*Multiply the weight of the body by the square of its velocity and divide the product by twice the acceleration due to the force of gravity.*

Or,
$$E = \frac{w\,v^2}{64.32}.$$

Thus, if a weight is raised a certain height, an amount of work is done equal to the product of the weight and the vertical height. If a weight is suspended at a certain height and allowed to fall, it will do the same amount of work in foot-pounds that was required to raise the weight to the height through which it fell.

EXAMPLE 1.—If a body weighing 25 pounds falls from a height of 100 feet, how much work can it do?

SOLUTION.—Work $= w\,h = 25 \times 100 = 2{,}500$ ft.-lb. Ans.

It requires the same amount of work or energy to stop a body in motion within a certain time as it does to give it that velocity in the same length of time.

EXAMPLE 2.—A body weighing 50 pounds has a velocity of 100 feet per second; what is its kinetic energy?

SOLUTION.—Kinetic energy $= \frac{w\,v^2}{64.32} = \frac{50 \times 100^2}{64.32} = 7{,}773.63$ ft.-lb. Ans.

EXAMPLE 3.—In the last example, how many horsepower will be required to give the body this amount of kinetic energy in 3 seconds?

SOLUTION.— 1 horsepower = 33,000 pounds raised 1 foot in 1 minute.

If 7,773.63 foot-pounds of work are done in 3 seconds, in 1 second there will be done $\frac{7{,}773.65}{3} = 2{,}591.21$ foot-pounds of work. 1 horsepower = 33,000 foot-pounds per minute = 33,000 ÷ 60 = 550 foot-pounds per second.

The number of horsepower required will be

$$\frac{2{,}591.21}{550} = 4.7113 \text{ H. P.} \quad \text{Ans.}$$

77. Potential energy *is latent energy; it is the energy that a body at rest is capable of giving out under certain conditions.*

If a stone is suspended by a string from a high tower, it has potential energy. If the string is cut, the stone will fall to the ground, and during its fall its potential energy will change into kinetic energy, so that at the instant it strikes the ground its potential energy is wholly changed into kinetic energy.

At a point equal to one-half the height of the fall, the potential and kinetic energies are equal. At the end of the first quarter the potential energy is three-fourths and the kinetic energy one-fourth; at the end of the third quarter the potential energy is one-fourth and the kinetic energy three-fourths.

A pound of coal has a certain amount of potential energy. When the coal is burned, the potential energy is liberated and changed into kinetic energy in the form of heat. The kinetic energy of the heat changes water into steam, which thus has a certain amount of potential energy. The steam acting on the piston of an engine causes it to move through a certain space, thus overcoming a resistance, changing the potential energy of the steam into kinetic energy, and thus doing work.

Potential energy, *then, is the energy stored within a body that may be liberated and produce motion, thus generating kinetic energy and enabling work to be done.*

78. The principle of **conservation of energy** teaches that energy, like matter, can never be destroyed. If a clock is put in motion, the potential energy of the spring is changed into kinetic energy of motion, which turns the wheels, thus producing friction.

The friction produces heat, which dissipates into the surrounding air, but still the energy is not destroyed—it merely exists in another form.

79. Work of Acceleration and Retardation.—The theoretical amount of work that must be done in order to start a body from a state of rest and accelerate it until it reaches a given velocity is equal to the kinetic energy of the body at the given velocity. Likewise, the theoretical amount of work that must be done on a moving body to retard it and finally bring it to rest is equal to the kinetic energy the moving body possessed at the moment retardation began. The work that must be done in changing the velocity of a body is equal to the difference in the kinetic energies at the initial and final velocities. Since the motion

of all bodies is opposed by some resisting force or forces, the actual amount of work required to give a body the given velocity will be the sum of the work of acceleration and the work required to overcome the outside resisting forces.

EXAMPLE 1.—A body weighing 1,000 pounds is started from rest and is to attain a velocity of 88 feet per second in 2 minutes, passing over a distance of 5,280 feet in that time. If a constant force of 120 pounds must be exerted to overcome the frictional resistances, what work must be done?

SOLUTION.—According to this article, the work required to accelerate the body is $\frac{1,000 \times 88^2}{64.32} = 120,398$ foot-pounds. As a constant force of 120 pounds must act through a distance of 5,280 feet to overcome the frictional resistances, the work done in overcoming friction is $5,280 \times 120 = 633,600$ foot-pounds. Then, the total amount of work done is $120,398 + 633,600 = 753,998$ ft.-lb. Ans.

EXAMPLE 2.—What horsepower will be required to do the work calculated in the last example?

SOLUTION.—As the work is done in 2 minutes, the horsepower is $\frac{753,998}{2 \times 33,000} = 11.424$ H. P., nearly. Ans.

FORCE OF A BLOW.

80. The questions are frequently asked, with what force will a falling hammer strike, or with what force will a projectile fired from a gun strike an object? These questions cannot be answered directly, as they are based on a misconception. A moving body possesses kinetic energy, or ability to do work, which ability can only be expressed in foot-pounds, but not in pounds of force, since the work done by the hammer or projectile in coming to rest is not a manifestation of force, but of energy.

Work is the product of force into distance; hence, if the amount of work a body has done or is capable of doing is known, the force can be determined for each case if, by some means, it is possible to determine exactly the distance in which the work is done. This distance depends on various resistances, such as that due to moving the object struck, the resistance to penetration, friction, the resistance

to shearing or deformation of the body, etc. The distance through which these resisting forces act is generally indeterminate, and since the average of the resisting forces varies generally with the distance, this average resisting force is also indeterminate; hence, the force that, acting through a distance, will absorb all the kinetic energy of the hammer or projectile cannot be determined for the practical reasons given.

COMPOSITION AND RESOLUTION OF FORCES.

81. According to Art. **44,** in order that forces may be compared with one another, three conditions must be fulfilled. These conditions may all be represented by a line; hence, we may represent forces by lines. Thus, in Fig. 2, let A be the point of application of the force, let the length of the line $A B$ represent its magnitude, and let the arrowhead indicate the direction in which the force acts; then, the line $A B$ fulfils the three conditions and the force is fully represented.

FIG. 2.

COMPOSITION OF FORCES.

82. When two or more forces act upon a body at the same time along lines that meet in a common point, their combined effect on the body may be obtained by an application of the principle of the **triangle of forces.**

In Fig. 3 (a), let A and B be two forces having the magnitudes and directions represented by the two lines. To find

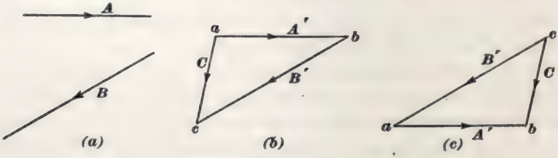


FIG. 3.

the effect due to the combined action of these two forces, draw in any convenient location a line parallel to either of the

lines representing the two forces, making it equal in length, to some scale, to the magnitude of the force. Mark upon it an arrowhead pointing in the same direction as the arrowhead on the line representing the force. Then, from one extremity of the line just drawn draw a line parallel to the line representing the second force and equal in length, to the same scale, to the magnitude of the second force, and mark the arrowhead upon it. It is essential that the second line be so drawn that when passing over the two lines with a pencil, commencing at the beginning of either force, the arrowheads will both point in the same direction in reference to the direction of motion of the pencil.

The second line may be drawn from either end of the first line, but its direction must be made to fulfil the above absolutely essential condition. Thus, in Fig. 3 (*b*), the line B' has been drawn from the right-hand extremity of A'. Starting at a with a pencil and moving toward b, the pencil will move in the direction in which the arrowhead on A' points. Passing over B' from b to c, the pencil will move in the direction in which the arrowhead on B' points; we thus see that the lines are drawn correctly in reference to each other.

In Fig. 3 (*c*), the line B' has been drawn from the left-hand extremity of A'. Starting at c and following up the lines, it is seen that in this case the arrowheads both point in the same direction relative to the direction of motion of the pencil, thus showing the lines to be located correctly.

The two lines having been drawn, complete the triangle by drawing the line C. This line, called the **resultant,** represents the combined effect of the two forces; it gives the direction along which the two forces will act when combined. The magnitude of their combined effect is found by measuring this line by the scale with which A' and B' were laid off. The resultant will always have a direction opposite in sense to that of the forces; that is, if we pass a pencil around the triangle in the direction in which the arrowheads on the lines A' and B' point, the arrowhead on C, representing the direction of action of the resultant, must point in a direction opposite to that in which the pencil moves.

83. In practice it is often desired to find not only the magnitude and direction, but also the *actual location of the resultant*, the magnitudes and lines of action of the two forces being known. (By location of the forces and resultant is meant the location of the lines along which the forces actually act.) This can readily be done by producing the lines giving the location of the forces until they meet and then drawing the triangle of forces with their point of intersection as the starting point.

ILLUSTRATION.—In Fig. 4 is shown a head-frame erected at the mouth of a deep, vertical shaft. The hoisting rope that leads to the

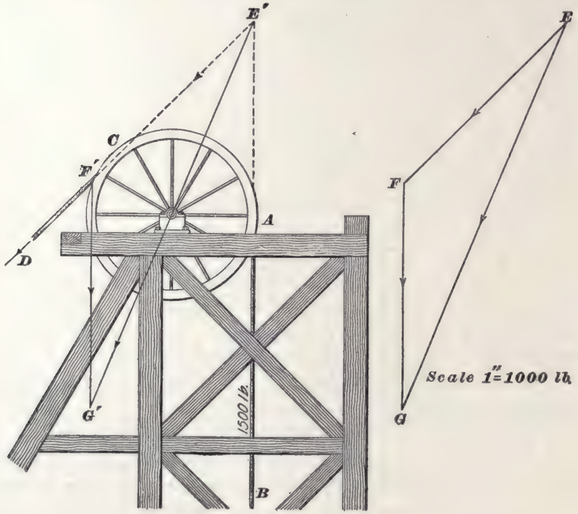


FIG. 4.

hoisting engine passes over a sheave at the top of the head-frame. A weight of 1,500 pounds hangs at the end of the rope. Neglecting the weight of the rope and sheave, what is the total pressure on the bearings of the sheave?

According to Art. **51**, the stress in both parts of the hoisting rope is 1,500 pounds. The part $A\,B$ of the rope supports the weight;

consequently, the force acting along $A B$ is downwards. The force acting along $C D$ is the pull exerted by the engine and is toward the engine. To find the resultant of these two forces, draw the triangle of forces. Choosing a scale of 1 inch per 1,000 pounds, draw the line $E F$ parallel to $C D$, making it $\frac{1500}{1000} = 1.5$ inches long. From F draw $F G$ parallel to $A B$ and $\frac{1500}{1000} = 1.5$ inches long. Join E and G; then $E G$ will be the resultant whose magnitude is measured by the same scale. Measuring $E G$, it is found to be 2.75 inches long. As each inch represents 1,000 pounds, the magnitude of the resultant is $2.75 \times 1{,}000 = 2{,}750$ pounds.

To find the actual location of the resultant in reference to $A B$ and $C D$, produce both lines, as shown in dotted lines, until they intersect at E'. Starting the triangle of forces at E', lay off $E' F' = 1.5$ inches. From F' draw $F' G' = 1.5$ inches and parallel to $A B$. Join $E' G'$. Upon measurement it is found to be 2,750 pounds. As inspection shows that the resultant passes through the center of the bearings, the total pressure on the bearings is 2,750 pounds. Ans.

RESOLUTION OF FORCES.

84. Since two forces can be combined to form a single resultant force, we may also treat a single force as if it were the resultant of two forces whose action upon a body will be the same as that of a single force. Thus, in Fig. 5, the force $O A$ may be resolved into two forces $O B$ and $B A$. If the force $O A$ acts upon a body moving or at rest upon a horizontal plane, and the resolved force $O B$ is vertical and $B A$ horizontal, $O B$, measured to the same scale as $O A$, is the magnitude of that part of $O A$ that pushes the body *downwards*, while $B A$ is the magnitude of that part of the force $O A$ that is exerted in pushing the body in a *horizontal* direction. $O B$ and $B A$ are called the **components** of the force $O A$, and when these components are vertical and horizontal, as in the present case, they are called the *vertical component* and the *horizontal component* of the force $O A$.

FIG. 5

85. It frequently happens that the position, magnitude, and direction of a certain force is known, and that it is desired to know the effect of the force in some direction other than that in which it acts. Thus, in Fig. 5, suppose that $O A$ represents, to some scale, the magnitude, direction, and line of action of a force acting upon a body at A, and that it is desired to know what effect $O A$ produces in the direction $B A$. Now $B A$, instead of being horizontal, as in the figure, may have any direction. To find the value of the component of $O A$ which acts in the direction $B A$, we use the following rule:

Rule 8.—*From one extremity of the line representing the given force, draw a line parallel to the direction in which it is desired that the component shall act; from the other extremity of the given force, draw a line perpendicular to the component first drawn, and intersecting it. The length of the component, measured from the point of intersection to the intersection of the component with the given force, will be the magnitude of the effect produced by the given force in the required direction.*

Thus, suppose $O A$, Fig. 5, represents a force acting upon a body resting upon a horizontal plane, and it is desired to know what *vertical pressure* $O A$ produces on the body. Here the desired direction is vertical; hence, from one extremity, as O, draw $O B$ parallel to the desired direction (vertical in this case), and from the other extremity draw $A B$ perpendicular to $O B$ and intersecting $O B$ at B. Then $O B$, when measured to the same scale as $O A$, will be the value of the vertical pressure produced by $O A$.

86. Tangential Pressure.—One of the most familiar applications of the principle of the resolution of forces occurring in steam engineering is the case of the connecting-rod and crank. When the piston is at the end of its stroke and the crankpin is in a line drawn through the center of the cylinder and crank-shaft, a position that is expressed by saying the engine is "on the center," the pressure of the connecting-rod on the crankpin acts directly against the

bearings of the shaft and there is no turning effect on the pin. After the pin leaves the center, the pressure exerted on it by the connecting-rod may be resolved into two components: One of these components acts in the direction of the center line PO of the crank (see Fig. 6) and merely

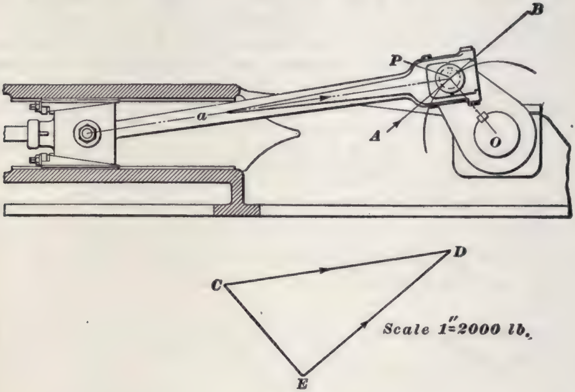


FIG. 6.

exerts a pressure on the bearings of the shaft; the other component acts along the line AB at right angles to the center line PO and *tangent* to the circle described by the crankpin. This is the force that tends to turn the crank and is called the **tangential pressure** on the crankpin.

When the crank is on the center, there is no tangential pressure on the pin and no tendency to turn the crank. The tangential pressure gradually increases as the pin leaves the center and becomes greatest at the point where the connecting-rod and crank are at right angles to each other; it then decreases until the next center is reached. At the position where the connecting-rod and crank are at right angles to each other, the tangential pressure on the pin is equal to the total pressure exerted on it by the connecting-rod, and there is no component in the direction of the center line PO of the crank.

EXAMPLE.—If a force of 3,000 pounds acts along the connecting-rod *a* in the direction of the arrow (see Fig. 6), what is the tangential pressure on the crankpin ?

SOLUTION.—As the tangential pressure is the pressure perpendicular to the crank, draw *A B* through the crankpin center at right angles to the center line of the crank; *A B* then represents the line along which the tangential pressure acts. Then, in any convenient location, draw *C D* parallel to the connecting-rod. Choosing a scale of 2,000 pounds = 1 inch, make th line *C D* 3,000 ÷ 2,000 = $1\frac{1}{2}$ inches long. From *D* draw an indefinite line *D E* parallel to *A B*, and draw *C E* perpendicular to *D E*. Now *E D*, measured to the scale adopted, will be the magnitude of the tangential pressure for the position of the crank and connecting-rod shown in the figure. Upon measurement, it is found to be 1.3 inches long. Then 1.3 × 2,000 = 2,600 lb., the tangential pressure. Ans.

87. When the total pressure on the piston rod of a steam engine is known, the force acting along the connecting-rod and the force acting upon the guides can be determined by the following application of the principle of the resolution of force:

Draw a line, as *A B*, Fig. 7, parallel to the line of motion of the crosshead *a*. Make its length, to some scale, equal in

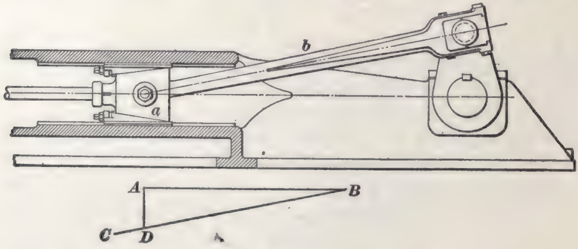


FIG. 7.

magnitude to the force impelling the piston. From one extremity of *A B* draw a line *B C* parallel to the center line of the connecting-rod *b*. From the other extremity of *A B* draw a line at a right angle to *A B*, producing it until it intersects the line *B C* at *D*. Then, *B D* represents the force acting along the connecting-rod and *A D* represents the force acting upon the guides.

FRICTION.

88. Friction is the resistance that a body meets with from the surface upon which it moves.

89. The ratio between the *resistance* to the motion of a body due to friction and the *perpendicular* pressure between the surfaces is called the **coefficient of friction.**

If a weight W, as in Fig. 8, rests upon a horizontal plane, and has a cord fastened to it passing over a pulley a, from

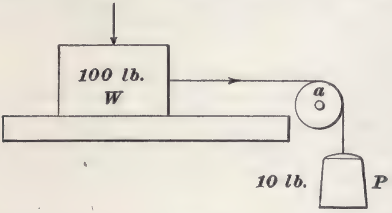


FIG. 8.

which a weight P is suspended, then, if P is just sufficient to start W, the ratio of P to W, or $\frac{P}{W}$, is the coefficient of friction between W and the surface it slides upon. The weight W is the perpendicular pressure, and P is the force necessary to overcome the resistance to the motion of W due to friction. If $W = 100$ pounds and $P = 10$ pounds, the coefficient of friction for this particular case would be $\frac{P}{W} = \frac{10}{100} = .1$.

90. Laws of Friction.—

1. *Friction is directly proportional to the perpendicular pressure between the two surfaces in contact.*

2. *Friction is independent of the extent of the surfaces in contact when the total perpendicular pressure remains the same.*

3. *Friction increases with the roughness of the surfaces.*

4. *Friction is greater between surfaces of the same material than between those of different materials.*

5. *Friction is greatest at the beginning of motion.*

6. *Friction is greater between soft bodies than between hard ones.*

7. *Rolling friction is less than sliding friction.*

8. *Friction is diminished by polishing or lubricating the surfaces.*

91. Law 1 shows why the friction is so much greater on journals after they begin to heat than before. The heat causes the journal to expand, thus increasing the pressure between the journal and its bearing, and, consequently, increasing the friction.

Law 2 states that, no matter how small may be the surface that presses against another, if the perpendicular pressure is the same the friction will be the same. Therefore, large surfaces are used where possible, not to reduce the friction, but to reduce the wear and diminish the liability of heating.

For instance, if the perpendicular pressure between a journal and its bearing is 10,000 pounds, and the coefficient of friction is .2, the amount of friction is $10,000 \times .2 = 2,000$ pounds. Suppose that the area receiving the pressure is 80 square inches, then the amount of friction for each square inch is $2,000 \div 80 = 25$ pounds.

If the area receiving the pressure had been 160 square inches, the friction would have been the same, that is, 2,000 pounds; but the friction per square inch would have been $2,000 \div 160 = 12\frac{1}{2}$ pounds, just one-half as much as before, and the wear and liability to heat would be one-half as great also.

92. The values of the coefficient of friction given in the following tables are average values determined by General Morin, many years ago. Under certain conditions the coefficient may be considerably less than is given in the tables; it varies greatly, but the variation depends on so many conditions and the numerous experiments that have been made have given such contradictory results that no definite rules have yet been derived for determining the

exact values under any condition. The student is, therefore, advised to use the values given in the tables, except where careful experiments have been made that give reliable values for the particular case under consideration. To find the force, in pounds, necessary to overcome friction, the coefficient taken from the table is multiplied by the perpendicular pressure, in pounds, on the surface considered. If the force acts at an angle to the surface, the perpendicular force can be found by resolving the given force into two components, one perpendicular and the other parallel to the surface.

TABLE I.

COEFFICIENTS OF FRICTION FOR PLANE SURFACES.

(Reduced from M. Morin's Data.)

Description of Surfaces in Contact.	State of the Surfaces.	Coefficient of Friction.
Wrought iron on cast iron.	Slightly greasy	.18
Wrought iron on bronze..	Slightly greasy	.18
Cast iron on cast iron....	Slightly greasy	.15
Cast iron on bronze......	Slightly greasy	.15
Bronze on bronze........	Dry	.20
Bronze on cast iron......	Dry	.22
Bronze on wrought iron..	Slightly greasy	.16
Cast iron, wrought iron, steel, and bronze sliding on one another or sliding on themselves.	Ordinary lubrication with lard, tallow, and oil	.07–.08
Cast iron, wrought iron, steel, and bronze sliding on one another or sliding on themselves.	Continuous lubrication	.05

TABLE II.

COEFFICIENTS OF FRICTION FOR JOURNAL FRICTION.

(Reduced from M. Morin's Data.)

Journals.	Bearings.	Lubricant.	Coefficient of Friction.	
			Ordinary Lubrication.	Continuous Lubrication.
Cast iron	Cast iron	Lard, olive oil, tallow	.07–.08	.03–.054
Cast iron	Bronze	Lard, olive oil, tallow	.07–.08	.03–.054
Wrought iron	Cast iron	Lard, olive oil, tallow	.07–.08	.03–.054
Wrought iron	Bronze	Lard, olive oil, tallow	.07–.08	.03–.054
Wrought iron	Lignum vitæ	Lard, olive oil	.11	
Bronze	Bronze	Oil	.10	
Bronze	Bronze	Lard	.09	

93. In the case of a weight sliding along a horizontal plane surface, the pressure is equal to the weight. When the surface is inclined, the weight acts vertically downwards, and the pressure perpendicular to the surface can be found by the principle of the resolution of forces. In many cases the pressure on the surfaces is due to the combined action of several forces that must be combined into one common resultant force.

94. The *work* that must be done in overcoming the resistance of friction depends on the distance through which the resistance is overcome. It may be calculated by the following rule:

Rule 9.—*Multiply the total pressure in pounds by the distance in feet and by the coefficient of friction.*

Let $W =$ work in foot-pounds;
$f =$ coefficient of friction;
$p =$ total pressure in pounds;
$d =$ distance in feet.

Then, $$W = p\,d\,f.$$

EXAMPLE.—The average perpendicular pressure on the guide of a steam engine due to the force impelling the piston is 2,500 pounds. The pressure due to the weight of the crosshead and connecting-rod is 400 pounds. The crosshead moves at the rate of 500 feet per minute; what horsepower is required to overcome the friction on the guides?

SOLUTION.—The total perpendicular pressure is $2{,}500 + 400 = 2{,}900$ pounds. Since the lubrication is usually intermittent, the coefficient of friction, for a brass slipper working on a cast-iron guide, may be taken as .08. The resistance being overcome through a distance of 500 feet each minute, the work done in overcoming friction is $2{,}900 \times 500 \times .08 = 116{,}000$ foot-pounds per minute. Then, the horsepower is $116{,}000 \div 33{,}000 = 3.52$ H. P., nearly. Ans.

95. In the case of a shaft rotating in a bearing, the distance through which friction is overcome each minute is found by multiplying the circumference of the journal by the number of revolutions per minute. For a shaft, or any other body rotating in a bearing, the force required to overcome friction, as calculated by multiplying the pressure by the coefficient of friction, is the force that must be applied *at the surface of the journal.*

96. Allowable Pressures.—It has been found by experience that when the pressure per unit of area exceeds a certain amount, the lubricant will be forced out and the bodies rubbing on each other will heat and, finally, seize.

The pressures that can safely be allowed on the bearings of steam engines, on guides, thrust bearings, crankpins, crosshead pins, etc. vary considerably, being dependent to a large extent on the character of the workmanship, the degree of finish, the variation of the pressure, the character of the lubrication, and the quality of the lubricant. With fair workmanship, the following pressures per square inch represent average practice in steam-engine work:

TABLE III.

PRESSURES PER SQUARE INCH ALLOWABLE IN STEAM-ENGINE WORK.

Engine Bearing.	Slow-Speed Stationary Engines. Pounds.	High-Speed Stationary and Marine Engines. Pounds.
Crankpins, iron......	800	400 to 600
Crankpins, steel......	1,200	
Wristpin.............	1,200	600 to 800
Main bearings.......	200	200
Guides	100	100
Thrust bearings......		60

97. For crankpins, wristpins, and guides, the allowable pressures given represent the pressures corresponding to the maximum load, which in the case of a wristpin and crankpin occurs when the crank, connecting-rod, and piston rod are in a straight line, and in the case of guides, when the connecting-rod and crank are at right angles to each other. In the case of pins and journals, the area to be considered in calculating the pressure on the bearing is the **projected area,** which is found by multiplying the length of the journal by its diameter.

EXAMPLES FOR PRACTICE.

1. A body weighs 90 pounds; what is its mass? Ans. 2.799, nearly.

2. What force will be required to accelerate a body at the rate of 2 feet per second, the body weighing 450 pounds, and the frictional resistances being equal to 10 per cent. of the weight of the body?
Ans. 72.99 lb., nearly.

3. What work is done in raising 950 pounds 17 feet?
Ans. 16,150 ft.-lb.

4. If an engine does 205,000 foot-pounds of work per minute, what is its horsepower? Ans. 6.21 H. P., nearly.

5. What is the kinetic energy of a shell fired from a cannon with a velocity of 1,800 feet per second, the shell weighing 1,000 pounds?

Ans. 50,373,135 ft.-lb., nearly.

6. Taking the coefficient of friction at .15, what horsepower will be required to pull 100 pounds at a uniform speed of 5 feet per second along a level surface?

Ans. .136 H. P.

CENTER OF GRAVITY.

98. *The center of gravity of a body is that point at which the body may be balanced, or it is the point at which the whole weight of a body may be considered as concentrated.*

This point is not always *in* the body; in the case of a horseshoe or a ring it lies outside of the substance of, but within the space enclosed by, the body.

In a moving body, the line described by its center of gravity is always taken as the path of the body. In finding the distance that a body has moved, the distance that the center of gravity has moved is taken.

The definition of the center of gravity of a body may be applied to a system of bodies if they are considered as being connected at their centers of gravity.

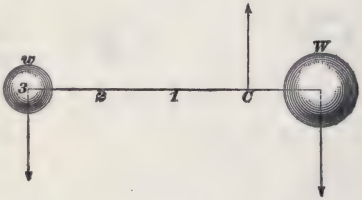


FIG. 9.

If w and W, Fig. 9, are two bodies of known weight, their center of gravity will be at C. This point C may be readily determined as follows:

Rule 10.—*The distance of the common center of gravity from the center of gravity of the large weight is equal to the weight of the smaller body multiplied by the distance between the centers of gravity of the two bodies, and this product divided by the sum of the weights of the two bodies.*

EXAMPLE.—In Fig. 9, $w = 10$ pounds, $W = 30$ pounds, and the distance between their centers of gravity is 36 inches; where is the center of gravity of both bodies situated?

SOLUTION.—Applying the rule, $10 \times 36 = 360$. $10 + 30 = 40$. $360 \div 40 = 9$ in., distance of center of gravity from center of large weight. Ans.

99. It is now very easy to extend this principle to find the center of gravity of any number of bodies, when their weights and the distances apart of their centers of gravity are known, by the following rule:

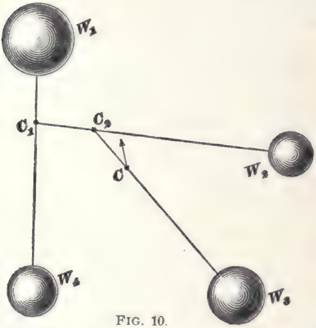


FIG. 10.

Rule 11.—*Find the center of gravity of two of the bodies, as W_1 and W_4 in Fig. 10. Assume that the weight of both bodies is concentrated at C_1, and find the center of gravity of this combined weight at C_1 and the weight of W_2. Let it be at C_2; then find the center of gravity of the combined weights of W_1, W_4, W_2 (concentrated at C_2), and W_3. Let it be at C; then C will be the center of gravity of the four bodies.*

100. To find the center of gravity of **any parallelogram:**

Rule 12.—*Draw the two diagonals, Fig. 11, and their point of intersection C will be the center of gravity.*

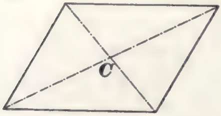


FIG. 11.

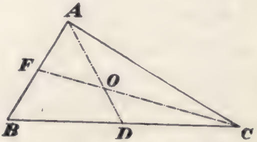


FIG. 12.

101. To find the center of gravity of a **triangle,** as ABC, Fig. 12:

Rule 13.—*From any vertex, as A, draw a line to the middle point D of the opposite side B C. From one of the other vertexes, as C, draw a line to F, the middle point of the opposite side A B; the point of intersection O of these two lines is the center of gravity.*

It is also true that the distance $D\,O = \frac{1}{3}\,D\,A$ and that $F\,O = \frac{1}{3}\,F\,C$; the center of gravity could have been found by drawing from any vertex a line to the middle point of the opposite side and measuring back from that side $\frac{1}{3}$ of the length of the line.

The center of gravity of **any regular plane figure** is the same as the center of the inscribed or circumscribed circle.

102. To find the center of gravity of **any irregular plane figure,** but of uniform thickness throughout, divide

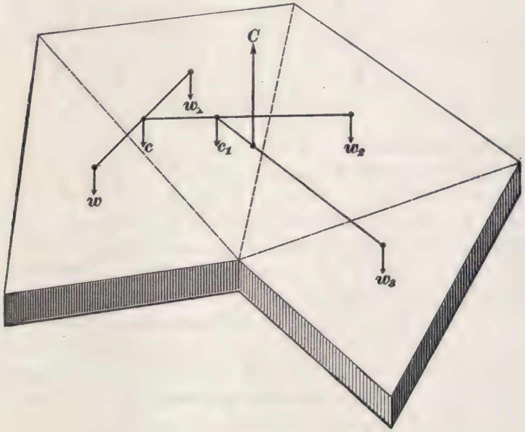


FIG. 13.

one of the parallel surfaces into triangles, parallelograms, circles, ellipses, etc., according to the shape of the figure; find the area and center of gravity of each part separately, and combine the centers of gravity thus found in the same

manner as in rule **11**, in this case, however, dealing with the *area* of each part instead of its weight. See Fig. 13.

103. Center of Gravity of a Solid.—In a body free to move, the center of gravity will lie in a vertical plumb-line drawn through the point of support. Therefore, to

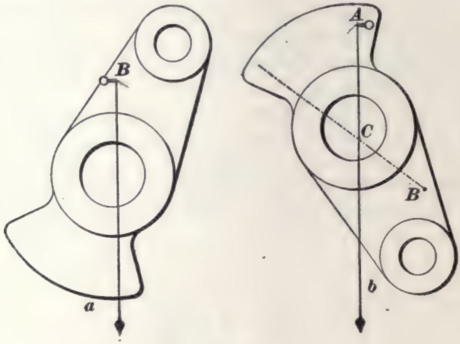


FIG. 14.

find the position of the center of gravity of an irregular solid, as the crank, Fig. 14, suspend it at some point, as B, so that it will move freely. Drop a plumb-line from the point of suspension, and mark its direction. Suspend the body at another point, as A, and repeat the process. The intersection C of the two lines will be directly over the center of gravity.

It is often desired to find the horizontal distance of the center of gravity from a given point of the body. In many cases this can readily be done by balancing the body on a knife edge, and then measuring, horizontally, the distance between the knife edge and the given point.

Since the position of the center of gravity depends wholly on the shape and weight of a body, it may be without the body, as in the case of a circular ring, whose center of gravity is the same as the center of the circumference of the ring.

By considering the symmetry of form, the position of the

center of gravity of homogeneous solids may often be determined without analysis, or it may be limited to a certain plane, line, or point. Thus, the center of gravity of a sphere, or any other regular body, is situated at its center; of a cylinder, in the middle of its axis; of a thin plate having the form of a circle or regular polygon, in the center of the figure; of a straight wire of uniform cross-section, in the middle of its length.

EXAMPLES FOR PRACTICE.

1. A spherical shell has a wrought-iron handle attached to it. The shell is 10 inches in diameter and weighs 20 pounds. The handle is $1\frac{1}{2}$ inches in diameter, and the distance from the center of the shell to the end of the handle is 4 feet. Where is the center of gravity? Take the weight of a cubic inch of wrought iron as .278 pound.

Ans. 13.612 in. from center of shell.

2. The distance between the centers of two bodies is 51 inches. The weights of the bodies being 20 and 73 pounds, where is the center of gravity? Ans. 10.968 in. from the center of the larger weight.

3. Weights of 5, 9, and 12 pounds lie in one straight line in the order named. Distance from the 5-pound weight to the 9-pound weight is 22 inches, and from the 9-pound weight to the 12-pound weight is 18 inches. Where is the center of gravity?

Ans. 13.923 in. from 12-lb. weight.

CENTRIFUGAL FORCE.

104. If a body is fastened to a string and whirled, so as to give it a circular motion, there will be a pull on the string that will be greater or less, according as the velocity increases or decreases. The cause of this pull on the string will now be explained.

Suppose that the body is revolved horizontally, so that the action of gravity upon it will always be the same. According to the first law of motion, a body put in motion tends to move in a straight line unless acted upon by some other force, causing a change in the direction. When the body moves in a circle, the force that causes it to move

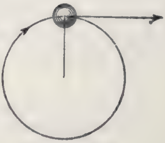


FIG. 15.

in a circle instead of in a straight line is exactly equal to the tension of the string. If the string were cut, the pulling force that drew it away from the straight line would be removed, and the body would then "fly off at a tangent"; that is, it would move in a straight line tangent to the circle, as shown in Fig. 15.

Since, according to the third law of motion, every action has an equal and opposite reaction, we call the force that acts as an equal and opposite force to the pull of the string the **centrifugal force,** and it acts *away* from the center of motion.

105. The other force, or tension, of the string is called the **centripetal force,** and it acts *toward* the center of motion. It is evident that these two forces, acting in opposite directions, tend to pull the string apart, and, if the velocity be increased sufficiently, the string will break. It is also evident that no body can revolve without generating centrifugal force.

106. To Find the Centrifugal Force of Any Revolving Body.—The value of the centrifugal force, expressed in pounds, of any revolving body is calculated by the following rule:

Rule 14.—*The centrifugal force equals the continued product of .00034, the weight of the body in pounds, the radius in feet* (*taken as the distance between the center of gravity of the body and the center about which it revolves*), *and the square of the number of revolutions per minute.*

Let F = centrifugal force in pounds;
W = weight of revolving body in pounds;
R = radius in feet of circle described by center of gravity of revolving body;
N = revolutions per minute of revolving body.

Then, $$F = .00034\,W R N^2.$$

EXAMPLE.—What is the tension in the string of Fig. 15, if the ball weighs 2 pounds and is revolved around at the rate of 500 revolutions per minute? The string is 2 feet long.

SOLUTION.—Applying the rule just given, we get

$$F = .00034 \times 2 \times 2 \times 500^2 = 340 \text{ lb. Ans.}$$

107. In flywheels, belt wheels, and pulleys the centrifugal force tends to tear the rim asunder; this tendency is resisted by the tenacity of the material of which the wheel is composed. Since the centrifugal force increases as the square of the number of revolutions, it will be seen that an apparently slight increase in the number of revolutions per minute may be sufficient to burst the wheel.

108. For solid cast-iron wheels and for built-up wheels of cast iron where the strength of the joint is equal to the strength of the rim, the greatest number of revolutions per minute that practice has indicated to be safe may be found by the following rule:

Rule 15.—*Divide 1,930 by the diameter of the wheel.*

Or,
$$N = \frac{1,930}{d};$$

where d = diameter of the wheel in feet;
N = number of revolutions per minute.

EXAMPLE.—What is the maximum number of revolutions allowable for a cast-iron flywheel 27 feet 6 inches in diameter ?

SOLUTION.—Applying rule **15**, we get

$$N = \frac{1,930}{27.5} = 70 \text{ rev. per min. Ans.}$$

EQUILIBRIUM.

109. When a body is at rest, all the forces that act upon it balance one another and are said to be in **equilibrium.** The most important force to be considered is the attraction of the earth, which acts upon every particle of a body. There are three states of equilibrium: *stable*, *unstable*, and *neutral.*

110. A body is in **stable equilibrium** when, if slightly rotated about its support in such manner as to change the

elevation of its center of gravity, it tends to return to its position of rest. Examples of bodies in a state of stable equilibrium are a cube resting on one of its sides, a cone resting on its base, a pendulum, etc. A body can only be in stable equilibrium when a rotation about its support raises the center of gravity.

111. A body is in **unstable equilibrium** when a rotation about its support, so as to change the elevation of its center of gravity, tends to make it fall farther from its position of rest. Examples of bodies in unstable equilibrium are a cube balanced on one of its edges, a cone standing upon its point, an egg balanced upon its end, etc. When a body is in unstable equilibrium, any rotation, no matter how slight, tends to lower its center of gravity.

112. A body is in **neutral equilibrium** when a rotation about its support does not change the elevation of its center of gravity. Examples of bodies in neutral equilibrium are a sphere of uniform density and a cone resting on its side.

113. A vertical line drawn through the center of gravity of a body is called the line of direction. So long as the line of direction falls within the base, the body will stand; when the line of direction falls outside of the base, the body will fall.

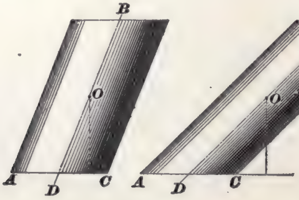


FIG. 16.

Let $A\,C\,B$, Fig. 16, be a cylinder whose base is oblique to the center line $B\,O\,D$, and let O be the center of gravity of

the cylinder. So long as the vertical line drawn through O falls between A and C, the cylinder will stand, but the instant it falls without the base, the cylinder will fall.

114. The center of gravity of a body has a tendency to always seek the lowest possible position.

MACHINE ELEMENTS.

THE LEVER, WHEEL, AND AXLE.

FUNDAMENTAL PRINCIPLES.

1. A **lever** is a bar capable of being turned about a pivot, or point, as in Figs. 1, 2, and 3.

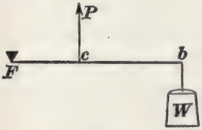


FIG. 1.

FIG. 2.

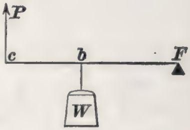


FIG. 3.

The object W to be lifted is called the **weight**; the force is represented by P*; and the point, or pivot F, is called the **fulcrum.**

* The force applied to a lever, screw, wheel and axle, or any similar machine element in order to raise a given weight, was formerly called the *power*, and the arm of a lever to which the force is applied was called the *power arm*; at present, however, the term power is universally used to represent the rate at which work is done, and, hence, its application to those cases where a simple force is meant often leads to a serious confusion of ideas regarding the relation between force, work, and power. To prevent this confusion, we will use the word power only in accordance with the definition given.

2. That part of the lever $F\,b$ between the fulcrum and the weight is called the **weight arm,** and the part $F\,c$ between the fulcrum and the force is called the **force arm.**

In order that the lever will be in equilibrium (balance), *the force multiplied by the force arm must equal the weight multiplied by the weight arm;* that is, $P \times F\ c$ must equal $W \times F\ b$.

3. If F is taken as the center of a circle, and arcs are described through b and c, it will be seen that if the weight arm is moved through a certain angle, the force arm will move through the same angle. Since in the same or equal angles the lengths of the arcs are proportional to the radii with which they were described, it is seen that the force arm is proportional to the distance through which the force acts, and the weight arm is proportional to the distance through which the weight moves. Hence, instead of writing $P \times F\,c = W \times F\,b$, we might have written $P \times$ (distance through which P acts) $= W \times$ (distance through which W moves). This is the general law of all machines, and can be applied to any mechanism from the simple lever up to the most complicated arrangement. When stated in the form of a rule it is as follows:

Rule 1.—*The force multiplied by the distance through which it acts equals the weight multiplied by the distance through which it moves.*

4. In the above rule, it will be noticed that there are four requirements necessary for a complete knowledge of the lever, viz.: the force, the weight, the force arm, or distance through which the force acts, and the weight arm, or distance through which the weight moves. If any three are given, the fourth may be found by letting x represent the requirement that is to be found, and multiplying the force by the force arm and the weight by the weight arm; then, dividing the product of the two known numbers by the number by which x is multiplied, the result will be the requirement that is to be found.

EXAMPLE.—If the weight arm of a lever is 6 inches long and the force arm is 4 feet long, how great a weight can be raised by a force of 20 pounds at the end of the force arm?

SOLUTION.—In this example, the weight is unknown; hence, representing it by x, we have, after reducing the 4 feet to inches, $20 \times 48 = 960$ = force multiplied by the force arm, and $x \times 6$ = weight multiplied by the weight arm. Dividing the 960 by 6, the result is 160 pounds, the weight. Ans.

5. If the distance through which the force acted or the weight had moved had been given instead of the force arm or weight arm, and it were required to find the force or weight, the process would have been exactly the same, using the given distance instead of the force arm or weight arm.

EXAMPLE.—If, in the above example, the weight had moved $2\frac{1}{2}$ inches, through what distance would the force have acted?

SOLUTION.—In this example, the distance through which the force acts is required. Let x represent the distance. Then, $20 \times x$ = distance multiplied by force, and $2\frac{1}{2} \times 160 = 400$ = distance multiplied by the weight. Hence, $x = \frac{400}{20} = 20$ inches = distance through which the force arm moves. Ans.

The ratio between the weight and the force is $160 \div 20 = 8$. The ratio between the distance through which the weight moves and the distance through which the force acts is $2\frac{1}{2} \div 20 = \frac{1}{8}$. This shows that while a force of 1 pound can raise a weight of 8 pounds, the 1-pound weight must move through 8 times the distance that the 8-pound weight does. It will also be noticed that the ratio of the lengths of the two arms of the lever is also 8, since $48 \div 6 = 8$.

6. The law that governs the straight lever also governs the bent lever, but care must be taken to determine the true lengths of the lever arms, which are, in every case, *the perpendicular distances from the fulcrum to the line of direction of the weight or force.*

Thus, in Figs. 4, 5, 6, and 7, $F c$ in each case represents the force arm and $F b$ the weight arm.

7. A **compound lever** is a series of single levers arranged in such a manner that when a force is applied to the first it is communicated to the second, and from that to the third, and so on.

Fig. 8 shows a compound lever. It will be seen that when a force is applied to the first lever at *P* it will be communicated to the second lever at *P*, from this to the third lever at *P*, and thus raise the weight *W*.

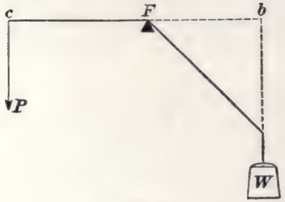


FIG. 4.

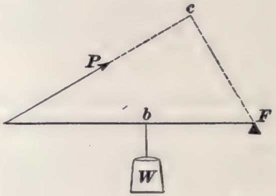


FIG. 5.

The weight that the force applied to the first lever could raise acts as the force of the second, and the weight that this force could raise through the second lever acts as the force

FIG. 6.

FIG. 7.

of the third lever, and so on, no matter how many single levers make up the compound lever.

In this case, as in every other, the force multiplied by the

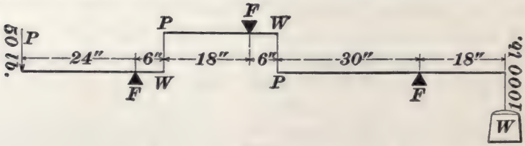


FIG. 8.

distance through which it acts equals the weight multiplied by the distance through which it moves.

Hence, if we move the P end of the lever, say 4 inches, and the end carrying the weight W moves $\frac{1}{5}$ inch, we know that the ratio between P and W is the same as the ratio between $\frac{1}{5}$ and 4; that is, 1 to 20, and, hence, that 10 pounds at P will balance 200 pounds at W, without measuring the lengths of the different lever arms. If the lengths of the lever arms are known, the ratio between P and W may be readily obtained from the following rule:

Rule 2.—*The continued product of the force and each force arm equals the continued product of the weight and each weight arm.*

EXAMPLE.—If, in Fig. 8, $P F = 24$ inches, 18 inches, and 30 inches, respectively, and $W F = 6$ inches, 6 inches, and 18 inches, respectively, how great a force at P will it require to raise 1,000 pounds at W? What is the ratio between W and P?

SOLUTION. — Let x represent the force; then, $x \times 24 \times 18 \times 30 = 12{,}960 \times x =$ continued product of the force and each force arm. $1{,}000 \times 6 \times 6 \times 18 = 648{,}000 =$ continued product of the weight and each weight arm; and, since $12{,}960 \times x = 648{,}000$,

$$x = \frac{648{,}000}{12{,}960} = 50 \text{ lb.} = \text{the force.} \quad \text{Ans.}$$

$$1{,}000 \div 50 = 20 = \text{ratio between } W \text{ and } P. \quad \text{Ans.}$$

8. The **wheel and axle** consists of *two cylinders of different diameters rigidly connected*, so that they turn

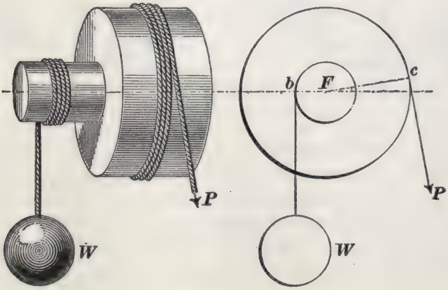


FIG. 9.

together about a common axis, as in Fig. 9. Then, as before, $P \times$ distance through which it acts $= W \times$ distance through which it moves; and, since these distances are proportional to the radii of the force cylinder and weight cylinder, $P \times Fc = W \times Fb$.

It is not necessary that an entire wheel be used; an arm, projection, radius, or anything that the force causes to revolve in a circle, may be considered as the wheel. Consequently, if it is desired to hoist a weight with a windlass, Fig. 10, the force is applied to the handle of the crank, and the distance between the center line of the crank-handle and the axis of the drum corresponds to the radius of the wheel.

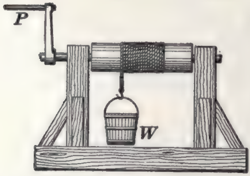


FIG. 10.

EXAMPLE.—If the distance between the center line of the handle and the axis of the drum in Fig. 10 is 18 inches and the diameter of the drum is 6 inches, what force will be required at P to raise a load of 300 pounds?

SOLUTION.— $P \times (18 \times 2) = 300 \times 6$, or $P = 50$. Ans.

EXAMPLES FOR PRACTICE.

1. The lever of a safety valve is of the form shown in Fig. 1, where the force is applied at a point between the fulcrum and the weight lifted. If the distance from the fulcrum to the valve is $5\frac{1}{2}$ inches and from the fulcrum to the weight is 42 inches, what total force is necessary to raise the valve, the weight being 78 pounds and the weight of valve and lever being neglected? Ans. 595.64 lb.

2. If, in Fig. 8, $PF = 10$ inches, 12 inches, 14 inches, and 16 inches, respectively, and $WF = 2$ inches, 3 inches, 4 inches, and 5 inches, respectively, (*a*) how great a weight can a force of 20 pounds raise? (*b*) What is the ratio between W and P? (*c*) If P moves 4 inches, how far will W move?

Ans. $\begin{cases} (a) & 4{,}480 \text{ lb.} \\ (b) & 224. \\ (c) & \frac{1}{56} \text{ in.} \end{cases}$

3. A windlass is used to hoist a weight. If the diameter of the drum on which the rope winds is 4 inches, and the distance from the center of the handle to the axis of the drum is 14 inches, how great a weight can a force of 32 pounds applied to the handle raise?

Ans. 224 lb.

PULLEYS.

9. Pulleys for the transmission of power by belts may be divided into two principal classes: (1) The *solid pulley* shown in Fig. 11, in which the hub, arms, and rim are one entire casting. (2) The *split pulley* shown in Fig. 12, which is cast in halves.

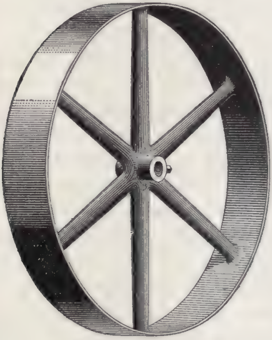


FIG. 11.

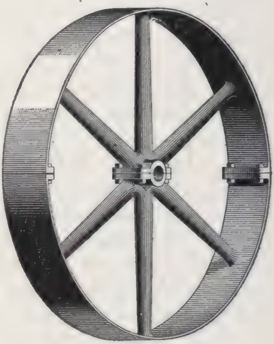


FIG. 12.

The latter style of pulley is more readily placed on and removed from the shaft than the solid pulley. Pulleys are generally cast in halves or parts when they are more than 6 feet in diameter; this is done on account of the shrinkage strain in large pulley castings, which renders them liable to crack as a result of the unequal cooling of the metal.

10. Of late years, wooden pulleys have come into extensive use. They are built of segments securely glued together, maple being the wood used. Wooden split pulleys can be procured that are fitted with removable bushings, thus allowing the same pulley to be readily adapted to various diameters of shafting. They are somewhat lighter than cast-iron pulleys.

11. Crowning Pulley Faces.—In Fig. 13, suppose the shafts *a* and *b* to be parallel. Let the pulley on the shaft *a* be cone-shaped, as shown. The right-hand side of the belt will be pulled ahead more rapidly than the left-hand side, because of the greater diameter and, consequently, greater speed of the right-hand end of the pulley. It has been observed that in this case the belt will leave its normal position, which is indicated by the dotted lines, and climb toward the part of the pulley that has the largest diameter, as shown in the illustration. This tendency of the belt to climb toward the high side is taken advantage of to make the belt stay on a pulley. Suppose the pulley on the shaft *a* is replaced with one formed of two equal cones, with the large diameter in the center. Then, each side of the belt will tend to climb toward the highest point, and the consequence is that the belt will stay in the center of the pulley.

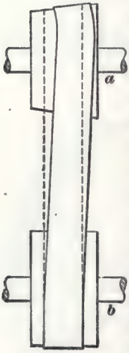


FIG. 13.

A pulley with its surface formed in this way is said to be **crowned.** The surface need not necessarily represent the frustums of two cones; it may simply be curved, as shown in Fig. 14. It is only required that the pulley be larger in diameter in the center.

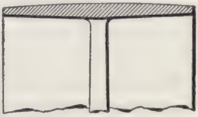


FIG. 14.

As to the proper amount of crowning necessary, the practice of the makers of pulleys differs considerably; usually, though, it is from $\frac{3}{16}$ to $\frac{1}{2}$ inch per foot of width of the pulley face.

12. Balancing Pulleys.—All pulleys that rotate at high speeds should be balanced. If they are not, the centrifugal force that is generated by the rotation of the pulley, on account of the center of gravity being outside the axis of rotation, will cause vibration; hence, pulleys are usually balanced in the manner shown

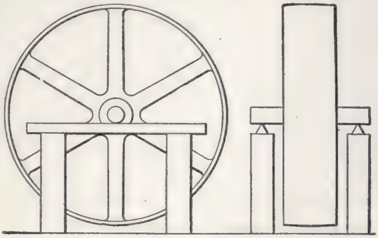


FIG. 15.

in Fig. 15. A closely fitting arbor, which is simply a truly cylindrical piece of iron or steel, is driven into the bore of the pulley, which is then placed on the so-called "balancing ways." These are two planed iron or steel bars, preferably planed to a knife edge. These bars are placed on convenient supports far enough apart to allow the pulley to go between them. The bars should be carefully leveled with a spirit level, so that both bars are in the same horizontal plane. When the arbor rests on the ways and the pulley is slightly rotated, the latter will quickly come to rest with the heavy part downwards. Now, either some metal must be removed from the heavy part or some weight added to the light part. For small pulleys, a convenient substance to use for finding the proper location and weight of the counterweight is ordinary glazier's putty. By repeated trials the proper weight of the counterweight is found, and then a mass of metal of convenient shape equal in weight to the putty is fastened to the light part of the pulley in whatever way is safe and convenient. The proper weight and location of the counterweight will have been obtained when the pulley will be at rest on the balancing ways in any position in which it is put. In other words, the pulley is balanced when it is in neutral equilibrium. This balance is called a **standing balance.** The method just explained answers very well for pulleys that are narrow in comparison

to their diameter. When pulleys with a wide face are run at a high speed, it is often found that serious vibrations are set up even when they are in perfect standing balance, thus showing that they do not possess the so-called **running balance.** No method has yet been found that will insure a running balance at all speeds, nor has any method become known by which it can be discovered directly where to apply the counterweight. The usual remedy for pulleys not possessing a running balance is to turn carefully the inside of the rim to run true with the outside of the pulley. Absence of running balance is, in all cases, due to an unequal distribution of metal in reference to the axis of the shaft, this unequal distribution being due to lack of homogeneity of the metal, to poor foundry work, or to poor lathe work.

13. Pulleys should run true in order that the strain, or tension, of the belt will be equal at all parts of the revolution, thus making the transmitting power equal. The smoother the surface of a pulley, the greater is its driving power.

The transmitting power of a pulley can be increased by covering its face with a leather or rubber band; this increases the driving power about one-quarter.

14. Relation Between Speeds of Drivers and Driven Pulleys.—The pulley that imparts motion to the belt is called the **driver**; that which receives the motion is called the **driven.**

The revolutions of any two pulleys over which a belt is run vary in an inverse proportion to their diameters; consequently, if a pulley 20 inches in diameter is driven by one 10 inches in diameter, the 20-inch pulley will make 1 revolution while the 10-inch pulley makes 2 revolutions, or they are in the ratio of 2 to 1. From this fact, the following formulas have been deduced:

Let $D =$ diameter of the driver;
$d =$ diameter of the driven;
$N =$ number of revolutions of the driver;
$n =$ number of revolutions of the driven.

NOTE.—The words revolutions per minute are frequently abbreviated to R. P. M.

15. To find the diameter of the driving pulley when the diameter of the driven pulley and the number of revolutions per minute of each is given:

Rule 3.—*The diameter of the driving pulley equals the product of the diameter and the number of revolutions of the driven pulley divided by the number of revolutions of the driving pulley.*

Or,
$$D = \frac{d\,n}{N}.$$

EXAMPLE.—The driving pulley makes 100 revolutions per minute, the driven pulley makes 75 revolutions per minute and is 18 inches in diameter; what is the diameter of the driving pulley?

SOLUTION.—Applying the rule just given and substituting, we have $D = \frac{18 \times 75}{100} = 13\frac{1}{2}$ in. Ans.

16. The diameter and number of revolutions per minute of the driving pulley being given, to find the diameter of the driven pulley, which must make a given number of revolutions per minute:

Rule 4.—*The diameter of the driven pulley equals the product of the diameter and the number of revolutions of the driving pulley divided by the number of revolutions of the driven pulley.*

Or,
$$d = \frac{D\,N}{n}.$$

EXAMPLE.—The diameter of the driving pulley is $13\frac{1}{2}$ inches and it makes 100 revolutions per minute; what must be the diameter of the driven pulley to make 75 revolutions per minute?

SOLUTION.—Applying rule 4 we have $d = \frac{13\frac{1}{2} \times 100}{75} = 18$ in. Ans.

17. To find the number of revolutions per minute of the driven pulley, its diameter and the diameter and the number of revolutions per minute of the driving pulley being given:

Rule 5.—*The number of revolutions of the driven pulley equals the product of the diameter and the number of revolutions of the driver divided by the diameter of the driven pulley.*

Or, $$n = \frac{D N}{d}.$$

EXAMPLE.—The driving pulley is 13½ inches in diameter and makes 100 revolutions per minute; how many revolutions will the driven pulley make in 1 minute if it is 18 inches in diameter?

SOLUTION.—Formula, $$n = \frac{D N}{d}.$$

Substituting, we have $n = \frac{13\frac{1}{2} \times 100}{18} = 75$ R. P. M. Ans.

18. To find the number of revolutions per minute of the driving pulley, its diameter and the diameter and the number of revolutions per minute of the driven pulley being given:

Rule 6.—*The number of revolutions of the driving pulley equals the product of the diameter and the number of revolutions of the driven pulley divided by the diameter of the driving pulley.*

Or, $$N = \frac{d n}{D}.$$

EXAMPLE.—The driven pulley is 18 inches in diameter and makes 75 revolutions per minute; how many revolutions will the driving pulley make in 1 minute if it is 13½ inches in diameter?

SOLUTION.—Formula, $$N = \frac{d n}{D}.$$

Substituting, we have $N = \frac{18 \times 75}{13\frac{1}{2}} = 100$ R. P. M. Ans.

BELTS.

19. A **belt** is a flexible connecting band that drives a pulley by its frictional resistance to slipping at the surface of the pulley. Belts are most commonly made of leather, cotton, or rubber, and are united in long lengths by cementing, riveting, or lacing.

20. Leather belts are made single and double. A single belt is one composed of a single thickness of leather; a double belt is one composed of two thicknesses of leather cemented and riveted together the whole length of the belt.

21. Cotton belts are in use to some extent, as are also belts made of a number of layers of duck sewed together and impregnated with a preparation rendering them waterproof. These belts, in accordance with the number of layers or "plys," are called two-ply, three-ply, etc. Four-ply cotton and duck belting is about equal to single leather belting, and eight-ply to double leather belting.

22. Rubber belts are especially adapted for use in damp or wet places; they will endure a great degree of heat or cold without injury, are quite durable, and are claimed to be less liable to slip than leather belts.

CALCULATIONS FOR BELTS.

23. To Find the Length of a Belt.—In practice, the necessary length for a belt to pass around pulleys that are already in their position on a shaft is usually obtained by passing a tape line around the pulleys, the stretch of the tape line being allowed as that necessary for the belt. The lengths of open-running belts for pulleys not in position can be obtained approximately as follows:

Rule 7.—*The length of a belt for open-running pulleys equals $3\frac{1}{4}$ times one-half the sum of the diameters of the pulleys plus 2 times the distance between the centers of the shaft.*

Let D = diameter of one pulley in inches;
d = diameter of the other pulley in inches;
L = distance between the centers of the shafts in inches;
B = length of the belt in inches.

Then,
$$B = 3\tfrac{1}{4}\left(\frac{D + d}{2}\right) + 2\,L.$$

EXAMPLE.—The distance between the centers of two shafts is 9 feet 7 inches; the diameter of the large pulley is 36 inches and the diameter of the small one is 14 inches; what is the necessary length of the belt?

SOLUTION.—Applying the rule just given and substituting the values given, we have, since 9 feet 7 inches = 115 inches,

$$B = 3\tfrac{1}{4}\left(\frac{36+14}{2}\right) + 2 \times 115 = 311\tfrac{1}{4} \text{ in., or } 25 \text{ ft. } 11\tfrac{1}{4} \text{ in. Ans.}$$

The length of crossed belts cannot be determined by any simple calculation, it being a rather difficult mathematical problem.

24. To Find the Width of Belts.—A belt should be wide enough to bear safely and for a reasonable length of time the greatest tension that will be put upon it. This will be the tension of the driving side. The safe tension for single belts may be taken as 60 pounds per inch of width; single belts average $\frac{3}{16}$ inch in thickness. The tension on the driving side, however, does not represent the force tending to turn the pulley. The force tending to turn the pulley, or the **effective pull,** is the difference in tension between the driving side and the slack side of the belt. The tension on the driving side depends on three factors: the *effective pull* of the belt, the *coefficient of friction* between the belt and pulley, and the size of the *arc of contact* of the belt on the smaller pulley.

25. The effective pull that may be allowed per inch of width for single leather belts with different arcs of contact (the arc in which the belt touches the smaller pulley), is given in Table I.

26. To Find the Arc of Contact.—The arc of contact in degrees, or as a fraction of the circumference, can be determined, practically, as follows: Stretch a string over the two pulleys to represent the belt. Then, take another string, wrap it around the small pulley and cut it off so that the ends meet. This represents the circumference of the small pulley. Now take a third string, hold one end at the

TABLE I.

ALLOWABLE BELT PULLS.

Arc Covered by Belt.		Allowable Effective Pull Per Inch of Width in Pounds.
Degrees.	Fraction of Circumference.	
90	.250	23.0
$112\frac{1}{2}$	.312	27.4
120	.333	28.8
135	.375	31.3
150	.417	33.8
$157\frac{1}{2}$	.437	34.9
180 or over	.500	38.1

beginning of the arc of contact, as shown by the string stretched around both pulleys, wrap it around the smaller pulley, and cut it off at the end of the arc of contact. The length of this last string represents the length of the arc of contact. We now have the proportion: *the length of the string representing the circumference* : *the length of the string representing the arc of contact* :: *360* (*the number of degrees in a circle*) : *the number of degrees in the arc of contact.* Whence, the number of degrees in the arc of contact equals the quotient obtained by dividing the product of the length of the arc of contact and 360 by the circumference of the pulley.

To obtain the fraction of the circumference, divide the length of the string representing the arc of contact by the circumference.

27. To use the table for finding the width of a single leather belt for transmitting a given number of horsepower, we have the following rule,

where C = allowable effective pull, from table;
H = horsepower to be transmitted;
W = width of single belt in inches;
V = velocity of belt in feet per minute.

Rule 8.—*Multiply the horsepower to be transmitted by 33,000, and divide this product by the product of the velocity of the belt and the allowable effective pull, as taken from the table. The quotient will be the width of the belt.*

Or,
$$W = \frac{33,000\,H}{V\,C}.$$

EXAMPLE.—What width of single belt is needed to transmit 20 horsepower, the arc of contact on the small pulley being 135° and the speed of the belt 1,500 feet per minute?

SOLUTION.—According to Table I, the allowable effective pull for 135° is 31.3 pounds. Then, applying rule **8**, we have

$$W = \frac{33,000 \times 20}{1,500 \times 31.3} = 14 \text{ in., nearly. Ans.}$$

28. To Find the Horsepower of a Belt.—The horsepower that a single belt will transmit is given by the following rule:

Rule 9.—*Multiply together the effective pull taken from the table, the width of the belt in inches, and the speed of the belt in feet per minute. Divide the product by 33,000.*

Or,
$$H = \frac{C\,W\,V}{33,000}.$$

29. Speed of Belts.—By applying rule **9** to the same belt running at different velocities, it will be seen that the higher the velocity, the greater is the horsepower that the same belt can transmit, and from rule **8** it will be seen that the higher the speed of the belt, the less may be its width to transmit a given horsepower. From this it follows that a belt should be run at as high a velocity as conditions will permit, the maximum velocity allowable for a laced belt being about 3,500 feet per minute for ordinary single leather and double leather belts. For belts spliced by

cementing, where the splice is practically as strong as the belt itself, the velocity may be as high as 5,000 feet per minute. Cases are on record where wide main belts have been run at as high a velocity as 6,000 feet per minute.

30. In choosing a proper belt speed, due regard must be paid to commercial conditions. While a high speed of the belt means a narrow and, consequently, a cheaper belt, the increased cost of the larger pulleys that may be required may offset the gain due to the high belt speed, at least as far as first cost is concerned.

For illustration, let the problem be to transmit 10 horsepower from one shaft to another. Let the revolutions of both the driven and the driving shaft be equal, and let the shafts make 200 revolutions per minute. Choosing a belt speed of 2,000 feet per minute, the width of a single belt to transmit the given horsepower will be, by rule **8**, say, $4\frac{1}{2}$ inches. The diameter of the pulley that at 200 revolutions per minute will give a belt speed of 2,000 feet per minute is $\frac{2,000 \times 12}{200 \times 3.1416} = 38\frac{1}{4}$ inches, nearly. Taking the price of a 38-inch cast-iron pulley, 5-inch face, as \$15, we have the price of two pulleys as \$30. Let the distance between the pulleys be 20 feet. Then, the length of belt, according to rule **7**, is 50 feet 4 inches. Taking 51 feet as the length of the belt, and the price of a single leather belt $4\frac{1}{2}$ inches wide at 50 cents per foot, the price of the belt will be \$25.50. Then, the cost of belt and pulleys, not counting freight or express charges, etc., will be \$25.50 + \$30 = \$55.50.

Choosing a belt speed of 3,500 feet per minute, the width of belt will be $2\frac{1}{2}$ inches, nearly. The proper diameter of the pulley is $\frac{3,500 \times 12}{200 \times 3.1416} = 66\frac{7}{8}$ inches, say, 67 inches. The length of the belt will be 59 feet, about. The price of the belt at 30 cents per foot is \$17.70. The price of two 67-inch pulleys $3\frac{1}{2}$-inch face is, say, \$80. Then, the total first cost is \$80 + \$17.70 = \$97.70, showing that in this

particular case the use of a low belt speed reduces the first cost by \$97.70 − \$55.50 = \$42.20.

The above illustration is not intended, and must not be construed, to be an argument against high belt speed; it simply shows the advisability of considering the commercial features in each and every case. In many cases it will be found that the narrow, high-speed belt is by far the more economical one to use.

31. Double belts are made of two single belts cemented and, usually, riveted together their whole length, and are used where much power is to be transmitted. As the effective pull for single belts, as given in Table I, is based primarily on the strength through the lace holes, a double belt, which is twice as thick, should be able to transmit twice as much power as a single belt, and in fact more than this, where, as is quite common, the ends of the belt are cemented instead of laced.

Where double belts are used on small pulleys, however, the contact with the pulley face is less perfect than it would be if a single belt were used, owing to the greater rigidity of the former. More work is, also, required to bend the belt as it runs over the pulley than in the case of the more pliable single belt, and the centrifugal force tending to throw the belt from the pulley also increases with the thickness. Moreover, in practice, it is seldom that a double belt is put on with twice the tension of a single belt. For these reasons, the width of a double belt required to transmit a given horsepower is generally assumed to be seven-tenths the width of a single belt to transmit the same power. On this basis, rules **8** and **9** become for double belts, by multiplying rule **8** by $\frac{7}{10}$ and dividing rule **9** by $\frac{7}{10}$, as follows:

Rule 10.—*To find the width of a double belt, multiply the horsepower to be transmitted by 23,100. Divide this product by the product of the velocity of the belt and the allowable effective pull, as taken from the table.*

Or,
$$W = \frac{23{,}100\,H}{V\,C}.$$

Rule 11.—*To find the horsepower that a double belt can transmit, multiply together the effective pull taken from the table, the width of the belt, and its velocity. Divide the product by 23,100.*

Or,
$$H = \frac{C\,W\,V}{23{,}100}.$$

EXAMPLE 1.—What width of double belt is required to transmit 20 horsepower, the arc of contact on the smaller pulley being 135° and the speed of the belt 1,500 feet per minute?

SOLUTION.—According to Table I, the effective pull is 31.3 pounds. Then, by rule **10**,

$$W = \frac{23{,}100 \times 20}{1{,}500 \times 31.3} = 10 \text{ in., nearly. Ans.}$$

EXAMPLE 2.—What horsepower can be transmitted by a 6-inch double belt running at 400 feet per minute with an arc of contact of 180°?

SOLUTION.—According to Table I, the effective pull is 38.1 pounds. Applying rule **11**, we have

$$H = \frac{38.1 \times 6 \times 400}{23{,}100} = 4 \text{ H. P., nearly. Ans.}$$

THE CARE AND USE OF BELTS.

32. Leather Belts.—It is a much disputed question as to which side of the belt should be run next to the pulley. The more common practice, it is believed, is to run the belt with the hair or grain side nearest the pulley. This side is harder and more liable to crack than the flesh side. By running it on the inside the tendency is to cramp or compress it as it passes over the pulley, while if it ran on the outside, the tendency would be for it to stretch and crack. The flesh side is the tougher side, but for the reason given above the life of the belt will be longer if the wear comes upon the grain side. The lower side of the belt should be the driving side, the slack side running from the top of the driving pulley. The sag of the belt will then cause it to encompass a greater length of the circumference. Long belts, running in any other direction than vertical, work

better than short ones, as their weight holds them more firmly to their work.

It is bad practice to use rosin to prevent slipping. It gums the belt, causes it to crack, and prevents slipping for only a short time. If a belt, in good condition, persists in slipping, a wider belt should be used. Sometimes larger pulleys on the driving and driven shafts are of advantage, as they increase the belt speed and reduce the stress on the belt. Belts may be kept soft and pliable by oiling them once a month with castor oil or neatsfoot oil.

33. The Flapping of Belts.—One of the most annoying troubles experienced with belting of all kinds is the violent flapping of the slack side. Flapping may be due to any one of several causes, or to a combination of them. The most usual cause is that one or both of the pulleys run out of true. The belt is then alternately stretched and released, and while this may not cause flapping at one speed, it will usually do so at a higher speed. If the belt is rather slack, tightening it somewhat may cure or alleviate the flapping. The most obvious and best remedy, but the most expensive, is to turn the pulleys to run true.

Pulleys being out of line with each other are another prolific source of flapping, especially when one or both run out of true. First, bring the pulleys in line; if this fails, tighten the belt if it is rather loose. If no improvement is noticed and it is not possible to turn the pulleys, try to lower the belt speed a little, either by the substitution of smaller pulleys or by changing the speed of the driving shaft, according to circumstances.

With belts running at speeds above 4,000 feet per minute, flapping may occur when the pulleys are perfectly true and in line with each other, even when the belt has the proper tension. This is believed to be due to air becoming entrapped between the face of the pulley and the belt. At any rate, it has been observed that perforating the belt with a series of small holes will cure this trouble. Perforated belts may now be bought in the market.

Lack of steadiness in running, due either to sudden variations in the speed of the engine, or sudden changes in the load of the machines driven by the belt, will produce a flapping that it is almost impossible to cure. The only known cure is to take such steps as will insure steady running, as for instance, increasing the weight of the flywheel on the engine, or placing a flywheel instead of a pulley on the driven machine.

The belt not being joined square will also cause flapping, especially when the belt is running at a rather high speed. The remedy is to unlace or unfasten the joint and make it square.

Too great a distance between the pulleys may also cause flapping. In general, the distance between the pulleys should not exceed 15 feet for belts up to 4 inches in width; 20 feet for belts above 4 and below 12 inches; 25 feet for belts above 12 inches and below 18 inches ; and 30 feet for larger belts. The distances here given are occasionally exceeded considerably, but as no experiments have ever been made public that would enable a fairly correct formula to be deduced for the distance between pulleys when the belt speed, width of belt, and effective pull are known, they must be taken as representing average practice.

A horizontal belt is, by many, considered to have the proper tension when it has about 1 inch of sag, while in motion, for every 8 feet between the pulleys. For belts other than horizontal, this should be less, there being no sag at all for vertical belts.

JOINING THE ENDS OF BELTS.

34. Lacing.—The ends of a belt may be joined by *lacing*, *sewing*, *riveting*, or *cementing*. Many ways of **lacing** belts are used. A very satisfactory method for belts up to 3 inches in width is shown in Fig. 16. Cut the ends of the belt square, using a sharp knife and a try square. Punch a row of holes according to the width of the belt, punching

corresponding holes exactly opposite each other in each end of the belt, using 3 holes in belts up to 2 inches wide, and 5 holes in belts between 2 and 3 inches wide. The number of holes in the row should always be uneven for the style of lacing shown. In the figure, *A* is the outside of the belt and *B* the side running nearest the pulley. The lacing should be drawn half-way through one of the middle holes from the under side, as at *1*; before going any further, it is well to see to it that the belt has no twists in it, or, in the case of a crossed belt, that it has not been given a wrong twist. The same side of the belt should run over both pulleys, which will be the case with a crossed belt if it has been twisted correctly. Having made sure that the belt is fair, pass the end of the lace on the upper side of the belt through *2* under the belt and up through *3*, back again through *2* and *3*, through *4* and up through *5*, where an incision is made in one side of the lacing, which forms a barb that will prevent the end from pulling through. Lace the right-hand side in the same manner. The lacing may advantageously be carried on at once to the right and left alternately.

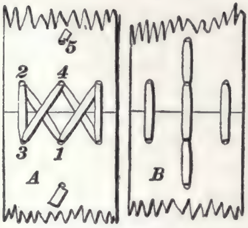


FIG. 16.

35. For belts wider than 3 inches, the lacing shown in Fig. 17 is a good one. As will be observed, there are two rows of holes. The number of holes in the row nearest the joint should exceed by one the number of holes in the second row. For belts up to $4\frac{1}{2}$ inches wide, use 3 holes in the row nearest the joint and 2 holes in the second row. For belts up to 6 inches wide, use 4 and 3 holes, respectively. For larger belts, make the total number of holes in each end either one or two more than the number of inches of width, with the object of getting an odd total number of holes. For example, for a 10-inch belt, the total number of

holes should be $10 + 1 = 11$. For a 13-inch belt, it should be $13 + 2 = 15$ holes. The outside holes of the first row should not be nearer the edges of the belt than $\frac{3}{4}$ inch, nor should the first row be nearer the joint than $\frac{7}{8}$ inch. The second row should be at least $1\frac{3}{4}$ inches from the end. In the figure, *A* is the outside and *B* the side nearest the pulley. Begin at one of the center holes in the outside row, as *1*, and continue through *2*, *3*, *4*, *5*, *6*, *7*, *6*, *7*, *4*, *5*, *2*, *3*, etc.

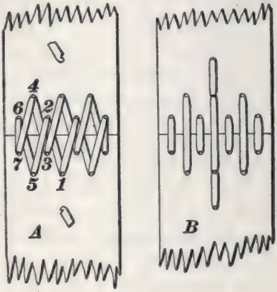


FIG. 17.

Another method is to begin the lacing on one side instead of in the middle. This method will give the rows of lacing on the under side of the belt the same thickness all the way across. The lacing should not be crossed on the side of the belt that runs next to the pulley.

Lacing affords convenient means of *shortening belts* when they stretch and of *increasing their tension.* If the belts are at all large, the ends of the belt should be drawn together by belt clamps.

36. Cementing.—Cementing makes probably the best kind of a joint ever devised. It has the serious disadvantage, however, that the stretch of the belt cannot be readily taken up, and, hence, tightening pulleys must be used when the center-to-center distance of the pulleys is not adjustable.

FIG. 18.

In dynamo driving, where endless belts are used to the exclusion of all others, the dynamo is usually mounted on a slide, so that the tension of the belt can be adjusted. For a cemented joint, the ends of the belt should be pared down with a very sharp knife to the form shown in Fig. 18, which shows a form that is recommended by belt manufacturers.

Warm the belt ends near a fire, apply the belt cement while hot, and press the joint together by two boards, one on the top and one on the bottom. Belt cement can be obtained of any dealer in engineer's supplies and full directions for using it will always be found on the can. These directions should be implicitly followed.

If belt cement cannot be obtained, a good cement may be made by melting together over a slow fire 16 parts of gutta percha, 4 parts of india rubber, 2 parts of pitch, 1 part of shellac, and 2 parts of linseed oil, by weight. Cut all ingredients very small, mix well, and use while hot.

37. Stretch of Belts.—New leather belts will stretch from one-fourth to one-half of an inch per foot of length, and hence must be taken up until the limit of stretch has been reached. Rubber belts are said to stretch continuously. Cotton and duck belts are said not to stretch with use.

38. Precautions to be Observed When Using Rubber Belts.—When rubber belts are used, animal oils or animal grease should never be used on them. If the belt should slip, it should be lightly moistened on the side nearest the pulley with boiled linseed oil.

EXAMPLES FOR PRACTICE.

1. The main line shaft is driven at 90 revolutions per minute and is to drive another shaft at 120 revolutions per minute; the latter shaft carries a pulley 20 inches in diameter. How large a pulley should be used on the main line shaft? Ans. $26\frac{2}{3}$ in.

2. The belt wheel of an engine is 10 feet in diameter and makes 65 revolutions per minute; the line shaft is to run at 150 revolutions per minute. What size pulley should be used? Ans. 52 in.

3. The driving pulley is 48 inches in diameter and makes 90 revolutions per minute. The driven pulley being 20 inches in diameter, how many revolutions per minute will the driven shaft make?
Ans. 216 R. P. M.

4. The belt wheel of an engine being 5 feet in diameter, the driven pulley 45 inches, and the driven shaft to make 90 revolutions per minute, what should be the number of revolutions of the belt wheel?
Ans. $67\frac{1}{2}$ R. P. M.

5. Two pulleys, 48 and 32 inches in diameter, respectively, are 10 feet 6 inches from center to center. What is the length of an open belt for these pulleys? Ans. 31 ft. 10 in.

6. A belt speed of 2,400 feet per minute having been chosen, what width of single belt will be needed to transmit 30 horsepower? The pulleys over which the belt runs are equal in size. Ans. 11 in., nearly.

7. What horsepower can be transmitted by a single belt 11 inches wide traveling at the rate of 1,000 feet per minute over a small pulley on which its arc of contact is 150°? Ans. 11¼ H. P., nearly.

8. With a belt speed of 1,300 feet per minute, what width of double belt will be required to transmit 5 horsepower over pulleys equal in diameter? Ans. 2½ in., nearly.

WHEEL WORK.

DRIVER AND FOLLOWER.

39. A combination of wheels and axles, as in Fig. 19, is called a train. The wheel in a train to which motion is imparted from a wheel on another shaft, by such means as a belt or gearing, is called the **driven wheel** or **follower;** the wheel that imparts the motion is called the **driver.**

It will be seen that the wheel and axle bears the same relation to the train that a simple lever does to the compound lever. Letting D_1, D_2, D_3, etc. represent the diameters of the driven wheels, and d_1, d_2, d_3, etc. the diameters of the different drivers, we have the following rule:

Rule 12.—*The continued product of the force and the diameters of the driven wheels equals the continued product of the weight, the diameter of the drum that moves the weight, and the diameters of the drivers.*

Or, $P \times D_1 \times D_2 \times D_3$, etc. $= W \times d_1 \times d_2 \times d_3$, etc.

Since the radii of several circles are in the same proportion to one another as the diameters of the same circles, it follows that the radii may be used instead of the diameters in the above rule.

EXAMPLE.—If the radius of the pulley A, Fig. 19, is 20 inches, of C 15 inches, and of E 24 inches, and the radius of the drum F is 4 inches, of the pinion D 5 inches, and of the pinion B 4 inches, how great a weight will a force of 1 pound applied at P raise?

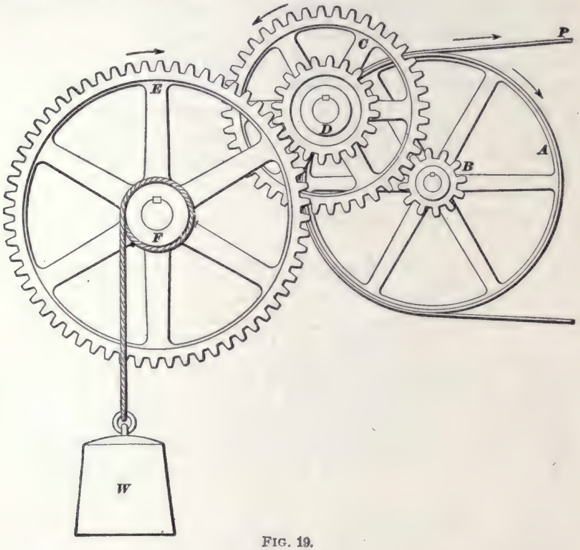


FIG. 19.

SOLUTION.—Applying rule **12**, we have

$$1 \times 20 \times 15 \times 24 = W \times 4 \times 5 \times 4,$$

or $$W = \frac{7{,}200}{80} = 90 \text{ lb. Ans.}$$

40. Although the combination of wheels in this example enables the lifting of a weight 90 times as great as the force applied, it has been necessary to exert the force through a distance 90 times the height through which the weight was raised. It is a universal law in the application of machines that *whenever there is a gain in power without a corresponding increase in the initial force, there is a loss in speed;* this is true of any machine.

In the example, if P were to move the entire 90 inches in 1 second, W would move only 1 inch in the same period of time.

41. Instead of using the diameter or radius of a gear, as in the last example, the number of teeth may be used when computing the weight that can be raised, or the velocity.

EXAMPLE.—The radius of the pulley A, Fig. 19, is 40 inches and that of F is 12 inches. The number of teeth in B is 9, in C 27, in D 12, and in E 36. If the weight to be lifted is 1,800 pounds, how great a force at P is it necessary to apply to the belt?

SOLUTION.—Let P represent the force; then, by the rule

$$P \times 40 \times 27 \times 36 = 1{,}800 \times 12 \times 9 \times 12,$$

or $$P \times 38{,}880 = 2{,}332{,}800.$$

Hence, $$P = \frac{2{,}332{,}800}{38{,}880} = 60 \text{ lb.},$$

= the amount of force necessary to apply to the belt. Ans.

GEAR-WHEELS.

42. A wheel that is provided with teeth that mesh with similar teeth on another wheel is called a **gear-wheel,** or **gear.** In Fig. 20 is shown a **spur gear.** On spur gears

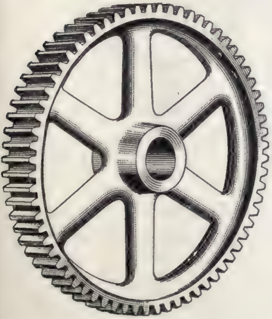


FIG. 20.

FIG. 21.

the teeth are usually parallel to the axis of the wheel or to its shaft.

Gears are said to be *in mesh* when the teeth of two wheels, respectively, engage each other or interlock.

43. In Fig. 21 is shown a pair of **bevel gears** in mesh. Of the two wheels shown, one is smaller than the other; when both wheels of a pair of bevel gears are of the same diameter they are called **miter gears.** In Fig. 22 is shown

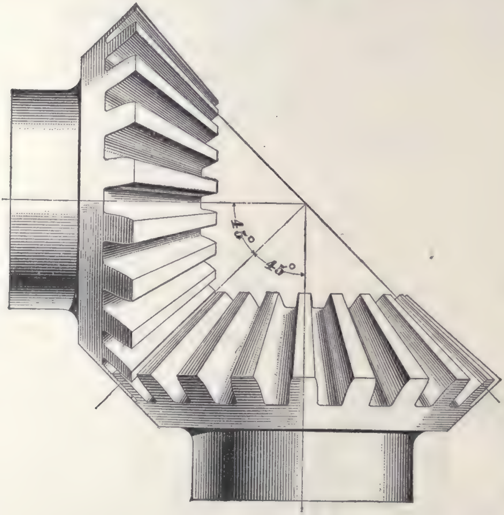


FIG. 22.

a pair of miter gears in mesh. It is obvious that the angle that the teeth of these gears make with the axis of the shaft must be 45°.

44. Of a pair of gear-wheels (either spur or bevel) having different diameters, the smaller is called a **pinion.**

In Fig. 23 is shown a revolving screw, or **worm,** as it is called, that meshes with a **worm-wheel.** It is used to transmit motion from one shaft to another at right angles to it.

As the worm is nothing else than a screw, each revolution given to it will rotate the wheel a distance equal to the pitch of the worm; consequently, if there are 40 teeth in the worm-wheel, a single-threaded worm must make 40 revolutions in order to turn the wheel once.

Fig. 23.

45. A **rack** is a straight bar that has gear-teeth cut on it. It may be considered as part of a gear-wheel in which the diameter is infinitely large. The teeth of racks are proportioned by the same rules as those of gear-wheels.

TEETH OF GEAR-WHEELS.

46. The object in designing the teeth of gear-wheels should be to so shape them that the motion transmitted will be exactly the same as with a corresponding pair of wheels, or cylinders, without teeth and running in contact without slipping. Such cylinders are called **pitch cylinders,** and are always represented on the drawing of a gear-wheel by a line called the **pitch circle** (see Fig. 24). The pitch circle is also called the **pitch line.**

The diameter of the pitch circle is called the **pitch diameter.** When the word "diameter" is applied to gears, it is always understood to mean the *pitch diameter*, unless especially stated as the "diameter over all" or "diameter at the root."

47. Circular and Diametral Pitch.—The distance from a point on one tooth to a corresponding point on the next tooth, *measured along the pitch circle*, is the **circular**

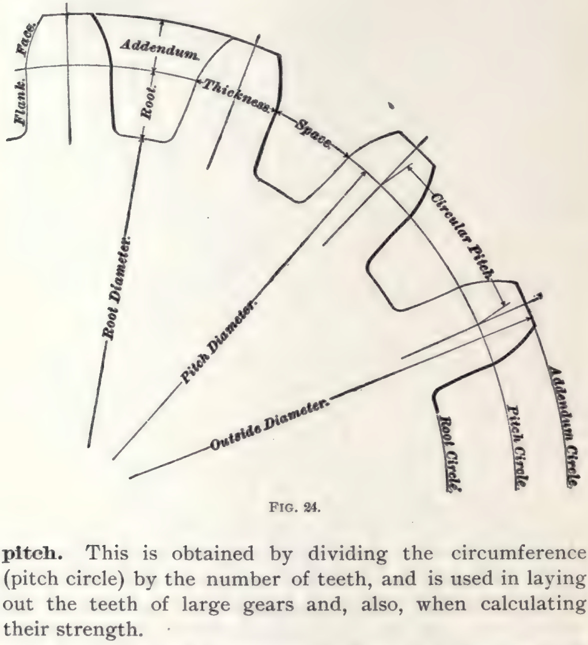


FIG. 24.

pitch. This is obtained by dividing the circumference (pitch circle) by the number of teeth, and is used in laying out the teeth of large gears and, also, when calculating their strength.

It would be very convenient to have the circular pitch expressed in manageable numbers like 1-inch, $\frac{3}{4}$ inch, etc.; but as the circumference of a gear is 3.1416 times its diameter, this requires awkward numbers for the diameters. Thus, a wheel of 40 teeth, 1-inch pitch, would have a circumference on the pitch circle of 40 inches and a diameter of 12.732 inches. Of the two it is more convenient, in the great majority of cases, to have the diameters expressed in numbers that can be easily handled. In order, however, to

have the pitch in a convenient form also, a pitch has been devised that is *expressed in terms of the diameter* and called the **diametral pitch.**

The diametral pitch is not a measurement like the circular pitch, but a *ratio. It is the ratio of the number of teeth in the gear to the number of inches in the diameter; or, it is the number of teeth on the circumference of the gear for 1 inch diameter of the pitch circle.* It is obtained by dividing the number of teeth by the diameter.

A gear, for example, has 60 teeth and is 10 inches in diameter. The diametral pitch is the ratio of 60 to $10 = \frac{60}{10} = 6$; this gear would be called a 6-pitch gear. From the definition it follows that teeth of any particular diametral pitch are of the same size and have the same width on the pitch line, whatever may be the diameter of the gear. Thus, if a 12-inch gear has 48 teeth, it will be 4 pitch. A 24-inch gear to have teeth of the same size will have twice 48, or 96 teeth, and as $96 \div 24 = 4$, has the same diametral pitch as before.

48. Other Definitions.—The other necessary definitions applying to the parts of a gear can be readily understood from Fig. 24. The thickness of the tooth and the width of the space are measured on the pitch circle. A tooth is composed of two parts, the **addendum,** or part outside of the pitch circle, and the **root,** which is inside.

A line through the outside end of the addendum is called the **addendum circle,** or **addendum line,** and one through the inside part of the root is called the **root circle,** or **root line.** The amount by which the width of the space is greater than the thickness of the tooth is called the **backlash,** or **side clearance.**

49. Proportions for Gear-Teeth.—With gears of large size, and often with cast gears of all sizes, the circular-pitch system is used. In these cases, it is usual to have the addendum, whole depth, and thickness of the tooth conform to arbitrary rules based on the circular pitch.

The usual proportions are for cast gears: Addendum $= .3 \times$ circular pitch. Root $= .4 \times$ circular pitch. Thickness of tooth $= .48 \times$ circular pitch.

The gears most often met with are the cut gears of small and medium size like those, for example, on machine tools, which are almost invariably diametral-pitch gears. The teeth are cut from the solid with standard milling cutters, proportioned with the diametral pitch as a basis. This system is also coming into use for cast gearing. In all diametral-pitch gears, the addendum, in inches, is made equal to 1 divided by the diametral pitch, and the working depth to twice the addendum. The end clearance is usually taken equal to $\frac{1}{8}$ the addendum for cut gears, though The Brown & Sharpe Manufacturing Company use $\frac{1}{10}$ the thickness of the tooth on the pitch line as the clearance. The side clearance, or "backlash," is barely enough to give a good working fit, and seldom exceeds $\frac{1}{50}$ the pitch.

Using the above proportions, a 4-pitch gear will have the addendum $= 1 \div 4 = \frac{1}{4}$ inch; the working depth will be $2 \times \frac{1}{4} = \frac{1}{2}$ inch; and the clearance, if made $\frac{1}{8}$ the addendum, $\frac{1}{8} \times \frac{1}{4} = \frac{1}{32}$ inch. The whole length of the tooth will be $\frac{1}{2} + \frac{1}{32} = \frac{17}{32}$ inch. The thickness of the tooth will be one-half the circular pitch, nearly. In a 10-pitch wheel, the addendum will be $\frac{1}{10}$ inch and the length of the tooth $\frac{17}{80}$ inch; in a $2\frac{1}{2}$-pitch, it will be $1 \div 2\frac{1}{2} = \frac{2}{5}$ inch and the length $\frac{17}{20}$ inch.

FORMS OF GEAR-TEETH.

50. The forms of teeth used in ordinary practice form part of certain curves known as the *epicycloid*, *hypocycloid*, and *involute*.

51. The Epicycloidal Tooth.—In the so-called epicycloidal tooth, which more properly is called a *cycloidal* tooth, the face of the tooth is part of an epicycloid, and the flank, part of a hypocycloid.

An **epicycloidal curve** is the path described by any point of a circle rolling, without slipping, on the outside of another circle. A **hypocycloid** is the path described by any point of a circle rolling, without slipping, on the inside of another circle.

Epicycloidal teeth can always be recognized by their appearance; they are formed by two curves that, commencing at the pitch circle, curve in opposite directions. Fig. 25 clearly exhibits the characteristic tooth form.

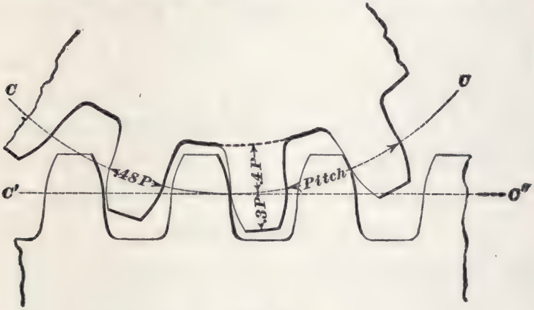


FIG. 25.

The use of epicycloidal teeth is being gradually abandoned, as they possess some practical defects, the chief defect being that the center-to-center distance of the two gears must be practically perfect in order to insure a uniform velocity of the driven gear.

52. Involute Teeth.—In Fig. 26 is shown the involute form of tooth, which is composed of but one curve.

The **involute** is the path described by any point of a string that is being wound on or off a cylinder, the cylinder being stationary. In the involute system, the sides of the teeth of the rack are straight lines, as shown in Fig. 26.

Involute teeth have two great advantages over epicycloidal teeth: (1) *They are stronger for the same pitch, as they are thicker at the root.* (2) *The gears may be spread*

slightly apart so that their pitch circles do not run tangent to

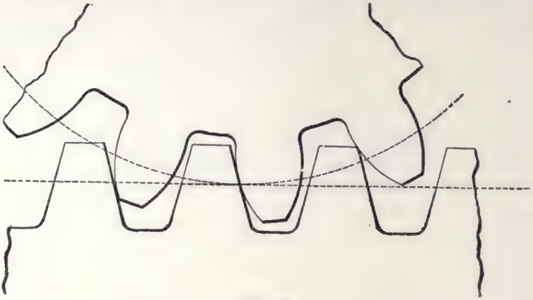


FIG. 26.

one another, without affecting the perfect action of the teeth to an appreciable extent.

GEAR CALCULATIONS.

53. The Circular-Pitch System.—For calculating the pitch diameter, number of teeth, etc. of gear-wheels, we have, for the circular-pitch system, the following rules,

where $P =$ circular pitch in inches;
$T =$ number of teeth;
$D =$ pitch diameter of the gear in inches.

54. To find the pitch diameter of a gear-wheel in inches, when the pitch and number of teeth are given:

Rule 13.—*The pitch diameter equals the product of the pitch and the number of teeth divided by 3.1416.*

Or,
$$D = \frac{PT}{3.1416}.$$

EXAMPLE.—What is the diameter of the pitch circle of a gear-wheel that has 75 teeth and whose pitch is 1.675 inches?

SOLUTION.—Applying rule **13,** we have

$$D = \frac{1.675 \times 75}{3.1416} = 40 \text{ in. Ans.}$$

55. To find the number of teeth in a gear-wheel when the diameter and pitch are given:

Rule 14.—*The number of teeth equals the product of 3.1416 and the diameter divided by the pitch.*

Or, $$T = \frac{3.1416\,D}{P}.$$

EXAMPLE.—The diameter of a gear-wheel is 40 inches and the pitch of the teeth is 1.675 inches; how many teeth are there in the wheel?

SOLUTION.—Applying the rule just given, we have

$$T = \frac{3.1416 \times 40}{1.675} = 75 \text{ teeth. Ans.}$$

56. To find the pitch of a gear-wheel when the diameter and the number of teeth are given:

Rule 15.—*The pitch of the teeth equals the product of 3.1416 and the diameter divided by the number of teeth.*

Or, $$P = \frac{3.1416\,D}{T}.$$

EXAMPLE.—The diameter of a gear-wheel is 40 inches and it has 75 teeth; what is the pitch of the teeth?

SOLUTION.—By rule **15**, we have

$$P = \frac{3.1416 \times 40}{75} = 1.675 \text{ in. pitch. Ans.}$$

57. The Diametral-Pitch System.—The diameter of the gear-wheel, the number of teeth, etc. are given by the following rules, where P_d = diametral pitch; D_o = outside diameter; N = number of teeth; and the other letters have the same meaning as in the three preceding rules.

58. To find the pitch diameter of the gear-wheel when the number of teeth and the pitch are given:

Rule 16.—*Divide the number of teeth by the diametral pitch.*

Or, $$D = \frac{N}{P_d}.$$

EXAMPLE.—A wheel is to have 40 teeth, 4 pitch; what is its pitch diameter?

SOLUTION.—By applying rule **16**, we have

$$D = \frac{40}{4} = 10 \text{ in. Ans.}$$

59. To find the diameter over all, that is, the diameter of the blank from which the gear-wheel is cut, the number of teeth and the diametral pitch being given:

Rule 17.—*Add 2 to the number of teeth and divide by the diametral pitch.*

Or, $$D_o = \frac{N+2}{P_d}.$$

EXAMPLE.—In the last example, what is the diameter over all the blank?

SOLUTION.—Applying the rule just given, we get

$$D_o = \frac{40+2}{4} = 10\tfrac{1}{2} \text{ in. Ans.}$$

60. The number of teeth and the outside diameter of the gear-wheel being known, to find the diametral pitch:

Rule 18.—*Add 2 to the number of the teeth and divide by the outside diameter.*

Or, $$P_d = \frac{N+2}{D_o}.$$

EXAMPLE.—A gear-wheel has 60 teeth and is $6\frac{1}{5}$ inches in diameter over all; what is the diametral pitch?

SOLUTION.—By applying rule **18**, we get

$$P_d = \frac{60+2}{6\frac{1}{5}} = 10. \text{ Ans.}$$

61. To find the diametral pitch, the number of teeth and the pitch diameter being known:

Rule 19.—*Divide the number of teeth by the pitch diameter.*

Or, $$P_d = \frac{N}{D}.$$

EXAMPLE.—A wheel has 90 teeth and its pitch diameter is 30 inches; what is the diametral pitch?

SOLUTION.—By rule **19**, we get

$$P_d = \frac{90}{30} = 3. \quad \text{Ans.}$$

62. The pitch diameter and the diametral pitch being given, to obtain the number of teeth:

Rule 20.—*Multiply the pitch diameter by the diametral pitch.*

Or, $$N = D P_d.$$

EXAMPLE.—How many teeth are there in a gear-wheel having a pitch diameter of 36 inches and a diametral pitch of 5?

SOLUTION.—Applying the rule just given, we get

$$N = 36 \times 5 = 180 \text{ teeth.} \quad \text{Ans.}$$

63. The diameter over all and the diametral pitch being given, to find the number of teeth:

Rule 21.—*Subtract 2 from the product of the outside diameter and the diametral pitch.*

Or, $$N = D_o P_d - 2.$$

EXAMPLE.—How many 10-pitch teeth has a gear-wheel having an outside diameter of $8\frac{1}{5}$ inches?

SOLUTION.—By rule **21**, we get

$$N = 8\tfrac{1}{5} \times 10 - 2 = 80 \text{ teeth.} \quad \text{Ans.}$$

64. The standard diametral pitches used for cut gears are as follows: 2, $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, 3, $3\frac{1}{2}$, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 36, 40, 48. Gears having a diametral pitch differing from those given here are, usually, either very large or very small.

65. Diameters and Distances Between Centers.—The distance between the centers of two gear-wheels being known and the ratio of their speeds, the diameter of the pitch circle of the smaller wheel is given by the following rule,

where A = center-to-center distance;
R = revolutions per minute of large gear;
r = revolutions per minute of small gear;
d = pitch diameter of small gear.

Rule 22.—*Multiply twice the center-to-center distance by the number of revolutions of the large gear, and divide the product by the sum of the revolutions of the large and small gear.*

Or, $$d = \frac{2\,A\,R}{R + r}.$$

EXAMPLE.—Given, the distance between centers $= 5\frac{1}{2}$ inches; the small gear is to make 24 revolutions for every 8 revolutions of the large gear. What is the diameter of each gear?

SOLUTION.—By rule **22**,

$$d = \frac{2 \times 5\frac{1}{2} \times 8}{8 + 24} = 2\tfrac{3}{4} \text{ in., the diameter of the small wheel.}$$

Then, as the center distance is equal to the sum of the radii of the two wheels, the radius of the large wheel is $5\frac{1}{2} - \frac{2\frac{3}{4}}{2} = 4\frac{1}{8}$ in., and its diameter is $4\frac{1}{8} \times 2 = 8\frac{1}{4}$ in. Ans.

66. Speed and Number of Teeth.—To calculate the number of teeth or the speed of one of two gear-wheels that are to gear together:

Let N = number of revolutions per minute of the driving wheel;
n = number of revolutions per minute of the driven wheel;
T = number of teeth in the driving wheel;
t = number of teeth in the driven wheel.

Rule 23.—*The number of teeth in the driving wheel equals the product of the number of teeth and number of revolutions of the driven wheel divided by the number of revolutions of the driving wheel.*

Or, $$T = \frac{t\,n}{N}.$$

EXAMPLE.—The driven wheel has 27 teeth and will make 66 revolutions per minute; if the driving wheel makes 99 revolutions per minute, how many teeth are there in the driving wheel?

SOLUTION.—Applying rule **23**, we have

$$T = \frac{27 \times 66}{99} = 18 \text{ teeth. Ans.}$$

67. The number of revolutions per minute of the driving wheel and the driven wheel and the number of teeth in the driving wheel being given, to find the number of teeth in the driven wheel:

Rule 24.—*The number of teeth in the driven wheel equals the product of the number of teeth and revolutions per minute of the driving wheel divided by the number of revolutions per minute of the driven wheel.*

Or,
$$t = \frac{TN}{n}.$$

EXAMPLE.—The driving wheel has 18 teeth and makes 99 revolutions per minute, and the driven wheel must make 66 revolutions per minute; how many teeth must there be in the driven wheel?

SOLUTION.—Applying the rule just given, we have

$$t = \frac{18 \times 99}{66} = 27 \text{ teeth. Ans.}$$

68. The number of teeth in the driving wheel and the driven wheel and the number of revolutions per minute of the driving wheel being given, to find the number of revolutions per minute of the driven wheel:

Rule 25.—*The number of revolutions per minute of the driven wheel equals the product of the number of teeth and number of revolutions of the driving wheel divided by the number of teeth of the driven wheel.*

Or,
$$n = \frac{TN}{t}.$$

EXAMPLE.—There are 18 teeth in the driving wheel and it makes 99 revolutions per minute; how many revolutions per minute will the driven wheel make if it has 27 teeth?

SOLUTION.—Applying rule **25**, we have

$$n = \frac{18 \times 99}{27} = 66 \text{ R. P. M. Ans.}$$

69. The number of teeth in the driving wheel and the driven wheel and the number of revolutions per minute of the driven wheel being given, to find the number of revolutions per minute of the driving wheel:

Rule 26.—*The number of revolutions of the driving wheel equals the product of the number of teeth and revolutions of the driven wheel divided by the number of teeth of the driving wheel.*

Or, $$N = \frac{t\,n}{T}.$$

EXAMPLE 1.—If there are 27 teeth in the driven wheel and if it makes 66 revolutions per minute, how many revolutions per minute will the driving wheel make if it has 18 teeth?

SOLUTION.—Applying the rule just given, we have

$$N = \frac{27 \times 66}{18} = 99 \text{ R. P. M.} \quad \text{Ans.}$$

EXAMPLE 2.—In Fig. 27, the crank-shaft makes 60 revolutions per minute; the governor pulley is 4 inches in diameter; the bevel gear on the governor pulley shaft has 19 teeth; the bevel gear that meshes with it and drives the governor has 30 teeth. The governor is to make 95 revolutions per minute: what should be the size of the pulley on the crank-shaft?

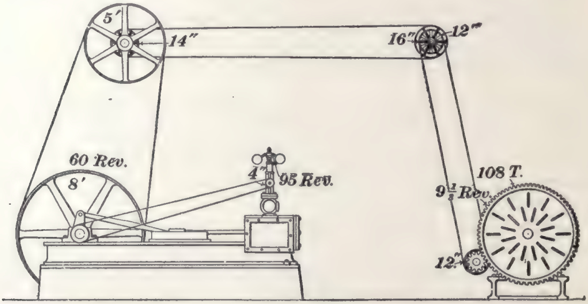


FIG. 27.

SOLUTION.—First determine the number of revolutions of the 4-inch pulley in order that the governor shall turn 95 times per minute. Applying rule **26**, $N = \frac{t\,n}{T} = \frac{30 \times 95}{19} = 150$ revolutions of gear on pulley shaft = revolutions of governor pulley. Now, applying rule **3**, the diameter of the pulley on the crank-shaft $= \frac{d\,n}{N} = \frac{4 \times 150}{60} = 10$ in. Ans.

EXAMPLE 3.—In Fig. 27, the flywheel is 8 feet in diameter and drives a 5-foot pulley on the main shaft. A 14-inch pulley on the main shaft drives a 16-inch pulley on the countershaft. A 12-inch pulley on the countershaft drives a 12-inch pulley on a shaft on which is a pinion that meshes into a large gear attached to the face plate of a large lathe, and which has 108 teeth. How many teeth must the pinion have in order that the face plate may make $9\frac{1}{3}$ revolutions per minute?

SOLUTION.—Applying rule **5**, to find the revolutions per minute of the main shaft, $\frac{8 \times 60}{5} = 96$ R. P. M. Applying the same rule again to find the revolutions of the countershaft, $\frac{14 \times 96}{16} = 84$ R. P. M. Applying it once more to find the revolutions of the pulley that turns the small gear, $\frac{12 \times 84}{12} = 84$ R. P. M. Applying rule **23**, $\frac{108 \times 9\frac{1}{3}}{84}$ $= 12$ teeth in pinion or driver. Ans.

EXAMPLES FOR PRACTICE.

1. The driving pulley makes 110 R. P. M. and is 21 inches in diameter; what should be the size of the driven pulley in order to make 385 R. P. M.? Ans. 6 in.

2. The main shaft of a certain shop makes 120 R. P. M. It is desired to have the countershaft make 150 R. P. M. There are on hand pulleys 16 inches, 24 inches, 28 inches, 35 inches, and 38 inches in diameter. Can two of these be used, or must a new pulley be ordered?
Ans. Use the 28-inch and 35-inch pulleys.

3. The pinion (driver) makes 174 R. P. M. and the follower 24 R. P. M.; how many teeth must the pinion have if the follower has 87 teeth? Ans. 12 teeth.

4. If an engine flywheel is 66 inches in diameter and makes 160 R. P. M., what must be the diameter of the pulley on the main shaft to make 128 R. P. M.? Ans. $82\frac{1}{2}$ in.

5. What is the pitch diameter of a gear whose circular pitch is $1\frac{1}{4}$ inches and has 28 teeth? Ans. 11.14 in.

6. How many teeth are there in a gear whose circular pitch is .7854 inch and which is 23 inches in diameter? Ans. 92 teeth.

7. What is the circular pitch of a gear whose diameter is 20.372 inches and which has 128 teeth? Ans. $\frac{1}{2}$ in.

8. What is the pitch diameter of a gear-wheel having 80 teeth, 7 diametral pitch? Ans. $11\frac{3}{7}$ in.

9. What is the over-all diameter of a gear-wheel of 9 diametral pitch and 47 teeth? Ans. $5\frac{4}{9}$ in.

10. What is the diametral pitch of a gear-wheel having 90 teeth and an outside diameter of $5\frac{3}{4}$ inches? Ans. 16 pitch.

11. How many teeth of 20 diametral pitch can be cut in a gear-wheel having an outside diameter of $2\frac{1}{10}$ inches? Ans. 40 teeth.

12. If the center-to-center distance of two gear-wheels is 6 inches and the small gear is to make 4 revolutions for 1 revolution of the large gear, what must be the diameter of the large gear?

Ans. 9.6 in.

13. In a pair of gear-wheels, the driver has 48 teeth and the driven wheel 56 teeth. If the driven wheel makes 98 revolutions per minute, how many revolutions must the driver make? Ans. $114\frac{1}{3}$ R. P. M.

14. In a train of gears, the drivers have 16, 30, 24, and 18 teeth, respectively; the followers have 12, 24, 36, 40 teeth, respectively. If the first driver makes 80 R. P. M., how many R. P. M. will the last follower make? Ans. 40 R. P. M.

FIXED AND MOVABLE PULLEYS.

70. Pulleys are also used for hoisting or raising loads, in which case the frame that supports the axle of the pulley is called the **block.**

71. A **fixed pulley** is one whose block is not movable, as in Fig. 28. In this case, if the weight W be lifted by

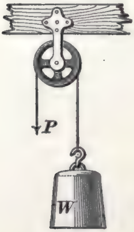


FIG. 28.

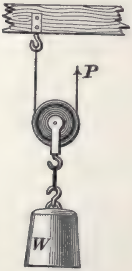


FIG. 29.

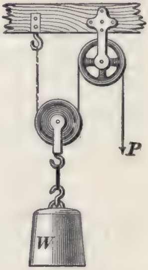


FIG. 30.

pulling down P, the other end of the cord W will evidently move the same distance upwards that P moves downwards; hence, P must equal W.

72. A **movable pulley** is one whose block is movable, as in Fig. 29. One end of the cord is fastened to the beam and the weight is suspended from the pulley, the other end of the cord being drawn up by the application of a force *P*. A little consideration will show that if *P* acts through a certain distance, say 1 foot, *W* will move through *half* that distance, or 6 inches; hence, a pull of 1 pound at *P* will lift 2 pounds at *W*.

The same would also be true if the free end of the cord were passed over a *fixed pulley*, as in Fig. 30, in which case the fixed pulley merely changes the direction in which *P* acts, so that a weight of 1 pound hung on the free end of the cord will balance 2 pounds hung from the *movable pulley*.

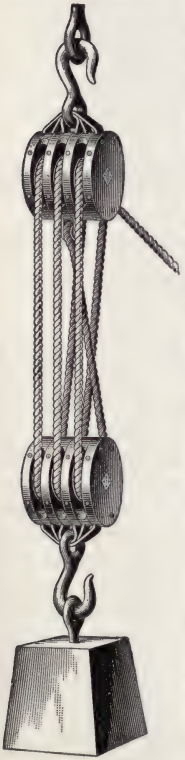


FIG. 31.

73. A **combination of pulleys,** as shown in Fig. 31, is sometimes used. In this case there are three movable and three fixed pulleys, and the amount of movement of *W*, owing to a certain movement of *P*, is readily found.

It will be noticed that there are 6 *parts* of the rope, not counting the free end; hence, if the movable block be lifted 1 foot, *P* remaining in the same position, there will be 1 foot of slack in each of the 6 parts of the rope, or 6 feet in all. Therefore, *P* must move 6 feet in order to take up this slack, or *P* moves 6 times as far as *W*. Hence, 1 pound at *P* will support 6 pounds at *W*, since the *force multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.* It will also be noticed that there are three movable pulleys, and that $3 \times 2 = 6$.

LAW OF COMBINATION OF PULLEYS.

74. Rule 27.—*In any combination of pulleys where one continuous rope is used, a load on the free end will balance a weight on the movable block as many times as great as itself as there are parts of the rope supporting the load, not counting the free end, assuming that there are no friction losses.*

The above law is good whether the pulleys are side by side, as in the ordinary block and tackle, or whether they are in line and beneath one another.

EXAMPLE.—In a block and tackle having 5 movable pulleys, how great a force must be applied to the free end of the rope to raise 1,250 pounds, assuming that there are no friction losses?

SOLUTION.—Since there are 5 movable pulleys, there must be 10 parts of the rope to support them. Hence, according to the above law, a force applied to the free end will support a load 10 times as great as itself, or the force $= \frac{1,250}{10} = 125$ lb. Ans.

75. Frictional Losses.—Owing to the friction of the sheaves and the friction and resistance to bending of the rope, it is not possible to move a weight with a block and tackle by applying the theoretical force calculated by the above rule. The actual pull required depends on such factors as the condition of the rope and the friction and number of the sheaves. By experiment it has been determined that under average conditions the load that can be raised with a block and tackle having 2 sheaves will average about 85 per cent. of the theoretical load; with 3 sheaves, about 80 per cent.; with 4 sheaves, about 75 per cent.; with 5 sheaves, about 70 per cent.; with 6 sheaves, about 66 per cent.; with 7 sheaves, about 63 per cent.; and with 8 sheaves, about 60 per cent. No records of tests with a larger number of sheaves have been made public; the decrease in the load that can be lifted will probably vary in about the same ratio.

Let f_a = the force applied at the free end of the rope;
c = the percentage of the actual load to the theoretical load, expressed decimally;
m = the number of parts of rope supporting the load;
l = the load to be raised.

Then, the force that is actually required to raise a given load, may be found approximately by

Rule 28.—*Divide the load by the product of the number of parts of the rope supporting the load and the per cent. corresponding to the number of sheaves used.*

Or, $$f_a = \frac{l}{m\,c}.$$

Example.—In an ordinary block and tackle having six parts of the rope supporting the load, as shown in Fig. 31, how great a force must actually be applied to raise 2,000 pounds?

Solution.—According to Art. **75,** c may be taken as .66, there being 6 parts of the rope. Applying rule **28,**

$$f_a = \frac{2{,}000}{6 \times .66} = 505 \text{ lb., nearly.} \quad \text{Ans.}$$

76. The load that may actually be raised with a block and tackle will be approximately given by the following rule:

Rule 29.—*Multiply the force to be applied to the free end of the rope by the number of parts of rope supporting the load and by the per cent. corresponding to the number of sheaves used.*

Or, $$l = f_a\,m\,c.$$

Example.—If there are enough men pulling on the free end of the rope of a block and tackle having four parts of the rope supporting the load to exert a force of 250 pounds, what weight may they expect to raise?

Solution.—There being 4 parts of the rope, by Art. **75,** $c = .75$. Then by rule **29,**

$$l = 250 \times 4 \times .75 = 750 \text{ lb.} \quad \text{Ans.}$$

77. If the free end of the hauling rope passes first around a stationary sheave, as in Fig. 31, it does not make any difference in what direction in the plane of the sheave the rope is pulled. If it passes first around a movable sheave, however, as in Fig. 29, the pull must be exerted in a line parallel to the line of action of the resistance, or a line joining the

centers of the movable and stationary sheaves, in order to obtain the maximum effect. If the rope pulls on the movable sheave at an angle, its effect will be a displacement sideways, instead of straight up.

78. The **Weston differential pulley block,** more commonly known as a **chain hoist,** is shown in Fig. 32. With the help of this device, one man can hoist a load far beyond what can be done with the ordinary block and tackle. There are two pulleys slightly different in size alongside of each other in the stationary block and rigidly connected so as to turn together. An endless chain passes over both pulleys and supports the movable block in one of its bights. The other loop, or bight, is free and forms the hauling part. This kind of a pulley block possesses the valuable property that the load can be stopped anywhere by simply ceasing to haul on the hoisting part of the free loop.

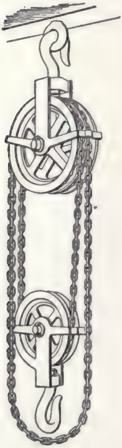


FIG. 32.

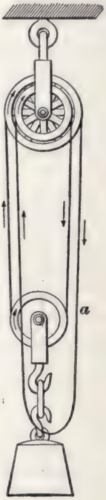


FIG. 33.

In Fig. 33 the differential pulley block is shown in diagrammatic form. Suppose that the part *a* of the free bight is pulled downwards, that is, in the direction of the arrow. Evidently, the upper pulleys will both turn in the direction of the hands of a watch, as indicated by the curved arrow. Then, the left-hand part of the bight supporting the movable pulley will move upwards, and the right-hand part downwards. But as the large and small pulley of the upper block move together, it follows that the left-hand side of the bight supporting the movable sheave will run faster

over the large pulley than the right-hand side of the bight will run downwards over the small pulley. The result is that the upward motion of the movable block will be equal to one-half the difference of the distances passed over in the same time by fixed points in both sides of the bight.

79. Differential chain hoists are invariably so constructed that one man can hoist any load up to the maximum capacity of the hoists. This maximum load will usually be found stamped on the upper block, and there is, hence, no need in practice of calculating the force that must be exerted to hoist a load with a chain hoist.

EXAMPLES FOR PRACTICE.

1. In a block and tackle with four parts of the rope supporting the load, (*a*) what force will be required to lift 300 pounds, assuming that there are no friction losses? (*b*) What force may be expected to be actually required, taking friction into account?

Ans. $\begin{cases} (a) & 75 \text{ lb.} \\ (b) & 100 \text{ lb.} \end{cases}$

2. A man weighing 120 pounds throws his whole weight on the free end of the rope of a block and tackle with six parts of the rope supporting the load. What weight can he raise, (*a*) assuming that there are no friction losses? (*b*) taking friction into account?

Ans. $\begin{cases} (a) & 720 \text{ lb.} \\ (b) & 475 \text{ lb., about} \end{cases}$

THE INCLINED PLANE.

80. An **inclined plane** is a slope, or flat surface, making an angle with another surface or base plane.

81. Three cases may arise in practice with an inclined plane having a horizontal base plane: (1) Where the force acts parallel to the plane, as

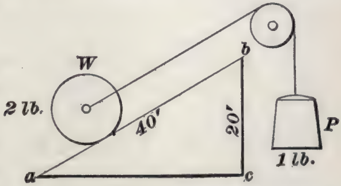


FIG. 34.

in Fig. 34. (2) Where the force acts parallel to the base, as in Fig. 35. (3) Where the force acts at an angle to the plane or to the base, as in Fig. 36.

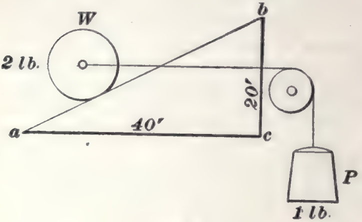


FIG. 35.

82. Relation Between Force and Weight.—In Fig. 34, the relation existing between the force and the weight is easily found. The weight ascends a distance equal to $c\,b$, or the height of the inclined plane, while the force acts through a distance equal to $a\,b$, or the length of the inclined plane. Therefore,

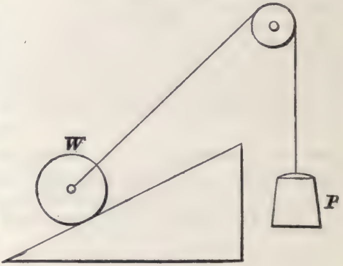


FIG. 36.

Rule 30.—*To find the force, multiply the weight by the height of the plane and divide the product by the length of the plane. To find the weight that can be raised, multiply the force by the length of the plane and divide this product by the height of the plane.*

Or, let $P =$ force;
$l =$ length of the plane;
$b =$ base of the plane;
$h =$ height of plane;
$W =$ weight.

Then, $$P = \frac{W h}{l} \text{ and } W = \frac{P l}{h}.$$

EXAMPLE 1.—The length of an inclined plane is 40 feet and its height is 5 feet; what force is required to sustain a weight of 100 pounds?

SOLUTION.—Applying rule **30**, we have

$$P = \frac{100 \times 5}{40} = 12\tfrac{1}{2} \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—The length of an inclined plane is 8 feet and its height is 15 inches; what weight will a force of 120 pounds support?

SOLUTION.—Applying the rule just given,

$$W = \frac{120 \times 96}{15} = 768 \text{ lb.} \quad \text{Ans.}$$

83. In Fig. 35, the force is supposed to act parallel to the base for any position of W; therefore, while W is moving from the level $a\,c$ to b, or through the height $c\,b$ of the inclined plane, P will move through a distance equal to the length of the base $a\,c$. When the force acts parallel to the base, we have

Rule 31.—*To find the force required, multiply the weight by the height of the plane and divide the product by the length of the base of the plane. To find the weight that can be raised, multiply the force by the length of the base of the plane and divide the product by the height of the plane.*

Or, $$P = \frac{W\,h}{b} \text{ and } W = \frac{P\,b}{h}.$$

EXAMPLE 1.—With a base 30 feet long and a height of 6 feet, what force will sustain a weight of 75 pounds?

SOLUTION.—By rule **31**,

$$P = \frac{75 \times 6}{30} = 15 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—A force of 12 pounds sustains a weight on a plane whose base is 6 feet long and whose height is 18 inches; find the weight.

SOLUTION.—Applying the rule just given, we have

$$W = \frac{12 \times 6}{1\tfrac{1}{2}} = 48 \text{ lb.} \quad \text{Ans.}$$

For Fig. 36, no rule can be given. The ratio of the power varies as the position of W varies; it may be determined by the principle of the resolution of forces. As W

shifts its position, the angle that its cord makes with the face of the plane varies, and the magnitude of the force P depends on this angle; the smaller the angle the less is P required to be in order to support a given weight on a given plane.

84. Applications of the Inclined Plane.—The wedge is a movable inclined plane that in various modifications serves for many different purposes. In the form of a stake wedge, as shown in Fig. 37, it serves to move heavy weights or to separate closely joined pieces of material. In the form of a cold chisel or knife, it serves to sever substances. In the form of a key, it serves to move the brasses in the different pin-joint connections of a steam engine. Familiar examples of inclined planes about machinery are the keys used for fastening a crank to the crank-shaft or a pulley to the shaft; in some designs of cross-heads, the cotters securing the crosshead to the piston rod are an example of an inclined plane; as are also the adjustable shoes in many designs of crossheads for steam engines.

FIG. 37.

85. Taper of Keys.—In keys, cotters, and stake wedges, it is customary to express the inclinations as a certain amount per foot, or taper per foot. Thus, in Fig. 38, the inclination would be $3 - 2 = 1$ inch in 12 inches, and the taper would be, as there are 12 inches in a foot, 1 inch per foot. The taper per foot can be ascertained by the following rule,

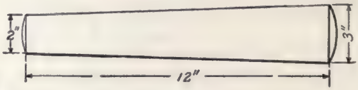


FIG. 38.

where $a =$ the large diameter in inches;
$b =$ the small diameter in inches;
$l =$ the length in inches;
$T =$ taper in inches per foot.

Rule 32.—*Subtract the small diameter from the large diameter, multiply the remainder by 12, and divide the product by the length of the key.*

Or,
$$T = \frac{(a - b)\ 12}{l}.$$

EXAMPLE.—A key measures 4 inches at the large end and 3 inches at the small end; if its length is 16 inches, what is the taper per foot?

SOLUTION.—Applying the rule just given, we get

$$T = \frac{(4 - 3) \times 12}{16} = \tfrac{3}{4} \text{ in. per ft.} \quad \text{Ans.}$$

86. Where tapered keys are used for adjusting brasses, it is often desirable to know how much the brasses will be closed in on the pin by a given movement of the key, or how much to back out a key that has been driven home until the brasses nip the pin, the clearance between the brasses and the pin necessary for satisfactory running having been decided on.

Let T = taper in inches per foot;
d = distance the key is driven in inches;
x = amount the brass is moved in inches.

Rule 33.—*To find the amount the brass has been moved, multiply the taper by the distance the key is driven and divide the product by 12. To find how far to back out a key, multiply the amount the brass is to be moved back by 12 and divide by the taper.*

Or,
$$x = \frac{T d}{12} \text{ and } d = \frac{12 x}{T}.$$

EXAMPLE.—In Fig. 39, the key has been driven home until the brasses nip the pin; it has been determined to run the brasses with a clearance of $\frac{1}{64}$ inch; how much must the key be backed out, its taper being $\frac{3}{4}$ inch per foot?

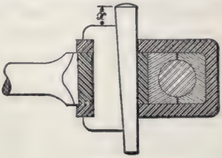


FIG. 39.

SOLUTION.—By rule **33**,

$$d = \frac{12 \times \frac{1}{64}}{\frac{3}{4}} = \tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

A ready way of measuring the distance the key is driven back is to measure from the top surface of the cotter to the upper edge of the key while the key is home. Add the amount the key is to be driven back, and move it until a rule shows it to have been driven the right amount. This method is preferable to making scriber marks on the flat sides of the key, as there is no liability of getting mixed up by the multiplicity of marks sure to be found after years of service.

87. Keys for fastening pulleys to shafts in order to cause them to rotate together belong usually to one of the three classes shown in Fig. 40. The one shown at *a* is a concave key that is hollowed out to fit the shaft. As it holds the pulley by friction alone, it is only suitable for light work. In case such a key slips, its holding power can be increased a little by hollowing out its concave side to a radius smaller than

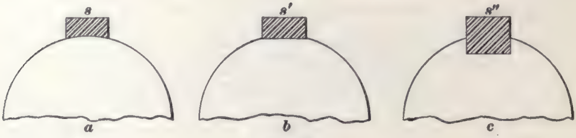


FIG. 40.

the radius of the shaft. The flat key is shown in Fig. 40 at *b*; a flat surface is cut on the shaft, and, consequently, the key is more effective than the concave key, as far as slipping under a rotative strain is concerned. The sunk key shown in Fig. 40 at *c* is the most effective, since it is impossible for the pulley to turn on the shaft without shearing off the key. Keys for fastening pulleys, etc. are usually given a taper of from $\frac{1}{8}$ to $\frac{1}{4}$ inch per foot.

EXAMPLES FOR PRACTICE.

1. An inclined plane is 30 feet long and 7 feet high; what force is required to roll a barrel of flour weighing 196 pounds up the plane, the friction being neglected? Ans. 45.7 lb.

2. What is the taper per foot of a stake wedge measuring $\frac{1}{32}$ inch at the small end and $\frac{1}{4}$ inch at the large end, its length being 4 inches?

Ans. $\frac{21}{32}$ in. per ft.

3. Given, a key having a taper of 1 inch per foot. How much must it be backed out to give the brasses a working clearance of $\frac{1}{32}$ inch?

Ans. $\frac{3}{8}$ in.

THE SCREW.

88. A **screw** is a cylinder having a helical projection winding around its circumference. This helix is called the **thread** of the screw. The distance that a point on the helix is drawn back or advanced in the direction of the length of the screw during one turn is called the **pitch** of the screw. The screw in Fig. 41 is turned in a nut *a* by means of a force applied at the end of the handle *P*. For one complete revolution of the handle, the screw will be advanced lengthwise an amount equal to the pitch. If the nut is fixed and a weight placed upon the end of the screw, as shown, it will be raised vertically, by one revolution of the screw, a distance equal to the pitch. During this revolution, the force at *P* will move through a distance equal to the circumference of a circle whose radius is *P F*. Therefore, *W* × *pitch of thread* = *P* × *circumference of circle through which P moves.*

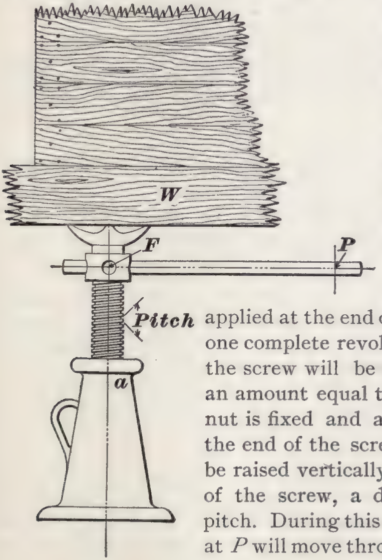


FIG. 41.

89. Owing to the friction between the nut and the screw and between the pivot of the screw and its cap, the weight that can be raised with a screw, or the force that can be

exerted by it in the direction of its axis, is far less than that calculated on the assumption that friction does not exist. The friction depends on the pitch, the condition of the thread on the screw and in the nut, and the nature of the lubricant.

From a long series of experiments, Mr. Wilfred Lewis has deduced a formula for determining a factor by which to multiply the weight that could be raised if there were no friction, so as to determine the probable actual weight that can be raised.

Let d = diameter of screw in inches;
p = pitch in inches;
E = the number by which to multiply the theoretical weight in order to obtain the approximate actual weight.

Rule 34.—*Add the diameter of the screw to the pitch and divide the pitch by this sum; the quotient so obtained will be the factor by which to multiply the theoretical weight in order to determine the approximate actual weight.*

Or,
$$E = \frac{p}{p + d}.$$

Example.—With a screw having 4 threads to the inch and 2 inches in diameter, what is the factor by which to determine how much of the theoretical load can probably be raised?

Solution.—Since there are 4 threads per inch, the pitch is $1 \div 4$, or $\frac{1}{4}$ inch. Then applying rule **34**, we get

$$E = \frac{\frac{1}{4}}{\frac{1}{4} + 2} = .11. \quad \text{Ans.}$$

90. The weight that can be lifted when friction is neglected, or the force that can be exerted by a screw when a given force is exerted on the lever, is given by the following rule,

where W = weight or force exerted by the screw;
P = force applied to the end of the lever;
p = pitch;
r = distance from the axis of the screw to the point of application of the force P.

Rule 35.—*Multiply the force applied to the lever, in pounds, by the distance of its point of application from the axis of the screw, in inches, and multiply this product by 6.2832; divide the result by the pitch in inches and the quotient will be the theoretical force that will be exerted by the screw.*

Or, $$W = \frac{6.2832\,P\,r}{p}.$$

91. To find the theoretical force that must be applied to the lever to raise a given weight or to exert a given force:

Rule 36.—*Multiply the weight or given force by the pitch and divide this product by the product of 6.2832 and the distance from the axis of the screw to the point on the lever where the force is applied.*

Or, $$P = \frac{W\,p}{6.2832\,r}.$$

92. To obtain the probable actual weight that may be raised, multiply the result obtained by rule **35** by the value obtained from rule **34.** To obtain the probable force that must be applied to the lever in order to lift a given weight, divide the result obtained from rule **36** by the value obtained from rule **34.**

Example 1.—A screw jack has a screw $1\frac{3}{4}$ inches diameter with 5 threads to the inch. If a man pulls at the end of the lever 14 inches from the axis with a force of 40 pounds, what weight may he expect to raise?

Solution.— 5 threads per inch = $\frac{1}{5}$-inch pitch.

By rule **35**, $W = \dfrac{6.2832 \times 40 \times 14}{\frac{1}{5}} = 17{,}593$ pounds nearly.

By rule **34**, $$E = \frac{\frac{1}{5}}{\frac{1}{5} + 1\frac{3}{4}} = .1, \text{ nearly.}$$

Then, as just stated, the probable weight that may be raised is

$$17{,}593 \times .1 = 1{,}759.3 \text{ lb.} \quad \text{Ans.}$$

Example 2.—The pull that an ordinary man can exert at the end of a lever is usually taken at 40 pounds; would it be possible for one man

to raise a weight of 3 tons with a jack-screw having 4 threads to the inch and $2\frac{1}{2}$ inches in diameter when the longest lever available is 24 inches?

SOLUTION.—According to rule **36,** the theoretical force that must be exerted is

$$\frac{3 \times 2000 \times \frac{1}{4}}{6.2832 \times 24} = 9.9 \text{ pounds, nearly.}$$

By rule **34,**
$$E = \frac{\frac{1}{4}}{\frac{1}{4} + 2\frac{1}{2}} = .091, \text{ nearly.}$$

Then, as just stated, the probable actual force required is 9.9. $9.9 \div .091 = 109$ pounds, about, or more than a single man may be expected to exert steadily. Ans.

SCREWS USED AS FASTENINGS.

93. In machine construction, screws are used more frequently as fastening devices than as a means of transmitting motion or producing pressure. Screws used as fastening devices may be divided into two general classes, which are distinguishable from each other by the form of thread used: (1) *Metal screws*, or screws intended to be screwed into metals. (2) *Wood screws*, or screws intended for wood.

94. Metal screws have either a sharp **V** thread or the United States standard thread shown in Fig. 42. In the

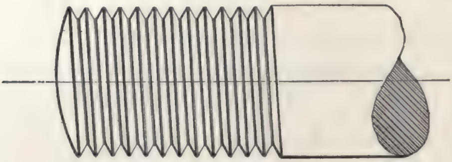


FIG. 42.

latter, as also in the sharp **V** thread, the sides of the thread form a 60° angle. In the United States standard thread, the thread is flattened at the top and bottom, as shown in

the figure. The advantage of this is that it makes a stronger screw for equal pitches and diameter.

95. Metal screws are divided by the trade into *machine bolts*, *capscrews*, and *machine screws*.

96. In **machine bolts,** or **bolts,** for short, one of which is shown in Fig. 43 (*a*), the shank and head are left rough, or just as they come from the bolt-heading machine. Machine bolts are regularly made in lengths varying by

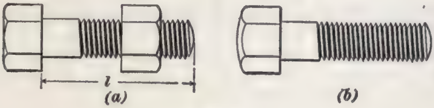


FIG. 43.

half inches from $1\frac{1}{2}$ to 8 inches. Above 8 inches and up to 20 inches the length varies by whole inches. The length of a machine bolt is always measured under the head, as the distance l in Fig. 43 (*a*).

97. The standard diameters in which machine bolts are regularly made are $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{7}{16}$, $\frac{1}{2}$, $\frac{9}{16}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, and 1 inch. Sizes differing from these are special and can only be obtained on special order. By the *diameter of a bolt* is always meant its diameter over the top of the thread. Machine bolts can be obtained with either hexagonal or square heads and with hexagonal or square nuts.

98. **Capscrews,** one of which is shown in Fig. 43 (*b*), are bolts with either hexagonal or square heads; they have the shanks turned and the heads finished all over by milling and turning. Unlike machine bolts, they are usually furnished without nuts, unless especially ordered.

99. Machine bolts and capscrews are made with a pitch of thread corresponding to the United States standard thread, as shown in Table II

TABLE II.

STANDARD THREADS PER INCH.

Diameter. Inches.	Threads per Inch.	Diameter. Inches.	Threads per Inch.
$\frac{1}{4}$	20	$1\frac{1}{4}$	7
$\frac{5}{16}$	18	$1\frac{3}{8}$	6
$\frac{3}{8}$	16	$1\frac{1}{2}$	6
$\frac{7}{16}$	14	$1\frac{5}{8}$	$5\frac{1}{2}$
$\frac{1}{2}$	13	$1\frac{3}{4}$	5
$\frac{9}{16}$	12	$1\frac{7}{8}$	5
$\frac{5}{8}$	11	2	$4\frac{1}{2}$
$\frac{3}{4}$	10	$2\frac{1}{4}$	$4\frac{1}{2}$
$\frac{7}{8}$	9	$2\frac{1}{2}$	4
1	8	$2\frac{3}{4}$	4
$1\frac{1}{8}$	7	3	$3\frac{1}{2}$

The only deviation from this table of threads is in $\frac{1}{2}$-inch square and hexagonal capscrews, where the usual number of threads for a sharp **V** thread is 12 per inch instead of 13, as called for. Capscrews are usually furnished with a sharp **V** thread, but can now be obtained with the United States standard thread. Machine bolts usually have the United States standard thread and the number of threads per inch called for in the table.

100. Machine screws are small screws with a slotted head. They are known to the trade as *flat heads*, shown in Fig. 44 (*a*); *round heads*, shown in Fig. 44 (*b*); *button heads*, shown in Fig. 44 (*c*), and *fillister heads*, shown in Fig. 44 (*d*).

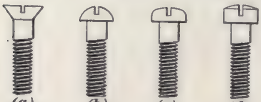

(a) (b) (c) (d)

FIG. 44.

101. Screws for uniting metals are made in a variety of forms that differ from those here shown to suit special

conditions. Some of these are setscrews, carriage bolts, tire bolts, boiler patch bolts, elevator bolts, etc. Most of these can only be obtained on special order, although some of the more common sizes may be found in stock.

102. **Wood screws** are either flat headed or round headed, with a slotted head. When wood screws have a square head for a wrench, instead of a slot for a screw-driver, they are known as *lagscrews*, or *coach screws.*

VELOCITY RATIO AND EFFICIENCY.

VELOCITY RATIO.

103. By the term **velocity ratio** is meant the ratio of the distance through which the force acts to the corresponding distance the weight moves. Thus, if the force acts through a distance of 12 inches while the weight moves 1 inch, the velocity ratio is 12 to 1, or 12; that is, P moves 12 times as fast as W, P representing the force and W the weight.

If the velocity ratio is known, the theoretical weight, that is, the weight that any machine will raise under the assumption that there are no frictional resistances, can be found by multiplying the force by the velocity ratio. If the velocity ratio is 8.7 to 1, or 8.7, $W = 8.7 \times P$, since $W \times 1 = P \times 8.7$.

104. From the preceding article it is seen that when the theoretical weight which may be lifted by a given force is known, the velocity ratio can be obtained by dividing this weight by the force. For example, if a force of 1 pound is applied to the handle of a screw whose pitch is $\frac{1}{4}$ inch, at a distance of 14 inches from the axis, it will lift, according to rule **35**, a theoretical weight equal to

$$W = \frac{6.2832 \times 1 \times 14}{\frac{1}{4}} = 351.86 \text{ pounds.}$$

Since a force of 1 pound lifts a theoretical weight of 351.86 pounds, the velocity ratio = 351.86 ÷ 1 = 351.86. This result can be verified by dividing the distance the point at which the force is applied moves in one revolution by the distance the screw advances in the same time. The point at which the force is applied travels in one revolution $2 \times 14 \times 3.1416 = 87.965$ inches. The screw advances in one revolution a distance equal to its pitch, or $\frac{1}{4}$ inch; hence, the velocity ratio = $87.965 \div \frac{1}{4} = 351.86$, the same value as obtained before.

EFFICIENCY.

105. The amount of work that can be obtained from a machine is always less than that required to operate it. This is due to the fact that some work is always lost in overcoming friction or other resistances. The ratio of the amount of useful work which may be obtained from a machine to the corresponding amount of work required to drive it is called the **efficiency** of the machine. In the case of the simple machines, previously described, the efficiency is the quotient found by dividing the weight which can be actually raised by that which may be raised theoretically.

EXAMPLE.—It is found that with a certain block and tackle having two movable sheaves a force of 50 pounds will lift a weight of 172 pounds; what is the efficiency?

SOLUTION.—According to rule **27**, the theoretical weight that can be raised, there being 4 parts of the rope supporting the load, is $4 \times 50 = 200$ lb. Then,

Efficiency $= \frac{172}{200} = .86$, or 86 per cent. Ans.

106. If the efficiency of a machine is known, the actual force required to raise a given load may be found by dividing the theoretical force required to raise the load by the efficiency. Thus, if it is found that a theoretical force of 85 pounds is required to lift a certain load by means of a screw, and the efficiency is 40 per cent., the actual force needed is $85 \div .40 = 212.5$ pounds.

107. If the efficiency is known, the weight that a certain force will raise may be found by multiplying together the force, velocity ratio, and the efficiency. Thus, if a certain machine has a velocity ratio of 6.5 and an efficiency of 78 per cent., a force of 140 pounds will raise a weight of $140 \times 6.5 \times .78 = 709.9$ pounds.

EXAMPLES FOR PRACTICE.

1. The distance from the axis of a screw to the point on the handle where the force is applied is 12 inches. The screw has 8 threads per inch and is 1 inch in diameter. (*a*) What force is necessary to raise a weight of 1,248 pounds, assuming that there is no friction? (*b*) What force will probably be actually required?

Ans. { (*a*) 2.07 lb., nearly. (*b*) 19 lb., nearly.

2. What weight can actually be raised with a screw jack having a screw $1\frac{1}{4}$ inches in diameter and 6 threads per inch, when a man is pulling with a force of 40 pounds at the end of a bar 16 inches long?

Ans. 2,838 lb., nearly.

3. In example 2, what is the velocity ratio? Ans. 603, nearly.

4. In example 2, what is the efficiency? Ans. $11\frac{3}{4}$ per cent.

MECHANICS OF FLUIDS.

HYDROSTATICS.

1. Hydrostatics treats of liquids at rest under the action of forces.

2. Liquids are very nearly *incompressible*. A pressure of 15 pounds per square inch compresses water less than $\frac{1}{20000}$ of its volume.

3. Hydrostatic Pressure.—Fig. 1 represents two cylindrical vessels of exactly the same size. The vessel a is fitted with a wooden block of the same size as, and free to move in, the cylinder; the vessel b is filled with water, whose depth is the same as the length of the wooden block in a. Both vessels are fitted with air-tight pistons P, each of whose areas are 10 square inches.

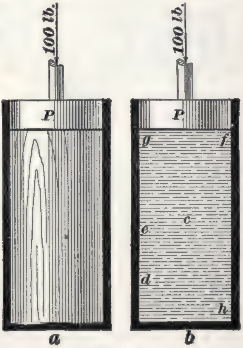


FIG. 1.

Suppose, for convenience, that the weights of the pistons, block, and water be neglected, and that a force of 100 pounds be applied to both pistons. The pressure per square inch will be $\frac{100}{10} = 10$ pounds. In the vessel a this pressure will be transmitted to the bottom of the vessel, and will be 10 pounds per square inch; it is easy to see that there will

be no pressure on the sides. In the vessel b an entirely different result is obtained. The pressure on the bottom will be the same as in the other case, that is, 10 pounds per square inch, but, owing to the fact that the molecules of the water are perfectly free to move, this pressure of 10 pounds per square inch is *transmitted in every direction with the same intensity;* that is to say, the pressure at any point c, d, e, f, g, h, etc., due to the force of 100 pounds, is exactly the same, and equals 10 pounds per square inch.

4. The foregoing fact may be easily proved experimentally by means of an apparatus like that shown in Fig. 2. Let the area of the piston a be 20 square inches; of b, 7 square inches; of c, 1 square inch; of d, 6 square inches; of e, 8 square inches; and of f, 4 square inches.

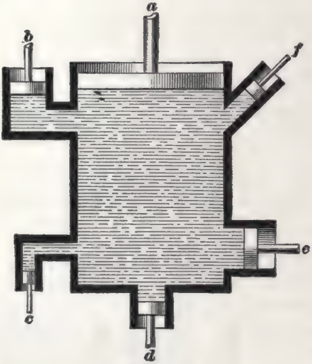


FIG. 2.

If the pressure due to the weight of the water be neglected, and a force of 5 pounds be applied at c (whose area is 1 square inch), a pressure of 5 pounds per square inch will be transmitted in all directions, and in order that there shall be no movement, a force of $6 \times 5 = 30$ pounds must be applied at d, 40 pounds at e, 20 pounds at f, 100 pounds at a, and 35 pounds at b.

If a force of 99 pounds were applied to a, instead of 100 pounds, the piston a would rise, and the other pistons b, c, d, e, and f would move inwards; but, if the force applied to a were 100 pounds, they would all be in equilibrium. If 101 pounds were applied at a, the pressure per square inch would be $\frac{101}{20} = 5.05$ pounds, which would be transmitted in all directions; and, since the pressure due to the load on c is

only 5 pounds per square inch, it is now evident that the piston *a* will move downwards, and the pistons *b*, *c*, *d*, *e*, and *f* will be forced outwards.

5. Pascal's Law.—The whole of the preceding may be summed up as follows:

The pressure per unit of area exerted anywhere on a liquid is transmitted undiminished in all directions and acts with the same force on all surfaces, in a direction at **right angles** *to those surfaces.*

This law, first discovered by Pascal, and accordingly named after him, is the most important one in **hydrostatics.** Its meaning should be thoroughly understood.

EXAMPLE.—If the area of the piston *e*, in Fig. 2, were 8.25 square inches and a force of 150 pounds were applied to it, what forces would have to be applied to the other pistons to keep the water in equilibrium, assuming that their areas were the same as given before?

SOLUTION.— $\frac{150}{8.25} = 18.182$ pounds per square inch, nearly.

$$\left.\begin{array}{l} 20 \times 18.182 = 363.64 \text{ lb.} = \text{force to balance } a. \\ 7 \times 18.182 = 127.274 \text{ lb.} = \text{force to balance } b. \\ 1 \times 18.182 = 18.182 \text{ lb.} = \text{force to balance } c. \\ 6 \times 18.182 = 109.092 \text{ lb.} = \text{force to balance } d. \\ 4 \times 18.182 = 72.728 \text{ lb.} = \text{force to balance } f. \end{array}\right\} \text{Ans.}$$

6. *The pressure due to the weight of a liquid may be downwards, upwards, or sidewise.*

7. Downward Pressure.—In Fig. 3 the pressure on the bottom of the vessel *a* is, of course, equal to the weight of the water it contains. If the area of the bottom of the vessel *b* and the depth of the liquid contained in it are the same as in the vessel *a*, the pressure on the bottom of *b* will be the same as on the bottom of *a*. Suppose the bottoms of the vessels *a* and *b* are 6 inches square, and that the part *c d* in the vessel *b* is 2 inches square, and that they are filled with water. Then, since 1 cubic foot of water weighs 62.5 pounds, the weight of 1 cubic inch of water is $\frac{62.5}{1,728}$ pound = .03617 pound. The number of cubic inches

in $a = 6 \times 6 \times 24 = 864$ cubic inches; and the weight of the water is $864 \times .03617 = 31.25$ pounds. Hence, the total pressure on the bottom of the vessel a is 31.25 pounds, or $\frac{31.25}{6 \times 6} = .868$ pound per square inch.

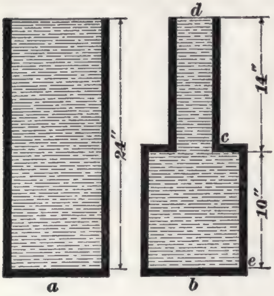


FIG. 3.

The pressure in b due to the weight contained in the part $b\,c$ is $6 \times 6 \times 10 \times .03617 = 13.02$ pounds.

The weight of the part contained in $c\,d$ is $2 \times 2 \times 14 \times .03617 = 2.0255$ pounds, and the weight per square inch of area in $c\,d$ is $\frac{2.0255}{4} = .5064$ pound.

8. According to Pascal's law, this weight (pressure) is transmitted equally in all directions, therefore, an extra weight of .5064 pound is imposed on every square inch of the bottom of $b\,c$; the area of this is $6 \times 6 = 36$ square inches, and the pressure on it due to the water contained in $c\,d$ is, therefore, $36 \times .5064 = 18.23$ pounds; thus, we have a total pressure on the bottom of vessel b of $13.02 + 18.23 = 31.25$ pounds, the same as in vessel a. As a result of the above law, there is also an upward pressure of .5064 pound acting on every square inch of the *top* of the enlarged part $b\,c$.

If an additional pressure of 10 pounds per square inch were applied to the upper surface of both vessels, the total pressure on each bottom would be $31.25 + (6 \times 6 \times 10) = 31.25 + 360 = 391.25$ pounds.

If this pressure were to be obtained by means of a weight placed on each piston (as shown in Fig. 1 at a and b), we would have to put a weight of $6 \times 6 \times 10 = 360$ pounds on the piston in vessel a, Fig. 3, and one of $2 \times 2 \times 10 = 40$ pounds on the piston in vessel b.

9. The general law for the downward pressure on the bottom of any vessel:

Rule 1.—*The pressure on the bottom of a vessel containing a fluid is independent of the shape of the vessel, and is equal to the weight of a column of the fluid, the area of whose base is equal to that of the bottom of the vessel and whose altitude is the distance between the bottom and the upper surface of the fluid, increased by the pressure per unit of area on the upper surface of the fluid multiplied by the area of the bottom of the vessel, in case there is any pressure on the surface.*

10. Suppose that the vessel *b*, in Fig. 3, were inverted, as shown in Fig. 4, the pressure on the bottom would still be .868 pound per square inch, but it would require a weight of 3,490 pounds on a piston at the upper surface to make the pressure on the bottom 391.25 pounds, instead of a weight of 40 pounds, as in the other case.

FIG. 4.

EXAMPLE.—A vessel filled with salt water, weighing .037254 pound per cubic inch, has a circular bottom 13 inches in diameter. The top of the vessel is fitted with a piston 3 inches in diameter, on which is laid a weight of 75 pounds; what is the total pressure on the bottom, if the depth of the water is 18 inches?

SOLUTION.—Applying the rule, we have

$13 \times 13 \times .7854 \times 18 \times .037254 = 89.01$ pounds, the pressure due to the weight of the water.

$\dfrac{75}{3 \times 3 \times .7854} = 10.61$ pounds per square inch, due to the weight on the piston.

$$13 \times 13 \times .7854 \times 10.61 = 1{,}408.29 \text{ pounds.}$$

$$1{,}408.29 + 89.01 = 1{,}497.3 \text{ lb.} = \text{total pressure. Ans.}$$

11. Upward Pressure.—In Fig. 5 is represented a vessel of exactly the same size as that shown in Fig. 4. There is no upward pressure on the surface *c* due to the weight of the water in the large part *c d*, but there is an upward pressure on *c* due to the weight of the water in the

small part $b\,c$. The pressure per square inch due to the weight of the water in $b\,c$ was found to be .5064 pound; the area of the upper surface c of the large part $c\,d$ is $(6 \times 6) - (2 \times 2) = 36 - 4 = 32$ square inches, and the total upward pressure due to the weight of the water is $.5064 \times 32 = 16.2$ pounds.

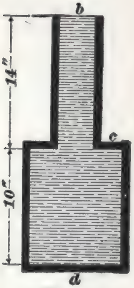


FIG. 5.

If an additional pressure of 10 pounds per square inch were applied to a piston fitting in the top of the vessel, the total upward pressure on the surface c would be $16.2 + (32 \times 10) = 336.2$ pounds.

12. General law for upward pressure:

Rule 2.—*The upward pressure on any submerged horizontal surface equals the weight of a column of the liquid whose base has an area equal to the area of the submerged surface and whose altitude is the distance between the submerged surface and the upper surface of the liquid, increased by the pressure per unit of area on the upper surface of the fluid multiplied by the area of the submerged surface, in case of any pressure on the upper surface.*

EXAMPLE.—A horizontal surface 6 inches by 4 inches is submerged in a vessel of water 26 inches below the upper surface; if the pressure on the water is 16 pounds per square inch, what is the total upward pressure on the horizontal surface?

SOLUTION.—Applying the rule, we get $6 \times 4 \times 26 \times .03617 = 22.57$ pounds for the upward pressure due to the weight of the water, and $6 \times 4 \times 16 = 384$ pounds for the upward pressure due to the outside pressure of 16 pounds per square inch.

Therefore, $384 + 22.57 = 406.57$ lb. = the total upward pressure. Ans.

13. Lateral (Sidewise) Pressure.—Suppose the top of the vessel shown in Fig. 6 is 10 inches square and that the projections at a and b are 1 inch square.

The pressure per square inch on the bottom of the vessel due to the weight of the liquid is $1 \times 1 \times 18 \times$ the weight of a cubic inch of the liquid.

The pressure at a depth equal to the distance of the upper surface of *b* is $1 \times 1 \times 17 \times$ the weight of a cubic inch of the liquid.

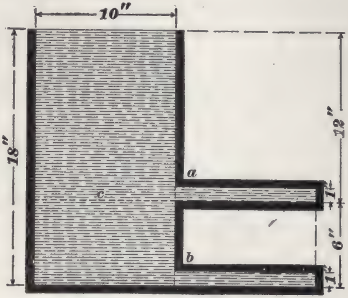


FIG. 6.

Since both of these pressures are transmitted in every direction, they are also transmitted laterally (sidewise), and the *pressure per unit of area on the projection b is a mean between the two*, and equals $1 \times 1 \times \frac{17 + 18}{2} \times$ the weight of a cubic inch of the liquid.

To find the lateral pressure on the projection *a*, imagine that the dotted line *c* is the bottom of the vessel; then, the conditions would be the same as in the preceding case, except that the depth is not so great.

The lateral pressure per square inch on *a* is thus seen to be $1 \times 1 \times \frac{11 + 12}{2} \times$ the weight of a cubic inch of the liquid.

14. General law for lateral pressure:

Rule 3.—*The pressure on any vertical surface due to the weight of the liquid is equal to the weight of a column of the liquid whose base has the same area as the vertical surface and whose altitude is the depth of the center of gravity of the vertical surface below the upper surface of the liquid.*

Any additional pressure is to be added, as in the previous cases.

EXAMPLE.—A well 3 feet in diameter and 20 feet deep is filled with water; what is the pressure on a strip of wall 1 inch wide, the top of which is 1 foot from the bottom of the well? What is the pressure on the bottom? What is the upward pressure per square inch 2 feet 6 inches from the bottom?

SOLUTION.—Applying the rule, the area of the strip is equal to its length (= circumference of well) multiplied by its height. The length $= 36 \times 3.1416 = 113.1$ inches; height $= 1$ inch; hence, area of

strip = 113.1 × 1 = 113.1 square inches. Depth of center of gravity of strip = (20 − 1) feet + ½ inch, the half width of strip = 228½ inches. Consequently, 113.1 × 228.5 × .03617 = 934.75 lb. = the total pressure on the strip. Ans.

The pressure on each square inch of the strip is $\frac{934.75}{113.1} = 8.265$ lb., nearly. Ans.

36 × 36 × .7854 × 20 × 12 × .03617 = 8,836 lb., the pressure on the bottom. Ans.

20 − 2.5 = 17.5. 1 × 17.5 × 12 × .03617 = 7.596 lb., the upward pressure per square inch 2 ft. 6 in. from the bottom. Ans.

15. The effects of lateral pressure are illustrated in Fig. 7. In the figure, *e* is a tall vessel having a stop-cock near its base and arranged to float upon the water, as shown. When this vessel is filled with water, the lateral pressures at any two points of the surface of the vessel opposite each other are equal. Being equal and acting in opposite directions, they destroy each other, and no motion can result; but, if the stop-cock is opened, there will be no resistance to the pressure acting on that part, and the water will flow out; at the same time, the reaction of the jet issuing from the stop-cock causes the vessel to move backwards through the water in a direction opposite to that of the issuing jet.

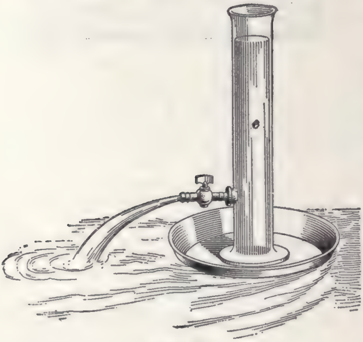


FIG. 7.

16. Liquids Influenced by Gravity.—Since the pressure on the bottom of a vessel due to the weight of the liquid is dependent only on the height of the liquid and not on the shape of the vessel, it follows that if a vessel has a number of radiating tubes (see Fig. 8) the water in each

tube will be on the same level, no matter what may be the shape of the tubes. For, if the water were higher in one tube than in the others, the downward pressure on the bottom due to the height of the water in this tube would be greater than that due to the height of the water in the other tubes. Consequently, the upward pressure would also be greater; the equilibrium would be destroyed and the water

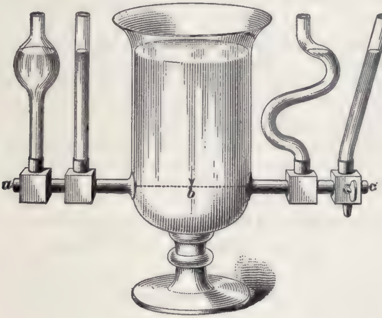


FIG. 8.

would flow from this tube into the vessel and rise in the other tubes until it was at the same level in all, when it would be in equilibrium. This principle is expressed in the familiar saying, *water seeks its level.*

An application of this principle is the glass water gauge used for showing the level of the water in a steam boiler or tank.

17. The above principle explains why city water reservoirs are located on high elevations and why water on leaving the hose nozzle spouts so high.

If there were no resistance by friction and air, the water would spout to a height equal to the level of the water in the reservoirs. If a long, vertical pipe, whose length was equal to the vertical distance between the nozzle and the level of the water in the reservoir, were attached to the nozzle, the

water would just reach the end of the pipe. If the pipe were lowered slightly, the water would trickle out.

Fountains, canal locks, and artesian wells are examples of the application of this principle.

EXAMPLE.—The water level in a city reservoir is 150 feet above the level of the street; what is the pressure of the water per square inch on the hydrant?

SOLUTION.—Applying rule **1**, $1 \times 150 \times 12 \times .03617 = 65.106$ lb. per sq. in. Ans.

NOTE.—In measuring the height of the water to find the pressure that it produces, the **vertical** height, or distance, between the level of the water and the point considered is always taken. This vertical height is called the **head.**

The weight of a column of water 1 inch square and 1 foot high is $62.5 \div 144 = .434$ pound, nearly. Hence, if the depth (head) be given, the pressure per square inch may be found by multiplying the depth in feet by .434. The constant .434 is the one ordinarily used in practical calculations.

18. In Fig. 9, let the area of the piston a be 1 square inch; of b, 40 square inches. According to Pascal's law, 1 pound placed on a will balance 40 pounds placed on b.

Suppose that a moves downwards 10 inches, then 10 cubic inches of water will be forced into the tube b. This will be distributed over the entire area of the tube b, in the form of a cylinder whose cubical contents must be 10 cubic inches, whose base has an area of 40 square inches, and whose altitude must be $\frac{10}{40} = \frac{1}{4}$ inch; that is, a movement of 10 inches of the piston a will cause a movement of $\frac{1}{4}$ inch in the piston b. This is another illustration of the well-known principle of machines: *The force multiplied by the distance through which it acts equals the weight multiplied by the distance through which it moves*, since, if 1 pound on the piston a represents the force P, the equivalent weight W on b may be obtained from the equation $W \times \frac{1}{4} = P \times 10$, whence $W = 40\,P = 40$ pounds.

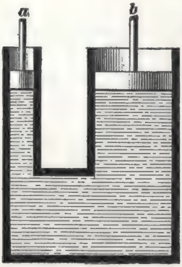


FIG. 9.

Another familiar fact is also recognized, for the velocity

ratio of P to W is 10 : $\frac{1}{4}$, or 40; and since in any machine the weight equals the force multiplied by the velocity ratio, $W = P \times 40$, and when $P = 1$, $W = 40$. An interesting application of this principle is the hydraulic jack, by the aid of which one man can lift a very large load.

19. A Watson-Stillman hydraulic jack is shown in section in Fig. 10 (a). In this illustration, a is a lever that is depressed by the operator when he desires to raise the load. This lever freely fits a rectangular hole or socket in a shaft b that extends clear through the ram head. The shaft b carries the crank c to which the piston d is hinged. A small valve is placed in the center of the piston and serves to open or close communication between the water space above and below the piston. The lower end of the ram is fitted with a small check valve, which normally is held closed by the small spring shown beneath the valve. The inside of the jack is filled with a mixture of alcohol and water, about 2 parts of alcohol to 3 parts of water, which prevents the freezing of the liquid when the jack is used in cold weather. The operation is as follows:

The jack being clear down, as shown in Fig. 10 (a), it is placed under the load and the operator alternately depresses and raises the lever through its full range, the motion being limited by a projection on the under side of the lever. The slightest depression of the lever closes the valve in the piston. The downward movement of the piston continuing, the water between the bottom of the piston and the end of the ram is subjected to a pressure that opens the valve in the end of the ram. This allows the water under pressure to pass into the space beneath the ram. The pressure so transmitted to the lower end of the ram forces it upwards. The lever having been depressed to its limit is again raised. This raises the piston, causes the piston valve to open, and allows the water above the piston to flow beneath it. The pressure of the water below the ram promptly closes the ram valve as soon as the piston commences to move upwards. The operation may now be repeated as often as required.

As will readily be seen, the ram cannot descend under the weight of the load, as the only water passage leading from

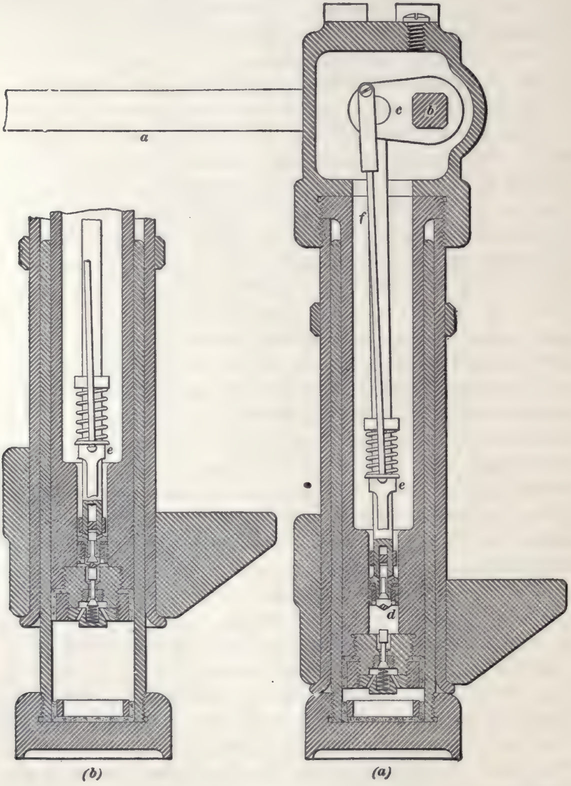


FIG. 10.

the space below the ram is closed by the ram valve, which is kept tightly closed by the pressure of the water beneath

it and also by the spring. Hence, in order to lower the ram, this ram valve must be opened by some means. As previously mentioned, the lever has a projection on its under side to limit its travel and the travel of the piston. This projection is made of such a length that when the lever is down as far as it will go, the piston is still a short distance from the upper end of the ram valve stem.

In order to lower the ram, the lever is withdrawn from its socket and inserted again with the projection upwards. It can now be depressed far enough to cause the end of the piston to strike and open the ram-head valve, as shown in Fig. 10 (*b*), where the parts of the jack are shown in the relative positions occupied while lowering. But before the water in the bottom of the ram can escape to the top, the valve in the piston must also be opened. This is accomplished by a heavy spring that forces the sleeve *e* downwards. The lower end of the sleeve has a cotter working in a slot in the piston rod; the cotter bearing against the top of the valve forces the latter downwards. The sleeve is connected to the crank *c* by the so-called lowering wire *f*, in such a manner that the cotter will not strike the piston valve while the jack is raising the load. The ram will descend as long as the lever is kept hard down; it can be stopped instantly by raising the lever slightly.

20. If the area of the piston is 1 square inch and the area of the ram is 10 square inches, the velocity ratio will be 10. If the length of the lever between the hand of the operator and the fulcrum is 10 times the length between the fulcrum and the piston, the velocity ratio of the lever will be 10, and the total velocity ratio of the hand to the piston will be $10 \times 10 = 100$. Taking the weight of the operator at 150 pounds, his whole weight to be thrown on the lever, the weight that can be raised is $150 \times 100 = 15{,}000$ pounds, or $7\frac{1}{2}$ tons. But if the average movement of the hand is 4 inches per stroke, it will require $\frac{100}{4} = 25$ strokes to raise the load on the jack a distance of 1 inch, and it is again seen that what is gained in force is lost in speed.

21. Other applications of this principle are seen in various hydraulic machines used in boiler shops. A familiar example is the hand test pump used in testing boilers under hydraulic pressure.

EXAMPLE 1.—A vertical cylinder is tested for the tightness of its heads by filling it with water. A pipe whose inside diameter is $\frac{1}{4}$ inch and whose length is 20 feet is screwed into a hole in the upper head and is then filled with water. What is the pressure per square inch on each head if the cylinder is 40 inches in diameter and 60 inches long?

SOLUTION.— $40^2 \times .7854 = 1,256.64$ square inches = area of head.
$1 \times 60 \times .03617 = 2.17$ pounds pressure per square inch on the bottom head due to the weight of the water in the cylinder.

$$(\tfrac{1}{4})^2 \times .7854 = .04909 \text{ square inch, the area of the pipe.}$$

$.04909 \times 20 \times 12 \times .03617 = .426$ pound = the weight of water in the pipe = the pressure on a surface area of .04909 square inch.

The pressure per square inch due to the water in the pipe is $\frac{1}{.04909} \times .426 = 8.68$ lb. per sq. in. upon the upper head. Ans.

The total pressure per square inch on the lower head is $8.68 + 2.17 = 10.85$ lb. Ans.

EXAMPLE 2.—In the other example, if the pipe be fitted with a piston weighing $\frac{1}{4}$ pound and a 5-pound weight be laid on it what is the pressure per square inch on the upper head?

SOLUTION.—In addition to the pressure of .426 pound on the area of .04909 square inch, there is now an additional pressure on this area of $5 + \frac{1}{4} = 5.25$ pounds, and the total pressure on this area is $.426 + 5.25 = 5.676$ pounds. $\frac{1}{.04909} \times 5.676 = 115.6$ lb. = the pressure per square inch. Ans.

22. When calculating the weight that can be raised with a hydraulic jack, no allowance was made for the power lost in overcoming the friction between the cup leathers of the piston and the ram, and the cup leather of the ram and the cylinder; this varies according to the condition of the leathers, and, of course, the smoothness of the ram and cylinder; when the leathers are in good condition, the loss is about 5 per cent. of the total pressure on the ram; when the leathers are old, stiff, and dirty, the loss may amount to 15 per cent. or more.

SPECIFIC GRAVITY.

23. The **specific gravity of a body** is the ratio between its weight and the weight of a like volume of water.

24. Since gases are so much lighter than water, it is usual to take the specific gravity of a gas as the ratio between the weight of a certain volume of the gas and the weight of the same volume of air.

EXAMPLE.—A cubic foot of cast iron weighs 450 pounds; what is its specific gravity, a cubic foot of water weighing 62.42 pounds?

SOLUTION.—According to the definition,

$$\frac{450}{62.42} = 7.21. \quad \text{Ans.}$$

TABLE I.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS METALS.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Platinum	21.50	1,343.8
Gold	19.50	1,218.8
Mercury	13.60	850.0
Lead (cast)	11.35	709.4
Silver	10.50	656.3
Copper (cast)	8.79	549.4
Brass	8.38	523.8
Wrought Iron	7.68	480.0
Cast Iron	7.21	450.0
Steel	7.84	490.0
Tin (cast)	7.29	455.6
Zinc (cast)	6.86	428.8
Antimony	6.71	419.4
Aluminum	2.50	156.3

TABLE II.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS WOODS.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Ash	.845	52.80
Beech	.852	53.25
Cedar	.561	35.06
Cork	.240	15.00
Ebony (American)	1.331	83.19
Lignum-vitæ	1.333	83.30
Maple	.750	46.88
Oak (old)	1.170	73.10
Spruce	.500	31.25
Pine (yellow)	.660	41.20
Pine (white)	.554	34.60
Walnut	.671	41.90

TABLE III.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS LIQUIDS.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Acetic acid	1.062	66.4
Nitric acid	1.217	76.1
Sulphuric acid	1.841	115.1
Muriatic acid	1.200	75.0
Alcohol	.800	50.0
Turpentine	.870	54.4
Sea water (ordinary)	1.026	64.1
Milk	1.032	64.5

25. The specific gravities of different bodies are given in printed tables; hence, if it is desired to know the weight of a body that cannot be conveniently weighed, *calculate its cubical contents and multiply the specific gravity of the body by the weight of a like volume of water, remembering that a cubic foot of water weighs 62.42 pounds.*

EXAMPLE 1.—How much will 3,214 cubic inches of cast iron weigh? Take its specific gravity as 7.21.

SOLUTION.—Since 1 cubic foot of water weighs 62.42 pounds, 3,214 cubic inches weigh

$$\frac{3{,}214}{1{,}728} \times 62.42 = 116.098 \text{ pounds.}$$

Then, $116.098 \times 7.21 = 837.067$ lb. Ans.

TABLE IV.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS GASES AT 32° F. AND UNDER A PRESSURE OF 1 ATMOSPHERE.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Atmospheric air	1.0000	.08073
Carbonic acid	1.5290	.12344
Carbonic oxide	.9674	.07810
Chlorine	2.4400	.19700
Oxygen	1.1056	.08925
Nitrogen	.9736	.07860
Smoke (bituminous coal)	.1020	.00815
Smoke (wood)	.0900	.00727
*Steam at 212° F.	.4700	.03790
Hydrogen	.0692	.00559

* The specific gravity of steam at any temperature and pressure compared with air at the same temperature and pressure is .622.

EXAMPLE 2.—What is the weight of a cubic inch of cast iron?

SOLUTION.— $\frac{62.42}{1,728} \times 7.21 = .26044$ lb. Ans.

One cubic foot of pure distilled water at a temperature of 39.2° Fahrenheit weighs 62.425 pounds, but *the value usually taken in making calculations is 62.5 pounds.*

EXAMPLE 3.—What is the weight in pounds of 7 cubic feet of oxygen?

SOLUTION.—One cubic foot of air weighs .08073 pound, and the specific gravity of oxygen is 1.1056, compared with air; hence,

$$.08073 \times 1.1056 \times 7 = .62479 \text{ lb., nearly. Ans.}$$

TABLE V.

SPECIFIC GRAVITY AND WEIGHT PER CUBIC FOOT OF VARIOUS SUBSTANCES.

Substance.	Specific Gravity.	Weight per Cubic Foot. Pounds.
Emery	4.00	250
Glass (average)	2.80	175
Chalk	2.78	174
Granite	2.65	166
Marble	2.70	169
Stone (common)	2.52	158
Salt (common)	2.13	133
Soil (common)	1.98	124
Clay	1.93	121
Brick	1.90	118
Plaster of Paris (average)	2.00	125
Sand	1.80	113

BUOYANT EFFECTS OF WATER.

26. In Fig. 11 is shown a 6-inch cube entirely submerged in water. The lateral pressures are equal and in opposite directions. The upward pressure acting on the lower surface of the cube is $6 \times 6 \times 21 \times .03617$; the downward pressure acting on the top of the cube is $6 \times 6 \times 15 \times .03617$; and the difference is $6 \times 6 \times 6 \times .03617$, which equals the volume of the cube in cubic inches times the weight of 1 cubic inch of water. That is, the upward pressure exceeds the downward pressure by the weight of a volume of water equal to the volume of the body.

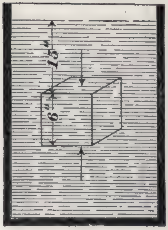


FIG. 11.

This excess of upward pressure over the downward pressure acts against gravity; that is, the water presses the body *upwards* with a greater force than it presses it *downwards*; consequently, *if a body is immersed in a fluid, it will lose in weight an amount equal to the weight of the fluid it displaces.* This is called the **principle of Archimedes,** because it was first stated by him.

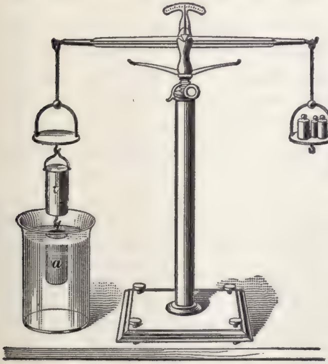


FIG. 12.

27. This principle may be experimentally demonstrated with the beam scales, as shown in Fig. 12. From one scale pan suspend a hollow cylinder of metal *t*, and below that a solid cylinder *a* of the same size as the hollow part of the upper cylinder. Put weights in the other scale pan until they exactly balance the two cylinders. If *a* be immersed in water, the scale pan

containing the weights will descend, showing that *a* has lost some of its weight. Now fill *t* with water, and the volume of water that can be poured into *t* will equal that displaced by *a*. The scale pan that contains the weights will gradually rise until *t* is filled, when the scales again balance.

If a body be lighter than the liquid in which it is immersed, the upward pressure will cause it to rise and project partly out of the liquid, until the weight of the body and the weight of the liquid displaced are equal. If the immersed body be heavier than the liquid, the downward pressure plus the weight of the body will be greater than the upward pressure, and the body will fall downwards until it touches bottom or meets an obstruction. If the weights of equal volumes of the liquid and the body are equal, the body will remain stationary and will be in equilibrium in any position or depth beneath the surface of the liquid.

28. An interesting experiment in confirmation of the above facts may be performed as follows: Drop an egg into a glass jar filled with fresh water. The mean density of the egg being a little greater than that of the water, it will fall to the bottom of the jar. Now, dissolve salt in the water, stirring it so as to mix the fresh and salt water. The salt water will presently become denser than the egg and the egg will rise. Now, if fresh water be poured in until the egg and water have the same density, the egg will remain stationary in any position that it may be placed below the surface of the water.

29. The principle of Archimedes gives a very easy and accurate method of finding the volume of an irregularly shaped body. Thus, subtract its weight in water from its weight in air and divide by .03617; the quotient will be the volume in cubic inches, or divide by 62.5 and the quotient will be the volume in cubic feet.

If the specific gravity of the body is known, its cubical contents can be found by dividing its weight by its specific gravity, and then dividing again by either .03617 or 62.5.

EXAMPLE.—A certain body has a specific gravity of 4.38 and weighs 76 pounds; how many cubic inches are there in the body?

SOLUTION.— $\frac{76}{4.38 \times .03617} = 479.72$ cu. in. Ans.

THE HYDROMETER.

30. Instruments called **hydrometers** are in general use for determining quickly and accurately the specific gravities of liquids and some forms of solids. They are of two kinds, viz.: (1) *Hydrometers of constant weight, as* **Beaume's;** (2) *hydrometers of constant volume, as* **Nicholson's.**

A hydrometer of constant weight is shown in Fig. 13. It consists of a glass tube, near the bottom of which are two bulbs. The lower and smaller bulb is loaded with mercury or shot, so as to cause the instrument to remain in a vertical position when placed in the liquid. The upper bulb is filled with air, and its volume is such that the whole instrument is lighter than an equal volume of water.

FIG. 13.

The point to which the hydrometer sinks when placed in water is usually marked, the tube being graduated above and below in such a manner that the specific gravity of the liquid can be read directly. It is customary to have two instruments, one with the zero point near the top of the stem for use in liquids heavier than water, and the other with the zero point near the bulb for use in liquids lighter than water.

These instruments are more commonly used for determining the degree of concentration or dilution of certain liquids, as acids, alcohol, milk, solutions of sugar, etc., rather than their actual specific gravities. They are then known as *acidometers*, *alcoholometers*, *lactometers*, *saccharometers*, *salinometers*, etc., according to the use to which they are put.

31. Hydrometers of constant volume are not in common use and are rarely found outside of laboratories.

HYDROKINETICS.

FLOW OF WATER IN PIPES.

32. Experience has demonstrated that for satisfactory work, the flow of water in the suction pipes of boiler feed-pumps and other comparatively small pumps should not exceed 200 feet per minute, and it should not be more than 500 feet in the delivery pipe for a duplex pump, or 400 feet for a single-cylinder pump.

Knowing the volume of water that is to flow through or to be discharged from a pipe in 1 minute, the area of the suction and delivery pipes can readily be determined.

33. The volume of water in cubic feet discharged from a pipe in 1 minute is equal to the velocity in feet per minute times the area of the pipe in square feet. Then, the area of the pipe equals $\frac{\text{volume in cubic feet per minute}}{\text{velocity in feet per minute}}$. As there are 144 square inches in a square foot, the area of the pipe in square inches is $\frac{144 \times \text{volume in cubic feet per minute}}{\text{velocity in feet per minute}}$.

34. If the volume is expressed in gallons per minute, then, as there are 7.48 gallons in a cubic foot, the area of the pipe in square inches will be $\frac{144 \times \text{volume in gallons*}}{7.48 \times \text{velocity in feet per minute}}$. Hence, the following rule,

where n = gallons per minute;
v = velocity in feet per minute;
A = area of pipe in square inches.

Rule 4.—*To find the area of a pipe in square inches, divide 19.25 times the number of gallons per minute by the velocity in feet per minute.*

Or,
$$A = \frac{19.25\, n}{v}.$$

* The gallon here referred to is the Winchester or wine gallon of 231 cubic inches capacity and in common use in the United States of America.

Example.—If a duplex pump is used, what area of feed-pipe is required for a boiler into which 25 gallons of water per minute is to be pumped?

Solution.—The allowable velocity being 500 feet, by applying rule 4, we get

$$A = \frac{19.25 \times 25}{500} = .9625 \text{ sq. in. Ans.}$$

35. The quantity of water, expressed in gallons, that will flow through a given pipe in 1 minute, its velocity being known, is given by the following rule:

Rule 5.—*Multiply the area of the pipe in square inches by the velocity in feet per minute. Divide the product by 19.25.*

Or,
$$n = \frac{A\,v}{19.25}.$$

Example.—How many gallons of water per minute will flow through a pipe having an area of 2 square inches, the velocity of flow being 450 feet per minute?

Solution.—Applying the rule just given, we get

$$n = \frac{2 \times 450}{19.25} = 46.75 \text{ gal. Ans.}$$

36. The velocity with which water will flow through the delivery pipe of a pump when the area of the water cylinder, the area of the delivery pipe, and the piston speed of the pump are known, is given by the following rule,

where v = velocity in feet per minute;
A = area of delivery pipe in square inches;
a = area of water piston in square inches;
S = piston speed in feet per minute.

Rule 6.—*Multiply the area of the water piston by the piston speed, and divide this product by the area of the delivery pipe.*

Or,
$$v = \frac{a\,S}{A}.$$

Example.—If the water piston of a pump has an area of 12 square inches and moves at a speed of 100 feet per minute, what will be the

velocity of the water in the delivery pipe if the latter has an area of 2 square inches?

SOLUTION.—Applying rule **6**, we get

$$v = \frac{12 \times 100}{2} = 600 \text{ ft. per min.} \quad \text{Ans.}$$

STANDARD PIPE DIMENSIONS.

37. The great majority of the pipes used about steam plants are made of wrought iron and are almost invariably made in accordance with the Briggs standard. It will be noticed that the diameter of the pipe by which it is known to the trade is not the *actual* diameter; hence, in calculating the amount of water that will flow through a pipe, the actual diameter or actual internal area must be taken from Table VI.

38. As wrought-iron pipes are not made in sizes differing from those given in the table, it will be apparent that only in rare instances can a pipe be selected that will have the area calculated by rule **4**. In practice the nearest commercial size of pipe would be selected.

EXAMPLE.—What commercial size of delivery pipe should be used for a single-cylinder pump to deliver 90 gallons of water per minute?

SOLUTION.—For a single-cylinder pump, the velocity of flow should not be more than 400 feet per minute. Then, applying rule **4**, we get

$$A = \frac{19.25 \times 90}{400} = 4.33 \text{ sq. in.}$$

According to Table VI, the commercial size of pipe having an area nearest this is $2\frac{1}{2}$ inches; therefore, a $2\frac{1}{2}$-inch pipe should be used. Ans.

Rule **4** will be found to agree quite closely with the practice of the leading pump manufacturers. In case they should, however, recommend a larger size of pipe, it is advisable to follow their advice.

Pipes made in accordance with the Briggs standard, when below 1 inch nominal size, are butt-welded and proved to 300 pounds per square inch by hydraulic pressure. Pipes above 1 inch are lap-welded and proved to 500 pounds.

TABLE VI.

TABLE OF STANDARD DIMENSIONS OF WROUGHT-IRON WELDED PIPES.

Nominal Diameter. Inches.	Actual Internal Diameter. Inches.	Actual Internal Area. Square Inches.	Actual External Diameter. Inches.	Number of Threads Per Inch.
$\frac{1}{8}$	.27	.057	.40	27
$\frac{1}{4}$	.36	.104	.54	18
$\frac{3}{8}$	.49	.192	.67	18
$\frac{1}{2}$	.62	.305	.84	14
$\frac{3}{4}$	.82	.533	1.05	14
1	1.05	.863	1.31	$11\frac{1}{2}$
$1\frac{1}{4}$	1.38	1.496	1.66	$11\frac{1}{2}$
$1\frac{1}{2}$	1.61	2.038	1.90	$11\frac{1}{2}$
2	2.07	3.355	2.37	$11\frac{1}{2}$
$2\frac{1}{2}$	2.47	4.783	2.87	8
3	3.07	7.388	3.50	8
$3\frac{1}{2}$	3.55	9.887	4.00	8
4	4.07	12.730	4.50	8
$4\frac{1}{2}$	4.51	15.939	5.00	8
5	5.04	19 990	5.56	8
6	6.06	28.889	6.62	8
7	7.02	38.737	7.62	8
8	7.98	50.039	8.62	8
9	9.00	63.633	9.62	8
10	10.02	78.838	10.75	8

PIPE FITTINGS.

39. In piping a steam plant, it is rarely possible to run the piping in a straight line, it being usually necessary to introduce one or more elbows or similar fittings to reach the point desired. The effect of **T** and **L** fittings is to increase the resistance to the flow of the water through the pipes,

thus requiring the pump to do more work for the same quantity of water delivered.

As pumps for boiler feeding are always built with a large steam cylinder, there is enough excess of power to allow the pump under ordinary conditions to force the required quantity of water through the pipe. On the suction side of the pump, however, the force impelling the water to flow into the pump is quite small, and a very slight resistance will be sufficient to interfere with the flow of the water into the pump. Hence, it is important that the suction pipe be as straight as possible; if it is impossible to make it straight, larger sizes of pipe should be used, or easy bends with as large a radius as possible should be substituted for the elbows. This applies especially to pumps that must lift the water more than 10 feet. It is impossible to lay down any hard-and-fast rules as to what numbers of elbows should not be exceeded in a suction and delivery pipe; judgment will have to be used. Generally speaking, they should be as few as possible.

EXAMPLES FOR PRACTICE.

1. Suppose a cylinder to be filled with water and placed in an upright position. If the diameter of the cylinder is 19 inches and its total length inside 26 inches, what will be the total pressure on the bottom when a pipe ½ inch in diameter and 12 feet long is screwed into the cylinder head and filled with water? The pipe is vertical.
Ans. 1,743.2 lb.

2. In the last example, what is the total pressure against the upper head? Ans. 1,476.6 lb.

3. In example 1, a piston is fitted to the upper end of the pipe and an additional force of 10 pounds is applied to the water in the pipe. What is the total pressure (*a*) on the bottom of the cylinder? (*b*) on the upper head?
Ans. (*a*) 16,184 lb. (*b*) 15,917 lb.

4. In example 3, what is the pressure per square inch in the pipe 2 inches from the upper cylinder head? Ans. 56.0656 lb. per sq. in.

5. A water tower 80 feet high is filled with water. A pipe 4 inches in diameter is so connected to the side of the tower that its center is 3 feet from the bottom. If the pipe is closed by a flat cover, what is the total pressure against the cover? Ans. 420 lb.

6. In the last example, what is the upward pressure per square inch 10 feet from the bottom of the tower? Ans. 30.3828 lb. per sq. in.

7. A cube of wood, one edge of which measures 3 feet, is sunk until the upper surface is 40 feet below the level of the water; what is the total force that tends to move the cube upwards? Ans. 1,687.5 lb.

8. A body weighs 72 pounds when immersed in water and 321 pounds in air; what is its volume in cubic feet? Ans. 3.984 cu. ft.

9. The specific gravity of a body being 9.5 and its weight 81 pounds, what is its volume in cubic inches? Ans. 235.73 cu. in.

10. What commercial size of pipe should be used for the suction and delivery pipes of a single-cylinder pump to deliver 50 gallons of water per minute? Ans. $\begin{cases} \text{Suction pipe, } 2\frac{1}{2} \text{ in.} \\ \text{Delivery pipe, 2 in.} \end{cases}$

11. How many gallons per hour, at a velocity of 400 feet per minute, will flow through a 1-inch pipe? Ans. 1,076 gal., nearly.

12. The water piston of a pump is $3\frac{1}{2}$ inches; the piston speed is 54 feet per minute; and a $1\frac{1}{2}$-inch delivery pipe is used; what is the velocity in the delivery pipe? Ans. 255 ft. per min., nearly.

PNEUMATICS.

PRESSURE OF GASES.

40. Pneumatics is that branch of mechanics that treats of the properties and pressures of gases.

41. The most striking feature of all gases is their great expansibility. *If we inject a quantity of gas, however small, into a vessel, it will expand and fill that vessel.* If a bladder or football be partly filled with air and placed under a glass jar (called a receiver), from which the air has been exhausted, the bladder or football will immediately expand, as shown in Fig. 14. The force that a gas always exerts when confined in a limited

FIG. 14.

space is called *tension*. The word **tension** in this case means pressure, and is only used in this sense in reference to gases.

42. As *water* is the most common type of fluids, so *air* is the most common type of gases. It was supposed by the ancients that air had no weight, and it was not until about the year 1650 that the contrary was proved. A cubic inch of air, under ordinary conditions, weighs .31 grain, nearly. At a temperature of 32° F. and a pressure of 14.7 pounds per square inch, the ratio of the weight of air to water is about 1 : 774; that is, air is only $\frac{1}{774}$ as heavy as water. It has been shown that if a body were immersed in water and weighed less than the volume of water displaced, the body would rise and project partly out of the water. The same is true, to a certain extent, of air. If a vessel made of light material is filled with a gas lighter than air, so that the total weight of the vessel and gas is less than the air they displace, the vessel will rise. It is on this principle that balloons are made.

PRESSURE OF THE ATMOSPHERE.

43. Since air has weight, it is evident that the enormous quantity of air that constitutes the atmosphere must exert a considerable pressure on the earth. This is easily proved by taking a long glass tube closed at one end and filling it with mercury. If the finger be placed over the open end so as to keep the mercury from running out and the tube inverted and placed in a cup of mercury, as shown in Fig. 15, the mercury will fall, then rise, and after a few oscillations will come to rest at a height above the top of the mercury in the cup equal to about 30 inches at sea level.

This height will always be the same under the same atmospheric conditions. Now, if the atmosphere has weight, it must press upon the upper surface of the mercury in the cup with equal intensity upon every square unit, except upon that part of the surface occupied by the tube. In order that there may be equilibrium, the weight of the mercury in the tube must be equal to the pressure of the air

upon a portion of the surface of the mercury in the cup equal in area to the inside of the tube. Suppose that the area of the inside of the tube is 1 square inch, then, since mercury is 13.6 times as heavy as water, the weight of the mercurial column is $.03617 \times 13.6 \times 30 = 14.7574$ pounds. The actual height of the mercury is a little less than 30 inches, and the actual weight of a cubic inch of distilled water is a little less than .03617 pound. When these considerations are taken into account, the average weight of the mercurial column at the level of the sea, when the temperature is 60° F., is 14.69 pounds, or practically 14.7 pounds. Since this weight, when exerted upon 1 square inch of the liquid in the glass, just produces equilibrium, it is plain that the pressure of the outside air is 14.7 pounds upon every square inch of surface.

FIG. 15.

44. Vacuum.—The space between the upper end of the tube and the upper surface of the mercury is called a **vacuum,** meaning that it is an entirely empty space and does not contain any substance—solid, liquid, or gaseous. If there were a gas of some kind there, no matter how small the quantity might be, it would expand and fill the space, and its tension would cause the column of mercury to fall and become shorter, according to the amount of gas or air present. The condition then existing in the space would be called a **partial vacuum.**

45. The Measurement of Vacuum.—The degree to which the air has been exhausted from a closed vessel in

which there is a partial vacuum is measured by the height to which a mercurial column in a vertical tube, whose top is connected to the vessel, will rise under the pressure of the atmosphere. Thus, when we say that there is a vacuum of 29 inches in a vessel, we mean that enough air has been exhausted from the vessel to enable the surrounding air to support a mercurial column 29 inches high, as shown in Fig. 16, where *A* is the vessel in which there is a partial vacuum. In other words, the pressure in the vessel is less than the pressure of the atmosphere by a column of mercury 29 inches in height.

FIG. 16.

When there is a partial vacuum inside a closed vessel, the air remaining in the vessel is under a tension, or pressure, less than its original tension of 14.7 pounds per square inch. This tension, in inches of mercury, is equal to the difference between the height at which the mercurial column connected to the vessel stands and that at which it would stand if the vacuum were perfect. Consider that the mercury column will be in equilibrium when the hydrostatic pressure on its base equals the atmospheric pressure. The hydrostatic pressure on the base is the sum of two pressures: (1) The pressure due to the weight of the mercury column; (2) the pressure in the space above the mercury.

From this it follows that if the atmospheric pressure and the pressure due to the weight of the mercurial column are given, the pressure above the column must be that due to their difference. As the atmosphere will force a mercurial column to a height of 30 inches when there is a perfect vacuum above it, it follows that to find the pressure in a vessel

in which there is a partial vacuum, the number of inches of height of the mercurial column is to be subtracted from 30. Thus, if the vacuum in a vessel is 26 inches, the pressure in the vessel is $30 - 26 = 4$ inches of mercury.

46. In practice it is convenient to always use the same unit of pressure, which is 1 pound per square inch. We know that 14.7 pounds per square inch will support a mercurial column 30 inches high. Hence, 1 inch of height of a mercurial column represents a pressure of $\frac{14.7}{30} = .49$ pound per square inch. Then, to reduce inches of mercury to pounds per square inch, multiply their number by .49.

Example.—A gauge shows a vacuum of 22 inches in a condenser. What is the absolute pressure in the condenser?

Solution.—The pressure, in inches of mercury, is $30 - 22 = 8$ inches, or $8 \times .49 = 3.92$ lb. per sq. in. Ans.

47. The Vacuum Gauge.—For engineering work, the glass tube of Fig. 16 would be a rather inconvenient form of gauge for measuring the vacuum. Hence, special metallic gauges, known as **vacuum gauges,** have been designed. Their dial is graduated to show the degree of vacuum, in inches of mercury. In steam-engineering work, they are used chiefly in connection with condensers for steam engines. It should be borne in mind that a vacuum gauge does not indicate the pressure in the vessel to which it is attached, but instead shows, in inches of mercury, how much the pressure has been *lowered below the atmospheric pressure.* In this respect a vacuum gauge differs essentially from a **pressure gauge,** which shows how much the pressure has been *increased* either above the pressure of a perfect vacuum, which is zero, as in case of a gauge registering the pressure of the atmosphere, or above the atmospheric pressure, as in case of the ordinary pressure gauge.

48. If the tube of Fig. 15 had been filled with a liquid lighter than mercury, the height of the column required to balance the atmospheric pressure would be greater. The

height of the column depends on the specific gravity of the liquid. Thus, if the liquid had a specific gravity of 1, as water, its height would be $\frac{30 \times 13.6}{1} = 408$ inches, or 34 feet. This means that if a tube be filled with water, inverted, and placed in a dish of water, in a manner similar to the experiment made with the mercury, the height of the column of water will be 34 feet.

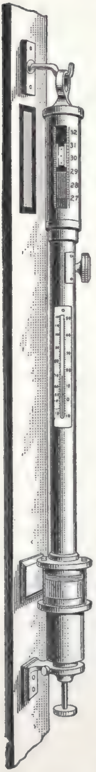


FIG. 17.

49. The Barometer.—As is well known, the atmosphere does not exert exactly the same pressure at all times; the pressure varies with conditions. As the height of the mercury in the glass tube of Fig. 15 depends chiefly on the atmospheric pressure, it is evident that an instrument constructed on this principle will indicate the varying pressure by the height of the column. Such an instrument is called a **mercurial barometer.**

50. A standard form of barometer is shown in Fig. 17. The barometer is simply a pressure gauge that registers the pressure of the air. In this case the cup and tube at the bottom are protected by a brass or iron casing. At the top of the tube is a graduated scale. Attached to the casing is an accurate thermometer for determining the temperature of the outside air at the time the barometric observation is taken. This is necessary, since mercury expands when the temperature is increased and contracts when the temperature falls; for this reason a standard temperature is assumed, and all barometer readings are reduced to this temperature. This standard temperature is usually taken at 32° F., at which temperature the height of the mercurial column is 30 inches under normal conditions at sea level. Another

correction is made for the altitude of the place above sea level, and a third correction for the effects of capillary attraction.

51. A mercurial barometer is not only a very bulky instrument, but is also quite heavy and, hence, is not very well adapted for transportation. To overcome these drawbacks, a form of portable barometer known as an **aneroid barometer,** has been designed, which operates on a somewhat different principle. Such a barometer is shown in

FIG. 18.

Fig. 18. The principal part of the aneroid barometer is a cylindrical, air-tight box of metal closed by a corrugated top of thin, elastic metal. The air is exhausted from the box, which is then sealed. Evidently, the pressure of the air on the outside of the cover will cause the cover to curve inwards, as the space inside of the cover is void of pressure, until the

resistance due to the elasticity of the cover, aided by the resistance of a spring beneath it, is equal to the force exerted by the air. Now, if the air pressure increases, the cover will be curved further inwards; if the air pressure decreases, the elasticity of the cover, aided by the spring beneath, will cause a reduction of its inward curvature. These movements of the cover are transmitted and multiplied by a combination of delicate levers that act on the index hand shown in the figure and cause it to move to the right or left over a graduated scale.

These barometers are compensated (self-correcting) for variations in temperature. The better grade of these instruments have two graduations; the inner graduation corresponds to the graduations of the mercurial barometer; that is, it reads to inches of mercury. The outer graduation represents elevations above and below sea level, the distance between each graduation line representing a difference in elevation of 10 feet.

These instruments are made in various sizes, from the size of a watch up to 8 or 10 inches in diameter. They are very portable, occupying but a small space, are very light, and are quite delicate.

52. Both the mercurial and the aneroid barometers operate on the same general principle; viz., that if two opposite forces act on a body, it will be in equilibrium when the forces are equal and will be set in motion when they are unequal, provided the difference in the magnitude of the two forces is sufficient to overcome the resistance.

The two styles of barometer differ from each other only in the method by which equilibrium is established. In the mercurial barometer, the weight of the mercurial column inside the tube equalizes the outside air pressure; in the aneroid barometer, the outside pressure is equalized by the resistance of the flexible cover and spring beneath it.

53. Variations of Pressure at Different Elevations. With air, as with water, the lower we get the greater is the pressure, and the higher we get the less is the pressure. At

the level of the sea, the height of the mercurial column is about 30 inches; at 5,000 feet above the sea, it is 24.7 inches; at 10,000 feet above the sea, it is 20.5 inches; at 15,000 feet, it is 16.9 inches; at 3 miles, it is 16.4 inches; and at 6 miles above the sea level, it is 8.9 inches.

Air being an elastic fluid, the density or weight of the atmosphere also varies with the altitude; that is, a cubic foot of air at an elevation of 5,000 feet above the sea level will not weigh as much as a cubic foot at sea level. This is proved conclusively by the fact that at a height of $3\frac{1}{2}$ miles the mercurial column measures but 15 inches, indicating that half the weight of the entire atmosphere is below that height. It is known that the height of the earth's atmosphere is at least 50 miles; hence, the air just before reaching the limit must be in an exceedingly rarefied state. It is by means of barometers that great heights are measured. The aneroid barometer has the heights marked on the dial, so that they can be read directly. With the mercurial barometer, the heights must be calculated from the reading.

54. Pressure in Different Directions.—The atmospheric pressure is everywhere present and presses all objects in all directions with equal force. If a book is laid upon the table, the air presses upon it in every direction with an equal average force of 14.7 pounds per square inch. It would seem as though it would take considerable force to raise a book from the table, since, if the size of the book were 8 inches by 5 inches, the pressure upon it would be $8 \times 5 \times 14.7 = 588$ pounds; but there is an equal pressure *beneath* the book that counteracts the pressure on the top. It would now seem as though it would require a great force to open the book, since there are two pressures of 588 pounds each acting in opposite directions and tending to crush the book; so it would, but for the fact that there is a layer of air beneath each leaf, which acts upwards and downwards with a pressure of 14.7 pounds per square inch.

If a piece of flat glass be laid upon a flat surface that has been previously moistened with water, it will require

considerable force to separate them; this is because the water helps to fill the pores in the flat surface and glass and thus creates a partial vacuum between the glass and the surface, thereby reducing the counter pressure beneath the glass.

55. Tension of Gases.—In Fig. 15, the space above the column of mercury was said to be a vacuum, and it was also stated that if any gas or air were present, it would expand and its tension would force the column of mercury downwards. If sufficient gas be admitted to cause the mercury to stand at 15 inches, the tension of the gas is evidently $\frac{14.7}{2} = 7.35$ pounds per square inch, since the pressure of the outside air, 14.7 pounds per square inch, now balances only 15 instead of 30 inches of mercury; that is, it balances only one-half as much as it would if there were no gas in the tube; therefore, the pressure (tension) of the gas in the tube is 7.35 pounds. If more gas be forced into the tube until the top of the mercurial column is just level with the mercury in the cup, the gas in the tube will then have a tension equal to the outside pressure of the atmosphere. Suppose that the bottom of the tube is fitted with a piston, and that the total length of the inside of the tube is 36 inches. If the piston be shoved upwards so that the space occupied by the gas is 18 inches long instead of 36 inches, the temperature remaining the same as before, it will be found that the tension of the gas within the tube is 29.4 pounds. It will be noticed that the volume occupied by the gas is only half that in the tube before the piston was moved, while the pressure is twice as great, since $14.7 \times 2 = 29.4$ pounds. If the piston be shoved up, so that the space occupied by the gas is only 9 inches instead of 18 inches, the temperature still remaining the same, the pressure will be found to be 58.8 pounds per square inch. The volume has again been reduced one-half and the pressure increased twofold, since $29.4 \times 2 = 58.8$ pounds. The space now occupied by the gas is 9 inches long, whereas,

before the piston was moved, it was 36 inches long; as the tube is assumed to be of uniform diameter throughout its length, the volume is now $\frac{9}{36} = \frac{1}{4}$ of its original volume, and its pressure is $\frac{58.8}{14.7} = 4$ times its original pressure. Moreover, if the temperature of the confined gas remains the same, the pressure and volume will always vary in a similar way.

56. Absolute and Gauge Pressures.—From the above explanation, it will be apparent that the pressure in the tube is the pressure above that of a perfect vacuum. Pressures reckoned above vacuum are called **absolute pressures.** The only pressure gauges that indicate absolute pressures are the mercurial and aneroid barometers; ordinary pressure gauges, such as the common steam gauge, indicate pressures above the pressure of the atmosphere. Pressures above that of the atmosphere are commonly called **gauge pressures.** Gauge pressures are changed to absolute pressures by adding 14.7 pounds per square inch to their readings. Truly speaking, the pressure indicated by a barometer, reduced to pounds pressure, should be added to the gauge pressure, since the value 14.7 pounds only represents the mean atmospheric pressure under normal conditions at *sea level.*

57. Mariotte's Law.—The law that states the effect of compressing and expanding gases is called **Mariotte's law** and is as follows:

The temperature remaining the same, the volume of a given quantity of gas varies inversely as the absolute pressure.

The meaning of the law is this: If the volume of a gas be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc. of its former value, the tension will be increased 2, 3, 5, etc. times, or if the outside pressure be increased 2, 3, 5, etc. times, the volume of the gas will be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc. of its original volume, the temperature remaining constant. It also means that if a gas is under a certain pressure, and this pressure is diminished to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$, etc. of its original intensity, the volume of

the confined gas will be increased 2, 4, 10, etc. times—its tension decreasing at the same rate.

Suppose 3 cubic feet of air to be under a pressure of 60 pounds per square inch in a cylinder fitted with a movable piston; then the product of the volume and pressure is $3 \times 60 = 180$. Let the volume be increased to 6 cubic feet; then the pressure will be 30 pounds per square inch, and $30 \times 6 = 180$, as before. Let the volume be increased to 24 cubic feet; it is then $\frac{24}{3} = 8$ times its original volume, and the pressure is $\frac{1}{8}$ of its original pressure, or $60 \times \frac{1}{8} = 7\frac{1}{2}$ pounds, and $24 \times 7\frac{1}{2} = 180$, as in the two preceding cases. It will now be noticed that if a gas be enclosed within a confined space and allowed to expand without losing any heat, *the product of the pressure and the corresponding volume for any one position of the piston is the same as for any other position.* If the piston were forced inwards so as to compress the air, the same results would be obtained.

58. Application of Mariotte's Law.—If the volume of the vessel and the pressure of the gas are known, and it is desired to know the pressure after the first volume has been changed:

Rule 7.—*Divide the product of the first, or original, volume and pressure by the new volume; the result will be the new pressure.*

Or, let p = original absolute pressure;
p_1 = final absolute pressure;
v = volume corresponding to the pressure p;
v_1 = volume corresponding to the pressure p_1.

Then,
$$p_1 = \frac{p\,v}{v_1}.$$

Example.—At the point of cut-off in a steam engine, the amount of steam in the cylinder is 862 cubic inches. The pressure at this point is 120 pounds per square inch. What will be the pressure of the steam when the piston has reached the end of its stroke and the volume is 1,800 cubic inches?

SOLUTION.—Applying the rule, we get

$$p_1 = \frac{862 \times 120}{1,800} = 57.47 \text{ lb. per sq. in. absolute. Ans.}$$

59. If it is required to determine the volume after a change in the pressure:

Rule 8.—*Divide the product of the original volume and pressure by the new pressure; the result will be the new volume.*

Or, using the same letters as before,

$$v_1 = \frac{p\,v}{p_1}.$$

EXAMPLE 1.—At the commencement of compression, the volume of the steam is 380 cubic inches and the pressure is 18 pounds per square inch. At the end of compression, the pressure is 112 pounds per square inch. What is the final volume?

SOLUTION.—Applying the rule, we get

$$v_1 = \frac{380 \times 18}{112} = 61.07 \text{ cu. in. Ans.}$$

EXAMPLE 2.—A vessel contains 10 cubic feet of air at a pressure of 15 pounds per square inch and has 25 cubic feet of air of the same pressure forced into it; what is the resulting pressure?

SOLUTION.—The original volume $= 10 + 25 = 35$ cubic feet; the original pressure is 15 pounds per square inch; the final volume is 10 cubic feet. Hence, applying rule **7**, we get

$$p_1 = \frac{35 \times 15}{10} = 52.5 \text{ lb. per sq. in. absolute. Ans.}$$

It must be remembered that in the preceding examples the temperature is supposed to remain constant. When the temperature changes during expansion and compression, the problem becomes a rather difficult one and cannot be solved by ordinary arithmetic.

EXAMPLES FOR PRACTICE.

1. A vessel contains 25 cubic feet of gas at a pressure of 18 pounds per square inch; if 125 cubic feet of gas having the same pressure are forced into the vessel, what will be the resulting pressure?

Ans. 108 lb. per sq. in.

2. The volume of steam in the cylinder of a steam engine at cut-off is 1.35 cubic feet and the pressure is 85 pounds per square inch; the pressure at the end of the stroke is 25 pounds per square inch. What is the new volume? Ans. 4.59 cu. ft.

3. A receiver contains 180 cubic feet of gas at a pressure of 20 pounds per square inch; if a vessel holding 12 cubic feet be filled from the larger vessel until its pressure is 20 pounds per square inch, what will be the pressure in the larger vessel?

Ans. $18\frac{2}{3}$ lb. per sq. in.

4. A spherical shell has a part of the air within it removed, forming a partial vacuum; if the outside diameter of the shell is 18 inches and the pressure of the air within is 5 pounds per square inch, what is the total pressure tending to crush the shell? Ans. 9,873.42 lb.

PNEUMATIC MACHINES.

THE AIR PUMP.

60. *The* **air pump** *is an instrument for removing air from a given space.* A section of the principal parts is

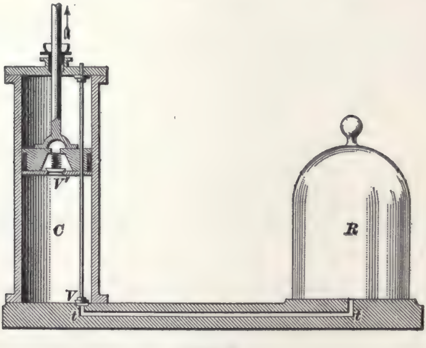


FIG. 19.

shown in Fig. 19 and the complete instrument in Fig. 20. The closed vessel R is called the **receiver,** and the space

that it encloses is that from which it is desired to remove the air. It is usually made of glass, and the edges are ground so as to be perfectly air-tight. When made in the form shown, it is called a **bell-jar receiver.** The receiver rests upon a horizontal plate, in the center of which is an opening communicating with the pump cylinder C by means of the passage $t\,t$. The pump piston accurately fits the cylinder, and has a valve V' opening upwards. Where the passage $t\,t$ joins the cylinder, another valve V is placed, which also opens upwards. When the piston is raised, the valve V' closes, and, since no air can get into the cylinder from

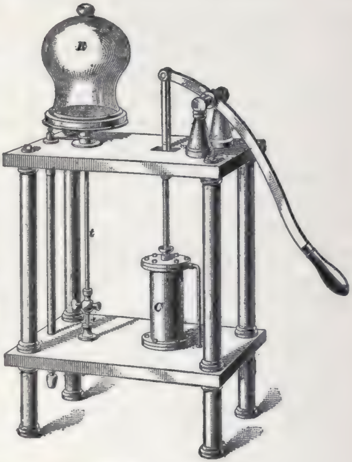


FIG. 20.

above, the piston leaves a vacuum behind it. The pressure upon V being now removed, the tension of the air in the receiver R causes V to rise; the air in the receiver and passage $t\,t$ then expands so as to occupy the additional space provided by the upward movement of the piston.

The piston is now pushed down, the valve V closes, the valve V' opens, and the air in C escapes. The lower valve V is sometimes supported, as shown in Fig. 19, by a metal rod passing through the piston and fitting it somewhat tightly. When the piston is raised or lowered, this rod moves with it. A button near the upper end of the rod confines its motion within very narrow limits, the piston sliding upon the rod during the greater part of the journey. In the complete form of the instrument shown

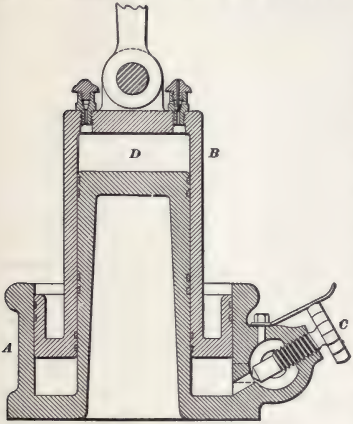
in Fig. 20, communication between receiver and pump is made by means of the tube t.

61. Degrees and Limits of Exhaustion.—Suppose that the volume of R and t together is four times that of C, Fig. 19, and that there are, say, 200 grains of air in R and t, and 50 grains in C when the piston is at the top of the cylinder. At the end of the first stroke, when the piston is again at the top, 50 grains of air in the cylinder C will have been removed, and the 200 grains in R and t will occupy the space R, t, and C. The ratio between the sum of the spaces R and t and the total space $R + t + C$ is $\frac{4}{5}$; hence, $200 \times \frac{4}{5} = 160$ grains = the weight of air in R and t after the first stroke. After the second stroke, the weight of the air in R and t would be $(200 \times \frac{4}{5}) \times \frac{4}{5} = 200 \times (\frac{4}{5})^2 = 200 \times \frac{16}{25} = 128$ grains. At the end of the third stroke, the weight would be $[200 \times (\frac{4}{5})^2] \times \frac{4}{5} = 200 \times (\frac{4}{5})^3 = 200 \times \frac{64}{125} = 102.4$ grains. At the end of n strokes the weight would be $200 \times (\frac{4}{5})^n$. It is evident that *it is impossible to remove all the air that is contained in R and t by this method.* It requires an exceedingly good air pump to reduce the tension of the air in R to $\frac{1}{50}$ inch of mercury. When the air has become so rarefied as this, the valve V' will not lift, and, consequently, no more air can be exhausted.

62. The Dashpot.—The pressure of the atmosphere is utilized in Corliss engines to close the steam valves rapidly. For this purpose a so-called **dashpot** is used. The dashpot of a Bullock-Corliss engine is shown in two positions in Fig. 21. It consists of the base A, which is fastened to the floor, and a plunger B pivoted to the end of a crank-arm keyed to the stem of the steam valve. The base is accurately turned and bored to fit the plunger; packing rings are fitted to both the plunger and the base in order to make an air-tight joint.

The annular space around the central stationary piston is in communication with the outside air by a small passage, the opening of which can be increased or decreased, at will,

by means of the valve *C*. In operation, the plunger is first in the position shown in Fig. 21 (*b*). It is then picked up by the valve gear of the engine and drawn up until it is in the position shown in Fig. 21 (*a*). The large piston on the lower end of the plunger is in communication with the atmosphere through the valve *C*, and hence the pressure of the atmosphere on both sides of the piston is equal. The upper part of the plunger, however, has an equal pressure on both its sides only when it is down, as in Fig. 21 (*b*). When it is drawn up, the air in the space *D* expands and a partial vacuum is formed. The valve gear next releases the plunger, and as the atmospheric pressure is much greater than the pressure within *D*, the plunger rapidly descends. During its descent, the large piston at the end of the plunger gradually compresses the air in the annular space beneath it, and is



(*a*)

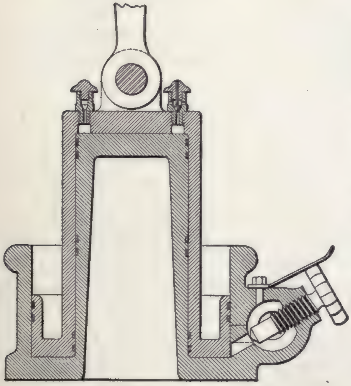

(*b*)

FIG. 21.

thus gradually brought to rest. The valve C serves to regulate the amount of compression and at the same time admits air during the up stroke of the plunger.

THE SIPHON.

63. Theory of the Siphon.—The action of the **siphon** illustrates the effect of atmospheric pressure. It is simply a bent tube having unequal branches, open at both ends, and is used to convey a liquid from a higher point to a lower over an intermediate point that is higher than either. In Fig. 22, A and B are two vessels, B being lower than A, and $A\ C\ B$ is the bent tube, or siphon. Suppose this tube to be filled with water and placed in the vessels as shown, with the short branch $A\ C$ in the vessel A. The water will flow from the vessel A into the vessel B as long as the level of the water in B is below the level of the water in A and the level of the water in A is above the lower end of the tube $A\ C$. The atmospheric pressure on the surfaces of A and B tends to force the water up the tubes $A\ C$ and $B\ C$. When the siphon is filled with water, each of these pressures is counteracted in part by the pressure of the water in that branch of the siphon that is immersed in the water on which the pressure is exerted. The atmospheric pressure opposed to the weight of the longer column of water will, therefore, be resisted more strongly than that opposed to the weight of the shorter column; consequently, the pressure exerted on the shorter column will be greater than that on the longer column, and this excess of pressure will produce motion.

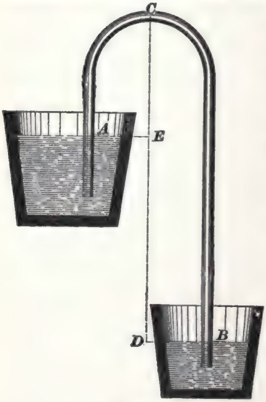


FIG. 22.

64. Let A = the area of the tube;

$h = D\,C$ = the vertical distance between the surface of the water in B and the highest point of the center line of the tube;

$h_1 = E\,C$ = the distance between the surface of the water in A and the highest point of the center line of the tube.

The weight of the water in the short column is $.03617 \times A\,h_1$, and the resultant atmospheric pressure tending to force the water up the short column is $14.7 \times A - .03617 \times A\,h_1$. The weight of the water in the long column is $.03617\,A\,h$, and the resultant atmospheric pressure tending to force the water up the long column is $14.7\,A - .03617\,A.h$. The difference between these two is $(14.7\,A - .03617\,A\,h_1) - (14.7\,A - .03617\,A\,h) = .03617\,A\,(h - h_1)$. But $h - h_1 = E\,D$ = the difference between the levels of the water in the two vessels. In the above, h and h_1 were taken in inches and A in square inches.

It will be noticed that the short column must not be higher than 34 feet for water, or the siphon will not work, since the pressure of the atmosphere will not support a column of water that is higher than 34 feet.

65. Fig. 23 shows the actual construction of a siphon. It is desired to convey the water from D to E. The point of discharge must always be lower than the point from which the water is taken, otherwise a siphon cannot work. The siphon consists of ordinary iron pipe jointed in any convenient manner so as to be air-tight. It has three valves A, B, and C. The suction end is provided with a perforated strainer c to keep out large particles of foreign matter.

In order to start the siphon, it is necessary to remove the air in the pipe. This is done by closing the valves A and B and opening the valve C, which should always be located at the highest point of the siphon. Water is then poured into the funnel above the valve C until the pipe is filled. When

no more water can be poured in, that is, when all the air has been driven out, the valve C is closed and the valves A and B are opened; the siphon will now start.

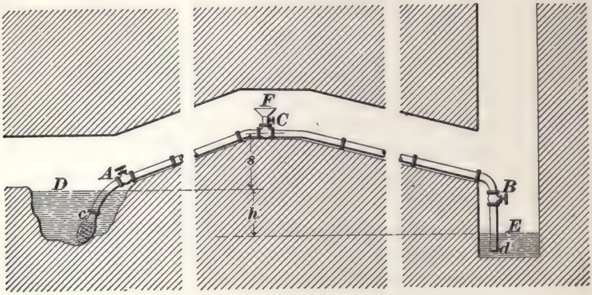


FIG. 23.

In practice, it has been found that the distance s must not exceed 28 feet at sea level, and that the siphon will work more satisfactory if it is less. The greater the distance h between the two water levels, the better the siphon will work.

66. In order that a siphon will work properly, it is necessary that air should be kept out of the pipe, or, if it does get in, means should be provided for its escape. The joints of the pipe must be perfectly air-tight. But some air will always be carried in with the water, and this air will collect at the highest point. The bad effects of this can be minimized by having a straight horizontal pipe at the highest point instead of a sharp bend.

67. A device that will remove the air is shown in Fig. 24. Here A is an air-tight vessel connected with the siphon by two small pipes B and C. The pipe B extends nearly to the top of A, while the pipe C barely enters the bottom. Each pipe has a valve D and E. A valve F and funnel G are placed on top of the vessel. When the siphon ceases to flow, which is an indication that air has collected (it is here

assumed that the suction end has not become uncovered), the valves D and E are closed and the valve F is opened. The vessel is now filled with water, the valve F is closed, and D and E are opened. The water will now flow through C into the siphon and force out the air collected there, which passes through B to the top of A. When all air is out of the siphon, D and E are shut and F is opened. The vessel A is now filled with water, F is shut, D and E are

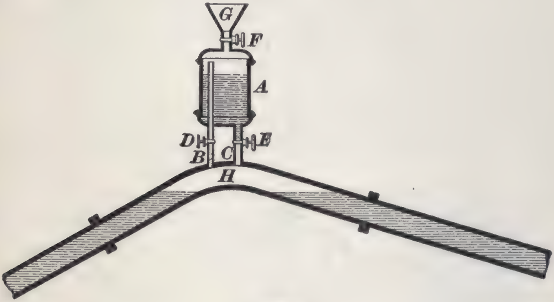


FIG. 24.

opened and left open. Any air that enters the siphon will, instead of collecting at H, ascend the pipe B and force out a small amount of water through C. This will continue until A is filled with air, when the valves D and E should be closed and A refilled. This arrangement may also be used to fill the siphon for the purpose of setting it to work.

The highest point of the water in A should not be more than 28 feet at sea level above the water level at the suction end.

PUMPS.

68. A **pump** is a machine for conveying liquids from one level to a higher level or for performing work equivalent to such an operation.

69. Classification of Pumps.—Pumps may be divided into three general divisions, according to the service they perform, viz., *suction pumps*, *lifting pumps*, and *force pumps*. They may also be divided into two general classes, *single-acting* and *double-acting pumps*, according as they take water on one side or on both sides of the water piston. According to the arrangement of the pump cylinders, they are classified as *simple*, *duplex*, or *triplex pumps*. As pumps displace the liquids in various ways, they may also be divided according to the method of displacement, into *reciprocating*, *centrifugal*, and *rotary pumps*. Reciprocating pumps only will be considered here.

70. The Suction Pump.—A section of an ordinary suction pump is shown in diagrammatic form in Fig. 25. Suppose the piston, or **bucket** as it is commonly termed, to be at the bottom of the cylinder and to be just on the point of moving upwards in the direction of the arrow. As the piston rises it leaves a partial vacuum behind it, and the atmospheric pressure on the surface of the water in the well causes it to rise in the pipe P for the same reason that the mercury rises in the barometer tube. The water rushes up the pipe and lifts the suction valve V, filling the empty space in the cylinder B caused by the displacement of the piston. When the piston has reached the end of its stroke, the water entirely fills the space between the bottom of the piston and the

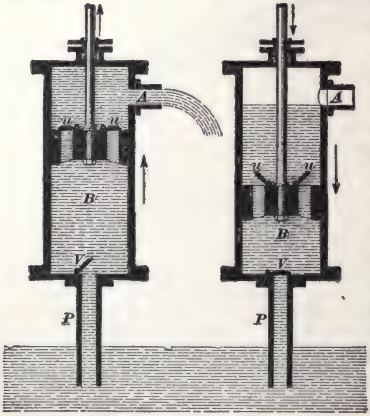


FIG. 25.

bottom of the cylinder, and also the pipe *P*. The instant the piston begins its down stroke, the water in the chamber *B* begins to flow back into the well, and its downward flow forces the valve *V* to its seat, thus preventing any further escape of the water. As the piston descends, the water must give way to it, and since the suction valve *V* is closed, the bucket valves *u*, *u* must open, and thus allow the water to pass through the piston, as shown in the right-hand figure. When the piston has reached the end of its downward stroke and commenced its upward movement again, the water flowing through the piston quickly closes the valves *u*, *u*. All the water resting on the top of the piston is then lifted by the piston on its upward stroke and discharged through the spout *A*; the valve *V* again opens and the water fills the space below the piston, as before.

71. It is evident that the distance between the piston when at the top of its stroke and the surface of the water in the well must not exceed 34 feet, the highest column of water that the pressure of the atmosphere will sustain, since, otherwise, the water in the pipe would not rise and fill the cylinder as the piston ascended. In practice, this distance should not exceed 28 feet. This is due to the fact that there is a little air left between the bottom of the piston and the bottom of the cylinder, a little air leaks through the valves, which are not perfectly air-tight, and a pressure is needed to raise the valve against its own weight, which, of course, acts downwards.

There are many varieties of the suction pump, differing principally in the construction of the valves and piston, but the principle is the same in all.

72. The Lifting Pump.—In some cases it is desired to raise water higher than it can be forced by the pressure of the atmosphere into the chamber of a simple suction pump, such as is shown in Fig. 25. To accomplish this the pump chamber with its bucket and valves are set at a distance above the supply not exceeding that to which the air will successfully force the water. A closed pipe *P′*, Fig. 26,

called the *delivery*, or *discharge*, *pipe*, is then led from the upper part of the chamber to the point where it is desired to deliver the water. Such a pump is often called a **lifting pump.**

In order to prevent the leakage of water around the piston rod, a stuffingbox S is provided. The lower end of the discharge pipe P' is sometimes fitted with a valve c to prevent the water flowing back into the pump chamber; this

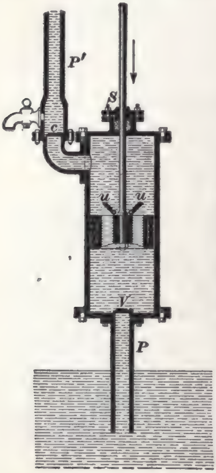


FIG. 26.

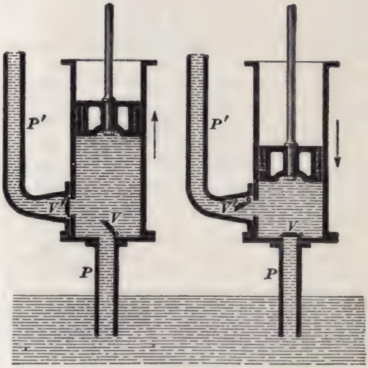


FIG. 27.

valve is not essential to the operation of the pump, however, since the valve V prevents the water in the chamber and discharge pipe discharging during the downward motion of the piston.

While it is customary to consider lifting pumps and suction pumps as two different types of pumps, there is in reality no difference in their operation, as a careful study of Figs. 25 and 26 will show. The only difference is that the water is discharged by a suction pump *at* the level of the

pump, while a lifting pump discharges the water *above* the level of the pump.

73. Force Pumps.—The force pump differs from the lifting pump in one important particular, that is, in the fact that its piston is solid. A section of a force pump is shown in Fig. 27. As the piston ascends, as shown in the left-hand figure, the pressure of the atmosphere forces the water up the suction pipe P; the water opens the suction valve V and flows into the pump cylinder. When the piston moves down, as shown in the right-hand figure, the suction valve is closed and the delivery valve V' opened. The water in the pump cylinder is now forced up the delivery pipe P'. When the piston again begins to move upwards, the delivery valve is closed by the water above it and the suction valve opened by the pressure of the atmosphere on the water below it.

74. Plunger Pumps.—When force pumps are used to convey water to great heights or to force water against heavy pressures, the great pressure of the water makes it extremely difficult to keep the water from leaking past the piston, and the constant repairing and renewal of the piston packing becomes a nuisance and involves serious expense. To obviate this drawback, plunger pumps have been designed, one of which is shown diagrammatically in Fig. 28. The action

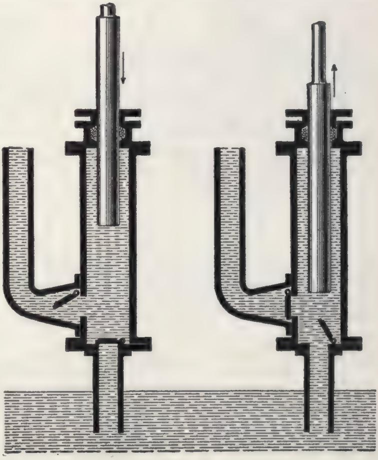


FIG. 28

does not differ in any way from that of the piston force pump. During the up stroke of the plunger, the suction valve is open and the delivery valve is closed; during the down stroke, the suction valve is closed and the delivery valve is open.

75. The force pumps shown so far are **single-acting,** that is, the water is forced into the delivery pipe only during the downstroke or forward stroke of the piston or plunger. Force pumps, either of the piston or plunger pattern, may be constructed so as tó force water into the delivery pipe both during the forward and return stroke. They are then called **double-acting.**

76. A **double-acting force pump** of the piston pattern is shown in Fig. 29. Such a pump has two sets of suction valves and delivery valves, one set for each side of the piston. With the piston moving in the direction of the arrow, the pressure of the atmosphere forces the water up the suction pipe P into the left-hand end of the pump cylinder, the left-hand suction valve opens and the left-hand delivery valve is closed. The piston in moving to the right, displaces the water in the right-hand end of the pump cylinder; as a consequence the right-hand suction valve is closed and the right-hand delivery valve opens. The water now flows up the delivery pipe P'. Imagine that the piston is at the end of its stroke and commences to move to the left. Its first movement

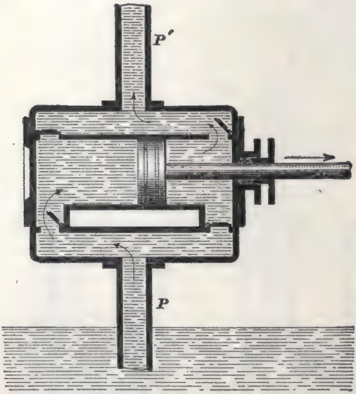


FIG. 29.

promptly closes the left-hand suction valve and opens the left-hand delivery valve. It also closes the right-hand delivery valve and opens the right-hand suction valve.

It is thus seen that with the arrangement given, which shows the principle of operation of all double-acting pumps, the piston will discharge water both during the forward and the return stroke. While the pump shown is a horizontal pump, it may be vertical as well.

STRENGTH OF MATERIALS.

GENERAL PRINCIPLES.

1. When a force is applied to a body, it changes either its form or its volume. A force, when considered with reference to the internal changes it tends to produce in any solid, is called a **stress.**

Thus, if we suspend a weight of 2 tons by a rod, the stress in the rod is 2 tons. This stress is accompanied by a lengthening of the rod, which increases until the internal stress or resistance is in equilibrium with the external weight.

2. Classification of Stresses.—Stresses may be classified as follows: *Tensile*, or *pulling stress; transverse*, or *bending stress; compressive*, or *pushing stress; shearing*, or *cutting stress; torsional*, or *twisting stress.*

3. A **unit stress** is the amount of stress on a unit of area, and may be expressed either in pounds or tons per square inch or per square foot; or, it is the load per square inch or per square foot on any body.

Thus, if 10 tons are suspended by a wrought-iron bar that has an area of 5 square inches, the unit stress is 2 tons per square inch, because $\frac{10}{5} = 2$ tons.

4. Strain is the deformation or change of shape of a body resulting from stress.

For example, if a rod 100 feet long is pulled in the direction of its length, and if it is lengthened 1 foot, it is strained $\frac{1}{100}$ its length, or 1 per cent.

5. Elasticity is the property by virtue of which a body regains its original form after the external force on it is withdrawn, provided the stress has not exceeded the elastic limit. It is a property possessed by all bodies.

Consequently, we see from this that all material is lengthened or shortened when subjected to either tensile or compressive stress, and the change of the length within the elastic limit is directly proportional to the stress.

For stresses within the elastic limits, materials are perfectly elastic, and return to their original length on removal of the stresses; but when their elastic limits are exceeded, the changes of their lengths are no longer regular, and a permanent *set* takes place. The destruction of the material has then begun.

6. The **measure of elasticity** of any material is the change of length under stress within the elastic limit.

7. The **elastic limit** is that unit stress under which the permanent set becomes visible.

8. The elasticity of wrought iron and of all grades of steel is practically the same; that is, *within the elastic limit* each material will change an equal amount of length under the same stress. The elastic limit, however, is not the same for steel as for iron; it is higher for soft steel than for wrought iron, and, in general, the harder and stronger the steel the higher will be its elastic limit. As a consequence, steel will lengthen or shorten more than wrought iron, and hard steel more than soft, before its elasticity or ability to return to its original dimensions is injured.

TENSILE STRENGTH OF MATERIALS.

9. The **tensile strength** of any material is the *resistance* offered by its fibers to being pulled apart.

10. The *tensile strength* of any material *is proportional to the area of its cross-section.*

Consequently, when it is required to find the safe tensile strength of any material, we have only to find the area at the minimum cross-section of the body, and multiply it by the load per square inch that it can safely carry, as given in the following table under the heading "Safe Loads."

NOTE.—The minimum cross-section referred to in the above paragraph is that section of the material which is pierced with holes, such as bolt or rivet holes in iron, or knots in wood, if there are any.

TABLE I.

TENSILE STRENGTH OF MATERIALS.

Material.	Ultimate Tensile Strength. Pounds per Square Inch.	Safe Loads. Pounds per Square Inch.		
		Sudden.	Gradual.	Steady.
Timber...........	10,000	600	900	1,200
Cast iron.........	16,000– 20,000	1,600	2,400	3,200
Gun-metal cast iron	30,000– 35,000	2,000	3,000	4,000
Wrought iron.....	45,000– 55,000	4,500	9,000	13,000
Extra soft steel...	45,000– 55,000	6,000	10,000	14,000
Flange steel.......	52,000– 62,000	6,000	10,000	14,000
Firebox steel......	52,000– 62,000	6,000	10,000	14,000
Machinery steel...	60,000– 75,000	6,500	11,000	15,000
Axle steel.........	75,000– 90,000	7,000	14,000	20,000
Hard steel (rail steel)...........	90,000–115,000	7,000	14,000	20,000
Tire steel.........	100,000–125,000	8,000	15,000	20,000
Crucible (tool) steel	125,000–180,000	15,000	23,000	30,000
Brass, cast........	14,000– 20,000	1,400	2,200	3,000
Bronze, cast......	30,000– 36,000	3,000	5,000	7,000
Tobin bronze.....	70,000– 80,000	7,000	11,000	14,000
Hard-drawn brass wire............	30,000– 40,000	2,500	3,700	5,000
Copper, rolled.....	30,000– 36,000	3,500	6,000	8,000

11. In metals, a high tensile strength in itself is no indication of the ability of the metal to stand repeated applications of sudden stresses, as a high tensile strength usually involves a lesser degree of ductility than is obtained in metals of lower tensile strength. Steel of a tensile strength higher than 75,000 pounds is seldom used merely on account of its superior strength, but rather on account of its hardness, which enables it to better withstand abrasion.

12. The loads given in Table I are conservative safe loads deducted from experience and observation. They are given for material free from welds for such material as can be welded. While it is possible to make a weld as strong as the solid bar, the chances of the weld being imperfect are so great that it is unsafe to rely on such a degree of strength in welded bars. Furthermore, the value of the weld is an uncertain quantity that cannot be determined by an ocular inspection or any other inspection short of actually pulling the weld apart in a testing machine. Hence, it is advisable to reduce the safe loads given in the above table by 25 per cent. when a welded bar is subjected to a tensile stress. When judging the safe load of timber, due allowance must be made for knot holes and sappy spots.

13. It is often rather hard to determine whether a stress is steady, gradually varying, or gradually applied, or sudden. For example, considering the shell of a boiler, it would appear on first thought as though the tensile stress in the shell plates was a steady stress. But looking further into the problem, it will become apparent that the stress varies with the steam pressure, which gradually fluctuates within narrow or wider limits. Hence, most designers would consider the stress in a boiler shell as a gradually applied stress. In a piston rod or connecting-rod the load is applied almost instantly as soon as the crank passes over the center, and, hence, the stress would be considered to be a suddenly applied stress. When in doubt, it is good policy to err on the side of safety, that is, to choose a smaller safe load per square inch of section.

For special work, experience has indicated safe loads for different materials that fall below, or are above, those given in the table. Examples of this will be given later on.

14. The safe load per square inch of section is often called the **working stress per square inch,** or the **working load per square inch.** Care should be taken not to confound these terms with *working load*, *working stress*, *safe load*, or *safe tensile strength*, which, when applied without the limitation as to the unit of area, refer to the safe load the whole bar can carry.

RULES AND FORMULAS FOR TENSILE STRENGTH.

15. Let W = safe load in pounds;
A = area of minimum cross-section;
S = working stress in pounds per square inch, as given in Table I.

Rule 1.—*The working load in pounds for any bar subjected to a tensile stress is equal to the minimum sectional area of the bar, multiplied by the working stress in pounds per square inch, as given in the table.*

Or, $$W = A S.$$

Example.—A bar of good wrought iron that is 3 inches square is to be subjected to a steady tensile stress; what is the maximum load that it should carry?

Solution.—According to the table, a working stress of 13,000 pounds may be allowed. Applying the rule, we have

$$W = 3 \times 3 \times 13{,}000 = 117{,}000 \text{ lb. Ans.}$$

Rule 2.—*The minimum sectional area of any bar subjected to a tensile stress should be equal to the load in pounds, divided by the working stress in pounds per square inch, as given in the table.*

Or, $$A = \frac{W}{S}.$$

Example.—What should be the area of a machinery-steel bar to carry a steady load of 108,000 pounds?

Solution.—According to the table, a safe load of 15,000 pounds per square inch of section may be allowed. Then, applying the rule,

$$A = \frac{108,000}{15,000} = 7.2 \text{ sq. in. Ans.}$$

Rule 3.—*The working stress in pounds per square inch is equal to the load in pounds divided by the minimum sectional area of the bar.*

Or,
$$S = \frac{W}{A}.$$

Example.—A piston rod 3 inches in diameter carries a piston 20 inches in diameter. With a steam pressure of 100 pounds to the square inch, what is the stress per square inch of section of the rod?

Solution.—The load on the piston is $20^2 \times .7854 \times 100 = 31,416$ pounds. The area of the rod is $3^2 \times .7854 = 7.0686$ square inches. Then, applying the rule just given, we have

$$S = \frac{31,416}{7.0686} = 4,444.4 \text{ lb. Ans.}$$

CHAINS.

16. **Chains** made of the same size iron vary in strength, owing to the different kinds of links from which they are made. It is a good practice to anneal old chains that have become brittle by overstraining. This renders them less liable to snap from sudden jerks. It reduces their tensile strength, but increases their toughness and ductility, which are sometimes more important qualities.

When annealing, care should be taken that a sufficient heat be applied, otherwise no benefit will be gained; the chains ought to be heated to a cherry red, say 1,300° F., at least.

17. The loads that can safely be lifted with chains are given in the following table. The safe load given is one-quarter of the steady load at which they may be expected to break. The table has been deduced from one published by the Lukens Iron and Steel Company.

TABLE II.

STRENGTH OF CHAINS.

Diameter of Rod of Which Link Is Made.	Safe Load. Pounds.	Diameter of Rod of Which Link Is Made.	Safe Load. Pounds.
$\frac{3}{16}$	430	1	12,500
$\frac{1}{4}$	770	$1\frac{1}{8}$	15,000
$\frac{5}{16}$	1,200	$1\frac{1}{4}$	18,000
$\frac{3}{8}$	1,750	$1\frac{3}{8}$	22,000
$\frac{7}{16}$	2,350	$1\frac{1}{2}$	26,000
$\frac{1}{2}$	3,100	$1\frac{5}{8}$	31,000
$\frac{9}{16}$	4,000	$1\frac{3}{4}$	36,000
$\frac{5}{8}$	4,800	$1\frac{7}{8}$	41,000
$\frac{11}{16}$	5,800	2	47,000
$\frac{3}{4}$	6,900	$2\frac{1}{4}$	56,000
$\frac{13}{16}$	8,000	$2\frac{1}{2}$	69,000
$\frac{7}{8}$	9,400	$2\frac{3}{4}$	84,000
$\frac{15}{16}$	11,000	3	100,000

STRENGTH OF ROPES.

HEMP ROPES.

18. The strength of hemp ropes does not depend entirely on the quality of the material and the cross-section of the rope, but to a large extent on the method of manufacture and the amount of twisting. A hemp rope is made up of fibers twisted together to form a **yarn,** several of which are laid up left-handed into **strands,** which in turn are twisted up right-handed into what is known as **plain-laid rope.**

Cable-laid or **hawser-laid rope** is left-handed rope of nine (9) strands.

Shroud-laid rope is formed by adding another strand to a plain-laid rope. Since the four strands on application of

strain would sink in and detract from the rope's strength by an unequal distribution of strain, they are laid up around a **heart,** which is a small rope, made soft and elastic, and about one-third the size of the strands.

19. Ordinary hemp rope is designated by its circumference; rope for transmission purposes, which is either iron, steel, or manila rope, is designated by its diameter. It is well to keep this in mind. The loads that can safely be hoisted with hemp ropes of different sizes is given in the following table prepared by Ensign F. R. Brainard, United States Navy.

TABLE III.

STRENGTH OF HEMP ROPES.

Circumference. Inches.	Safe Load. Pounds.	Circumference. Inches.	Safe Load. Pounds.
1.00	65	5.75	2,680
1.25	125	6.00	2,975
1.50	185	6.50	3,470
1.75	255	7.00	3,965
2.00	320	7.50	4,570
2.25	410	8.00	5,175
2.50	500	8.50	5,910
2.75	610	9.00	6,640
3.00	715	9.50	7,370
3.25	850	10.00	8,105
3.50	980	10.50	8,970
3.75	1,135	11.00	9,840
4.00	1,285	11.50	11,015
4.25	1,450	12.00	12,190
4.50	1,610	12.50	13,365
4.75	1,750	13.00	14,540
5.00	1,990	13.50	15,845
5.25	2,190	14.00	17,150
5.50	2,385	14.50	18,450

20. The above table applies to new ropes or ropes in first-class condition. If the rope is used for a block and tackle, the bending of the rope around the pulleys will cause a rapid deterioration of the inside, owing to the chafing and sliding of the strands and yarns upon one another. The outside appearance of a rope used for block and tackle is in itself no indication of its quality; ropes are frequently found that appear perfectly sound when judged by their outside appearance and yet are entirely unsafe. To inspect the rope, take it in both hands and untwist it sufficiently to expose the inner surfaces that have been chafing against one another. If a considerable number of broken fibers are found, the safe load should be reduced by 50 per cent. If a rather large quantity of the fibers have been reduced to powder, the rope should be condemned and preferably destroyed immediately, in order to remove any temptation to use it. A rope in this condition is liable to give way suddenly under a very light load.

Ropes used for slings and lashings, as a general rule, are ruined by the external chafing they receive and, hence, their safety can be determined by their outward appearance.

21. **Slings** are chiefly used for attaching tackles to machine parts during the erection and repair of machinery. They are made by taking a piece of rope a little more than twice the required length of the rope and joining the ends by splicing, the splice known as a **short splice** being usually employed. It should be distinctly understood that this splice is not intended nor adapted for ropes that are to pass over a pulley.

22. **Splicing a Sling.**—To make a short splice in a sling, untwist the strands for some distance from each end of the rope. Take an end in each hand and lay the strands within one another, as shown in Fig. 1 (*a*). The strands *1'*, *2'*, and *3'* may be tied to the left-hand end for convenience in handling. Now take the strand *1*, pass it over strand *2'* and under strand *1'*, as shown in Fig. 1 (*b*); then, draw it

tight. Next, take strand *2*, pass it over *3′* and under *2′*, drawing it up tightly. Now take strand *3*, pass it over *1′* and under *3′*; then draw it up. The splice will now appear as shown in Fig. 1 (*c*). Weave the strands *1′*, *2′*, and *3′* into the left-hand rope in the same manner. Continue weaving the right-hand and left-hand strands alternately until the length of the splice is about 6 times the diameter of the rope. Draw the strands tight and subject the splice to a strain. Then cut off the projecting ends of the strands.

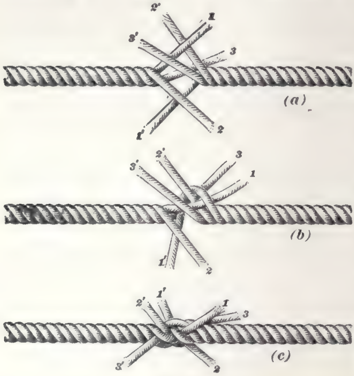


FIG. 1.

23. Use of Slings.—The weight that can safely be lifted by a sling depends not only on the size of the rope of which it is made, but also on the manner in which it is attached to the hook of the tackle block. Thus, in Fig. 2, the sling is simply hooked over the hook.

Owing to the sharp bend over the hook, the strength of the sling is greatly reduced, and the direct load for this manner of attachment should not exceed the safe load given in the table for one rope of equal circumference, although the load is borne by two parts of the rope.

When the sling is attached to the hook by doubling over, as shown in Fig. 3, its greatest strength is realized, and for the two parts of the rope parallel to each other, the stress in the rope will be equal to half the load, as the load is supported by two ropes. Then, with a sling doubled over as shown, a load twice as great can safely be hoisted than when applied as shown in Fig. 2.

In the foregoing statement, it is assumed that the sling is fastened to the work without a sharp bend, the equivalent of a single loop over a hook. The stress in the two parts of a sling will be least when they are parallel to each other; hence, a sling will then lift the greatest load with safety. The stress becomes greater when the angle between the two parts of the sling is increased; its magnitude for any angle can be determined by the method of the triangle of forces.

24. A quick way of attaching a rope to a hook is that shown in Fig. 4, and known as a **Blackwall hitch.** When

FIG. 2.

FIG. 3.

FIG. 4.

making this hitch, always make it as shown in the figure; that is, let the detached end lie between the hook and the end sustaining the load.

25. The proper size of a rope for a block and tackle is fixed by the size of the groove turned in the pulleys. Then, from the table, take the safe working load of the rope, and using this as the force applied to the free end of the rope, the maximum load a given block and tackle can lift safely can be calculated by the rule given in connection with the subject of pulleys.

MANILA ROPE.

26. Manila rope is used chiefly for hoisting purposes and for power transmission. The so-called "Stevedore" manila rope has the fibers lubricated with a mixture of tallow and plumbago during the process of manufacture. This lubrication prevents internal chafing and wear to a large extent.

Manila rope for hoisting and power-transmission purposes is usually made with four strands, that is, shroud laid. Hoisting rope is ordered by *circumference;* transmission rope by *diameter*.

27. The working stress on manila hoisting rope should not exceed that given in the table below, which was published by C. W. Hunt & Company.

TABLE IV.

STRENGTH OF MANILA HOISTING ROPE.

Circumference of Rope. Inches.	Working Stress.
3	350
$3\frac{1}{2}$	500
4	650
$4\frac{1}{2}$	800
5	1,000

Under ordinary conditions, with a working stress not exceeding that given in the table, a manila hoisting rope will last until a total load of about 6,000 tons has been hoisted with it. Under exceptionally favorable circumstances, where the rope runs over large sheaves, a manila rope has hoisted 20,000 tons. Manila ropes for hoisting purposes should not be spliced, as it is difficult to make a splice that will not pull out while running over the sheaves,

and the increased wear to be obtained from splicing a broken hoisting rope is usually very small.

28. Manila rope for power transmission is made from $\frac{1}{2}$ inch up to 2 inches in diameter. The greatest horsepower can be transmitted by a rope when the rope speed is about 5,000 feet per minute. When run at greater speed, the effect of the centrifugal force in the rope rapidly decreases the horsepower that can be transmitted. The size of the pulley has an important bearing on the wear of the rope; in general, it should not be smaller than 40 times the diameter of the rope, and as much larger as it can conveniently be made.

WIRE ROPES.

29. Wire rope is made of iron and steel wire. It is stronger than hemp rope, and, to carry the same load, is of smaller diameter.

In substituting steel for iron rope, the object in view should be to gain an increase of wear from the rope, rather than to reduce the size. A steel rope to be serviceable should be of the best obtainable quality, because ropes made from low grades of steel are inferior to good iron ropes.

30. Rules for the strength of wire ropes:

Let W = maximum of working load in pounds;
C = circumference of rope in inches.

Rule 4.—*The maximum working load in pounds that should be allowed on any iron-wire rope is equal to the square of the circumference of the rope in inches, multiplied by 600.*

Or, $$W = 600\, C^2.$$

EXAMPLE.—What is the maximum load in pounds that should be carried by an iron rope whose circumference is $4\frac{1}{2}$ inches?

SOLUTION.—Applying the rule just given, we have

$$W = 600 \times 4.5^2 = 12{,}150 \text{ lb.} \quad \text{Ans.}$$

Rule 5.—*The circumference of any iron-wire rope in inches is equal to the square root of the maximum working load in pounds, multiplied by .0408.*

Or, $C = .0408 \sqrt{W}.$

Example.—A maximum working load of 12,150 pounds is to be carried by an iron-wire rope; what should be the minimum circumference of the rope?

Solution.—Applying rule **5**, we have

$$C = .0408 \sqrt{12{,}150} = 4\tfrac{1}{2} \text{ in. Ans.}$$

Rule 6.—*The above rules and formulas are also made applicable when computing the safe strength of steel-wire rope by substituting the constant 1,000 for the constant 600, and .0316 for .0408.*

Example.—What is the maximum load in pounds that should be carried by a steel-wire rope, the circumference of which is $4\frac{1}{2}$ inches?

Solution.—Applying rule **4**, we have

$$W = 1{,}000 \times 4.5^2 = 20{,}250 \text{ lb. Ans.}$$

Example.—A maximum working load of 10,485 pounds is to be carried by a steel-wire rope; what should be the minimum circumference of the rope?

Solution.—Applying rule **5**, we have

$$C = .0316 \sqrt{10{,}485} = 3.24 \text{ in. Ans.}$$

EXAMPLES FOR PRACTICE.

1. What should be the diameter of a machinery-steel piston rod of a steam engine to resist tension, if the piston is 19 inches in diameter and the pressure is 85 pounds per square inch? Ans. $2\frac{1}{4}$ in., nearly.

2. What safe load will a cast-iron bar of rectangular cross-section $7\frac{1}{2}$ in. by $3\frac{1}{2}$ in. support if subjected to shocks? The bar is in tension. Ans. 42,000 lb.

3. What is the stress per square inch on a piece of timber 8 inches square that is subjected to a steady pull of 60,000 pounds? Ans. 937.5 lb. per sq. in.

4. What should be the allowable working load for a steel-wire rope whose circumference is $3\frac{3}{4}$ inches? Ans. 14,062.5 lb.

5. What should be the circumference of an iron-wire rope to support a load of 20,000 pounds? Ans. $5\frac{3}{4}$ in., nearly.

CRUSHING STRENGTH OF MATERIALS.

31. The **crushing strength** of any material is the resistance offered by its fibers to being pushed together.

32. To obtain only compression, the length of a rod should not be more than five times greater than its least diameter or its least thickness when it is a rectangular rod. Such a rod is called a **short column.**

If a bar is long compared with its cross dimensions, the load, if sufficiently great, will cause it to bend sidewise under the compressive force, and we have, then, not only compression, but compression compounded with bending.

33. When a bar or rod is longer than five times its least diameter or least thickness, it is called a **long column,** or **long pillar.**

34. The shape of the *ends* of a column has great influence on its strength. In Fig. 5 are shown three columns with differently shaped ends.

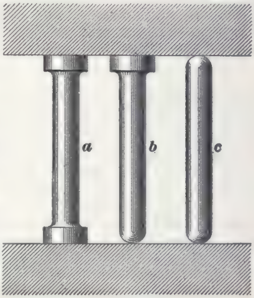


FIG. 5.

It has been proved by the aid of higher mathematics that, theoretically, a pillar, as *a*, having flat or fixed ends, is 4 times as strong as one that has round or movable ends, as the pillar *c*, and $1\frac{7}{9}$ times as strong as one having one flat and one round end, as the pillar *b*; *b* is thus $2\frac{1}{4}$ times as strong as *c*. It has also been found that if three pillars *a*, *b*, *c*, which have the same cross-section, are to carry the same load and be of equal strength, their lengths must be as the numbers 2, $1\frac{1}{2}$, and 1, respectively.

In practice, however, the ends of the pillars *b* and *c* are not generally made as shown by the figure, but have holes at their ends into which pins are fitted that are fastened to some other piece. In such cases, it has been found that *a* is

2 times as strong as *c* and that *b* is $1\frac{1}{2}$ times as strong as *c*. That is, in actual practice, a column fixed as at *c* is really $\frac{1}{2}$ as strong as one fixed as at *a*, instead of being only $\frac{1}{4}$ as strong, as given above.

An example of a column fixed at both ends is the piston rod of a steam engine. The valve stem of a slide-valve engine may be considered as an example of a column fixed at one end and movable at the other. The connecting-rod of a steam engine is a good example of a column having two movable ends.

35. Average values of the crushing strength of various materials used in engineering are given in the table below.

TABLE V.

CRUSHING STRENGTH OF MATERIALS.

Material.	Crushing Strength. Pounds per Sq. In.	Safe Loads. Pounds.		
		Sudden.	Gradual.	Steady.
Cast iron	80,000	7,000	11,000	14,000
Wrought iron	45,000	3,500	4,500	6,000
Machinery steel	60,000	4,000	6,000	8,000
Cast brass	9,000	800	1,200	1,500
Timber, dry	7,000	600	850	1,100
Timber, wet	3,000	250	380	500
Stone	6,000	....	....	200
Brick	2,000	....	....	100

STRENGTH OF COLUMNS.

36. In practice, columns subjected to a compressive stress are made of cast iron, wrought iron, steel, or timber. For short columns, the area, etc. can be calculated by the same rules that have been given for tensile stresses, substituting the working loads per square inch given in the above table.

37. The **safe working load on long columns** is given by the following rule, which is applicable to columns the length of which does not exceed 40 times their least diameter or their least thickness when rectangular. The rule given applies to columns uniform throughout their length. The rules for tapering columns involve considerable mathematical knowledge and are so rarely used in practice that they will not be given here.

Let C = safe crushing load, as given in the table;
S = sectional area in square inches;
L = length of column in inches;
d = least thickness of rectangular column, or diameter of round column, or dimension indicated by the arrowheads in the following table, in inches;
W = safe working load in pounds;
A = area of the two flanges in square inches;
B = area of the web in square inches;
a = constant corresponding to the cross-section of the column, as given in Tables VI, VII, and VIII.

Rule 7.—*The safe working load of a long column, in pounds, is equal to the safe working load corresponding to the kind of stress given in Table V multiplied by the sectional area of the column, in square inches, and the product divided by 1 plus the quotient obtained by dividing the square of the length of the pillar in inches by the square of the diameter (or least thickness, if rectangular) multiplied by the value of a.*

Or,
$$W = \frac{C\,S}{1 + \frac{L^2}{a\,d^2}}.$$

38. When using this formula, first obtain the value of C from Table V. Next, calculate the area of the cross-section of the pillar. Then find the value of a from one of the last three tables. Finally, be sure that the length of the column has been reduced to inches before substituting in the formula.

TABLE VI.

CONSTANTS FOR WROUGHT-IRON AND STRUCTURAL-STEEL PILLARS.

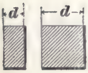
Cross-Section of Pillar.		When Both Ends of the Pillar Are Flat or Fixed.	When One End of the Pillar Is Flat or Fixed and the Other Round or Movable.	When Both Ends of the Pillar Are Round or Movable.
	Round.	2,250	1,500	1,125
	Square or Rectangle.	3,000	2,000	1,500
	Thin Square Tube.	6,000	4,000	3,000
	Thin Round Tube.	4,500	3,000	2,250
	Angle With Equal Sides.	1,500	1,000	750
	Cross With Equal Arms.	1,500	1,000	750
	I Beam.	$3{,}000 \times \frac{A}{A+B}$	$2{,}000 \times \frac{A}{A+B}$	$1{,}500 \times \frac{A}{A+B}$

TABLE VII.

CONSTANTS FOR CAST-IRON PILLARS.

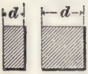
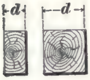
Cross-Section of Pillar.		When Both Ends of the Pillar Are Flat or Fixed.	When One End of the Pillar Is Flat or Fixed and the Other Round or Movable.	When Both Ends of the Pillar Are Round or Movable.
	Round.	281.25	187.5	140.625
	Square or Rectangle.	375.00	250.0	187.500
	Thin Square Tube.	750.00	500.0	375.000
	Thin Round Tube.	562.50	375.0	281.250
	Angle With Equal Sides.	187.50	125.0	93.750
	Cross With Equal Arms.	187.50	125.0	93.750
	I Beam.	$375 \times \frac{A}{A+B}$	$250 \times \frac{A}{A+B}$	$125 \times \frac{A}{A+B}$

TABLE VIII.

CONSTANTS FOR WOODEN PILLARS.

Cross-Section of Pillar.		When Both Ends of the Pillar Are Flat or Fixed.	When One End of the Pillar Is Flat or Fixed and the Other Round or Movable.	When Both Ends of the Pillar Are Round or Movable.
	Round.	187.5	125.00	93.75
	Square or Rectangle.	250.0	166.66	125.00
	Hollow Square Made of Boards.	500.0	333.33	250.00

To find the proper value of a in any example, first turn to the table dealing with the material in question, and find the figure corresponding to the given cross-section; in the horizontal line containing this are three numbers corresponding to the different conditions of the ends of the column. From these numbers select the one corresponding to the given conditions of the column to be calculated, and this will be the required value of a.

NOTE.—If the length of the pillar is given in feet, be sure to reduce it to inches before substituting in the formula.

39. Rule **7** cannot be transformed to determine directly the diameter or area of a column of a given cross-section required to sustain a stated load. An area and a corresponding value of d must be assumed and the safe load

corresponding to these values calculated. If the safe load is smaller than the load to be sustained, a larger area and value of d must be chosen. If the calculated safe load is larger, a somewhat smaller area and value of d may be tried.

When applying the rule given to a column subject to wear, as, for instance, the piston rod of an engine, the diameter of the rod should be increased somewhat over that given by calculation in order to allow for subsequent truing up. This allowance may be from $\frac{1}{8}$ to $\frac{1}{4}$ of an inch, according to the judgment of the designer.

40. A rough-and-ready rule for the size of a piston rod is to make it one-sixth the diameter of the cylinder of a simple engine. This rule of thumb allows us to choose a value of S and d that will answer very well indeed for the first trial.

ILLUSTRATION.—What size machinery-steel piston rod is required for a simple engine having a piston 36 inches in diameter to carry a steam pressure (gauge) of 100 pounds per square inch; the rod is 70 inches long?

According to the rule given, an approximate size of the piston rod will be $36 \div 6 = 6$ inches. The area corresponding to this is $6^2 \times .7854 = 28.27$ square inches. From Table VI, $a = 2,250$. The piston rod being subjected to suddenly applied stresses, by Table V, the allowable safe working load per square inch is 4,000 pounds. The working load on the piston rod is $36^2 \times .7854 \times 100 = 101,787.8$ pounds. Applying rule **7** and substituting values, we get

$$W = \frac{4,000 \times 28.27}{1 + \frac{70^2}{2,250 \times 6^2}} = 106,629 \text{ pounds,}$$

which is a safe load on a 6-inch piston rod under the given conditions. This tends to show that the rod is rather large.

Now try a $5\frac{7}{8}$-inch rod. The area of the rod is $5\frac{7}{8}^2 \times .7854 = 27.1$ square inches, nearly.

Applying rule **7** again and substituting the new values of S and d, we get

$$W = \frac{4,000 \times 27.1}{1 + \frac{70^2}{2,250 \times 5\frac{7}{8}^2}} = 102,264 \text{ pounds, nearly.}$$

It is thus seen that a $5\frac{3}{8}$-inch rod is just about right. Most designers would allow $\frac{1}{8}$ inch for truing up, thus making the rod 6 inches in diameter.

In this particular case the size of rod found by calculation agrees with that given by the rough-and-ready rule. This cannot be in every case, however, as will readily be seen if a steam pressure of 150 pounds be substituted for 100 pounds and the rod be calculated for this pressure. It will then be found that a rod larger than 6 inches will be required.

41. In calculating the size of piston rods in actual work, it is good practice to assume at least a steam pressure of 100 pounds for the calculation, even though it is intended to carry a lower pressure. An extra margin of strength is thus provided which may be needed in the future, as cases are very frequent where engines designed for 75 pounds have ultimately been run at 100 pounds pressure.

42. In engineers' examinations, candidates for license are often asked to calculate the size of the piston rod for a given engine. A rule that is easily remembered is the following, which assumes the piston rod to be a short column and makes allowance for failure by bending by reducing the allowable working stress given in the table by 10 per cent., thus making the safe working stress for steel piston rods 3,600 pounds per square inch and, say, 3,100 pounds for iron.

This rule will give fairly good results within the limits of ordinary practice and will usually satisfy an examiner. For an actual design, however, rule **7**, which is more rational, is preferable.

Let S = area of piston rod;
l = load in pounds on the rod = area of piston $\times$ steam pressure;
c = safe working load in pounds.

Rule 8.—*Multiply the area of the piston by the steam pressure and divide the product by the safe working load corresponding to the material. The quotient will be the required area.*

Or, $$S = \frac{l}{c}.$$

EXAMPLE.—What size steel piston rod is required for a 36-inch cylinder with a steam pressure of 100 pounds?

SOLUTION.—The safe working load to be used in applying rule 8 is 3,600 pounds.

Then, $$S = \frac{36^2 \times .7854 \times 100}{3,600} = 28.27 \text{ square inches.}$$

The corresponding diameter is $\sqrt{\frac{28.27}{.7854}} = 6$ in. Ans.

EXAMPLES FOR PRACTICE.

1. A round wrought-iron column 4 inches in diameter and 60 inches long is subjected to a steady load. The column is fixed at one end and movable at the other. What load will it sustain?

Ans. 65,564 lb., nearly.

2. A solid machinery-steel column with both ends hinged is $4\frac{1}{2}$ inches in diameter and 10 feet long. It is subjected to a suddenly applied stress. What is the safe working load? Ans. 38,979 lb.

3. A rectangular wooden column is 14 feet long. One end is fixed and one end is movable. If the cross-section is 12 in. $\times$ 8 in., what is its safe load for a steady stress? Ans. 28,963 lb.

TRANSVERSE STRENGTH OF MATERIALS.

43. The **transverse strength** of any material is the resistance offered by its fibers to being broken by bending. As, for example, when a beam, bar, rod, etc. which is supported at its ends is broken by a force applied between the supports.

The transverse strength of any beam, bar, rod, etc. is proportional to the product of the square of its depth

multiplied by its width; consequently, it is more economical to increase the depth than the width.

44. The safe load that can be carried by beams, bars, rods, etc. of uniform cross-section depends on the material, the manner in which the beam is supported and loaded, and on the shape of the cross-section. A beam that is rigidly fixed at one end and free at the other, when subjected to a transverse stress, is called a **cantilever.**

45. The safe working loads on beams supported and loaded in different ways can be found by applying the corresponding formula given in Table IX. In the formulas given, $W=$ the safe load in pounds, and $l=$ length of beam in inches to be taken as shown in the illustrations of Table IX. To apply the formulas to a beam, the values of S must be taken from Table X; and the value of R is to be computed by the formulas given in Table XI. It may then be substituted. Attention is called to the fact that it is absolutely necessary to reduce the length of the beam to inches; furthermore, the load W on a uniformly loaded beam is the total load for the length l, and not the load per unit of length.

In the table giving the value of R, the letter A denotes the whole area of the section in square inches, that is, taking the hollow cylinder, for example, it denotes the area corresponding to its outside diameter. The letter a stands for the area of the hollow part of the section. In the formulas where A is applied to a solid section, it stands for the area of the section.

The values of the constant S have been determined from practical experience and are safe, conservative values for the conditions stated.

EXAMPLE 1.—A cast-iron, solid, rectangular beam 8 inches deep and 4 inches wide rests upon two supports 8 feet apart. What steady load will it safely support when the load is applied at the middle?

SOLUTION.—According to Table IX, the formula to be used is $W=\frac{4\,S\,R}{l}$. For a steady stress on cast iron, Table X gives 7,500 as

TABLE IX.

FORMULAS FOR STRENGTH OF BEAMS.

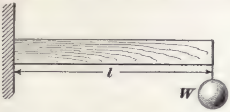
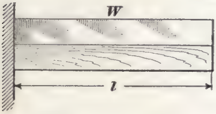
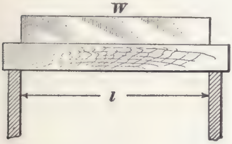
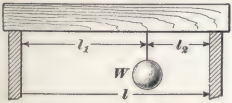
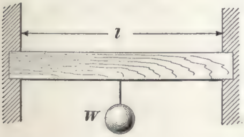
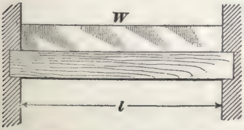
Manner of Supporting Beams.	Remarks.	Formula for Safe Load. Pounds.
	Cantilever load W at free end.	$W = \frac{SR}{l}$
	Cantilever, uniformly loaded. W = total load.	$W = \frac{2SR}{l}$
	Simple beam resting on two supports, load W at middle.	$W = \frac{4SR}{l}$
	Simple beam resting on two supports, uniformly loaded. W = total load on length l.	$W = \frac{8SR}{l}$
	Simple beam resting on two supports, single load W not in the middle.	$W = \frac{SR}{\frac{l_1 l_2}{l}}$
	Beam rigidly fixed at both ends, load W in middle.	$W = \frac{8SR}{l}$
	Beam rigidly fixed at both ends, uniformly loaded. W = total load on length l.	$W = \frac{12SR}{l}$

TABLE X.

VALUES OF S.

Material.	Nature of Load.		
	Suddenly Applied.	Gradually Applied.	Steady.
Cast iron	2,250	3,000	7,500
Wrought iron	4,000	6,000	13,700
Structural steel	5,000	7,500	16,800
Brass	1,100	1,500	3,600
White pine	320	480	960
Yellow pine	500	730	1,460
Hemlock	240	360	720
Oak	400	600	1,200

the value of S. For a solid rectangular beam, Table XI gives $R = \frac{b\,h^2}{6}$. Substituting the width and depth of the beam in this last formula, we get $R = \frac{4 \times 8^2}{6} = 42.67$, nearly. The length of the beam in inches is $8 \times 12 = 96$ inches. Substituting all the values in the formula for the safe working load, we get

$$W = \frac{4 \times 7{,}500 \times 42.67}{96} = 13{,}334 \text{ lb., nearly. Ans.}$$

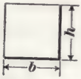
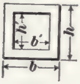
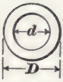
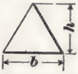
EXAMPLE 2.—An I beam having a depth of 10 inches and an area of section of 7.5 square inches is to be used as a cantilever to bear a varying load that is to be uniformly distributed. The beam is made of steel and is 100 inches long. What safe load will it carry?

SOLUTION.—From Table IX, it appears that the proper formula to be used is $W = \frac{2\,S\,R}{l}$. According to Table X, the value of S for steel for a gradually applied load is 7,500. By Table XI for an I beam, $R = \frac{A\,h}{3.33}$. Substituting values in this last formula, we get $R = \frac{7.5 \times 10}{3.33} = 22.52$, nearly. Substituting values in the formula for the safe working load, we get

$$W = \frac{2 \times 7{,}500 \times 22.52}{100} = 3{,}378 \text{ lb. Ans.}$$

TABLE XI.

VALUES OF *R*.

Section.	*R*.
	$\frac{b\,h^2}{6}$
	$\frac{b\,h^3 - b'\,h'^3}{6\,h}$
	$\frac{A\,D}{8}$
	$\frac{A\,D^2 - a\,d^2}{8\,D}$
	$\frac{b\,h^2}{24}$
	$\frac{A\,h}{7.2}$
	$\frac{A\,h}{6.5}$
	$\frac{A\,h}{9.5}$
	$\frac{A\,h}{8}$
	$\frac{A\,h}{3.33}$
	$\frac{A\,h}{3.67}$

46. While the formulas given in Table IX will allow the safe load on a given beam to be calculated, they cannot readily be transformed to give directly the size of beam required under given conditions to carry a given load.

In practice, when it is required to determine the size of beam, the length between supports is known. After the shape of cross-section of the beam has been chosen, dimensions for the beam may be assumed and its safe load for the given dimensions calculated. If the load thus found falls below the load the beam is to carry, larger dimensions must be chosen. In case of rolled sections, such as angle irons, **T** irons, channels, or **I** beams, catalogues of manufacturers should be consulted and standard sizes chosen. These catalogues usually contain the area of the section for different weights and dimensions of rolled sections.

47. The values of R given in Table XI *apply only when the dimension marked h is vertical.* If the beam is placed in any other position, R will have a different value, which can only be calculated by the aid of higher mathematics. Beams rigidly fixed at both ends are rarely met with in practice, and it is safer to assume that the beam merely rests on two supports.

EXAMPLES FOR PRACTICE.

1. What weight will a yellow-pine rectangular beam carry safely under a steady load when used as a cantilever and uniformly loaded? The beam is 9 feet 6 inches long, 16 inches deep, and 4 inches wide. Ans. 4,371 lb.

2. What weight would the beam in example 1 carry safely if it rested on two supports? Ans. 17,484 lb.

3. What weight can safely be carried at the end of a wrought-iron round cantilever 3 inches in diameter when the load is suddenly applied 4 feet 2 inches from the support of the cantilever? Ans. 212 lb.

4. What steady load can be carried safely by the same beam given in the preceding example? Ans. 726 lb.

SHEARING, OR CUTTING, STRENGTH OF MATERIALS.

48. The **shearing strength** of any material is the resistance offered by its fibers to being cut in two. Thus, the pressure of the cutting edges of an ordinary shearing machine, Fig. 6, causes a shearing stress in the plane *a b*. The unit shearing force may be found by dividing the force *P* by the area of the plane *a b*.

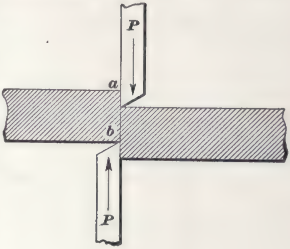


FIG. 6.

Fig. 7 shows a piece in double shear; here the central piece *c d* is forced out while the ends remain on their supports *M* and *N*.

The shearing strength of any body is directly proportional to its area.

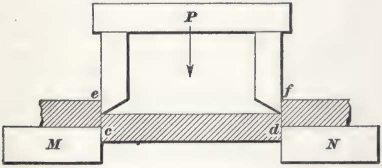


FIG. 7.

49. In Table XII. are given the greatest and the safe shearing strengths per square inch of different kinds of materials.

50. Rules for Shearing.—In general, the force required to shear a piece of material in double shear will be *twice* that required for single shear. This does not apply to all cases, however; a notable exception are rivets in double shear. Experiments have determined that the force required to shear both iron and steel rivets in double shear averages about 1.85 times that required for single shear.

TABLE XII.

SHEARING STRENGTH OF MATERIALS.

Material.	Average Ultimate Shearing Strength.		Safe Loads. Sudden Load.		Safe Loads. Gradual Load.		Safe Loads. Steady Load.	
	Parallel to Grain.	Across Grain.	Parallel to Grain.	Across Grain.	Parallel to Grain.	Across Grain.	Parallel to Grain.	Across Grain.
Cast iron	25,000	....	2,500	...	3,700	...	5,000	...
Wrought-iron rivets	38,000	....	3,700	...	5,600	...	7,500	...
Soft-steel rivets	44,000	....	4,400	...	6,600	...	8,800	...
Ordinary wrought iron	44,000	....	4,400	...	6,600	...	8,800	...
Structural steel	52,000	....	5,000	...	7,500	...	10,000	...
White pine	300	2,500	30	250	45	375	60	500
Yellow pine	400	4,500	40	450	60	675	80	900
Hemlock	250	2,500	25	250	38	375	50	500
Spruce	300	3,000	30	300	45	450	60	600
Oak	600	5,000	60	500	90	750	120	1,000

Let a = area of cross-section in square inches;
S = shearing stress as given in table;
W = load in pounds.

Then, to find the safe load or the load that will shear the material:

Rule 9.—*Multiply the area of the section by the shearing stress.*

Or, $$W = a S.$$

In applying this rule, it should be remembered that when the safe load the material can bear is required, the value of S is to be taken from the column headed "Safe Loads," selecting the safe load corresponding to the nature of the load. When it is desired to find the load at which the material will fail, the value of S is to be taken from the column headed "Ultimate Shearing Strength."

EXAMPLE.—If the beam in double shear shown in Fig. 7 is rectangular and measures 4 in. $\times$ 2 in., what steady safe load would you allow if the beam were made of structural steel?

SOLUTION.—According to the table, a safe load of 10,000 pounds per square inch of section may be allowed. Applying rule **9** and multiplying by 2, since the beam is in double shear, we get

$$W = 4 \times 2 \times 10{,}000 \times 2 = 160{,}000 \text{ lb.} \quad \text{Ans.}$$

51. To find the area required for a material subjected to a shearing stress:

Rule 10.—*Divide the load by the shearing stress.*

Or, $$a = \frac{W}{S}.$$

In applying rule **10**, it should be remembered that for double shear the result is to be divided by 2 to obtain the area of the beam.

EXAMPLE.—A white pine beam is subjected to a suddenly applied shearing stress by a load of 4,000 pounds, the beam being in double shear and the load being applied across the grain. What should be the area of the beam to bear this stress safely?

SOLUTION.—By the table, $S = 250$. Then, by rule **10**, we get

$$a = \frac{4,000}{250} = 16 \text{ square inches for single shear.}$$

Dividing the result by 2 for double shear, we get $16 \div 2 = 8$ sq. in. area. Ans.

EXAMPLES FOR PRACTICE.

1. A hemlock beam 8 in. $\times$ 2 in. is in single shear parallel to the grain. What load will cause it to fail by shearing? Ans. 4,000 lb.

2. A wrought-iron rivet 1 inch in diameter is in double shear. At what load will it be likely to shear? Ans. 55,214 lb., nearly.

TORSION.

52. When a force is applied to a beam, bar, or rod in such a manner that it tends to twist it, the stress thus produced is termed **torsion.** Torsion manifests itself in the case of rotating shafts, such as line shafts and engine shafts.

LINE SHAFTING.

53. A **line of shafting** is one continuous run, or length, composed of lengths of shafts joined together by couplings.

54. The **main line of shafting** is that which receives the power from the engine or motor and distributes it to the other lines of shafting or to the various machines to be driven.

Line shafting is supported by hangers, which are brackets provided with bearings, bolted either to the walls, posts, ceilings, or floors of the building. Short lengths of shafting, called **countershafts,** are provided to effect changes of speed and to enable the machinery to be stopped or started.

55. Shafting is usually made cylindrically true, either by a special rolling process, when it is known as **cold-rolled shafting,** or else it is turned up in a machine called a **lathe.** In the latter case it is called **bright shafting.** What is

known as **black shafting** is simply bar iron rolled by the ordinary process and turned where it receives the couplings, pulleys, bearings, etc.

56. The diameter of bright turned shafting increases by $\frac{1}{4}$ inch up to about $3\frac{1}{2}$ inches in diameter; above this diameter it increases by $\frac{1}{2}$ inch. The actual diameter of a bright shaft is $\frac{1}{16}$ inch less than the commercial diameter, it being designated from the diameter of the ordinary round bar iron from which it is turned. Thus, a length of what is called 3-inch bright shafting is really only $2\frac{15}{16}$ inches in diameter.

Cold-rolled shafting is designated by its commercial diameter; thus, a length of what is called 3-inch shafting is 3 inches in diameter.

57. In the following table is given the maximum distance between the bearings of some continuous shafts that are used for the transmission of power:

TABLE XIII.

DISTANCE BETWEEN BEARINGS.

Diameter of Shaft. Inches.	Distance Between Bearings. Feet.	
	Wrought-Iron Shaft.	Steel Shaft.
2	11	11.50
3	13	13.75
4	15	15.75
5	17	18.25
6	19	20.00
7	21	22.25
8	23	24.00
9	25	26.00

Pulleys from which considerable power is to be taken should always be placed as close to a bearing as possible.

58. The diameters of the different lengths of shafts composing a line of shafting may be proportional to the quantity of power delivered by each respective length. In this connection it is to be observed that the positions of the various pulleys in reference to the bearings must be taken into consideration in deciding upon the size of a shaft. Suppose, for example, that a piece of shafting delivers a certain amount of power; then, it is obvious that the shaft will deflect or bend less if the pulley transmitting that power be placed close to a hanger or bearing than if it be placed midway between the two hangers or bearings. It is impossible to give any rule for the proper distance of bearings that could be used universally, as in some cases the requirements demand that the bearings be nearer than in others.

Wherever possible it is advisable to have the main line of shafting run through the center of the room, or at least far enough from either wall to allow countershafts to be placed on either side of it. When this is done, power may be taken off the main shaft from either side by alternate pulleys, and the deflection caused in the main shaft in one direction by one pulley will be counteracted by the deflection caused in the opposite direction by the next pulley.

If the work done by a line of shafting is distributed quite equally along its entire length and the power can be applied near the middle, the strength of the shaft need be only half as great as would be required if the power were applied at one end.

59. *To compute the horsepower that can be transmitted by a shaft of any given diameter:*

Let $D =$ diameter of shaft;
$R =$ revolutions per minute;
$H =$ horsepower transmitted;
$C =$ constant given in Table XIV.

TABLE XIV.

CONSTANTS FOR LINE SHAFTING.

Material of Shaft.	No Pulleys Between Bearings.	Pulleys Between Bearings.
Steel or cold-rolled iron.........	65	85
Wrought iron..................	70	95
Cast iron......................	90	120

In the above table the bearings are supposed to be spaced so as to relieve the shaft of excessive bending; also, in the third vertical column, an average number and weight of pulleys and power given off is assumed.

60. Rules and Formulas.—In determining the above constants, allowance has been made to insure the stiffness as well as strength of the shaft. Cold-rolled iron is considerably stronger than ordinary turned wrought iron; the increased strength is due to the process of rolling, which seems to compress the metal and so make it denser, not merely skin deep, but practically throughout the whole diameter. We have, then, the following

Rule 11.—*The horsepower that a shaft will transmit equals the product of the cube of the diameter and the number of revolutions, divided by the value of C for the given material.*

Or, $$H = \frac{D^3 \times R}{C}.$$

EXAMPLE.—What horsepower will a 3-inch wrought-iron shaft transmit which makes 100 revolutions per minute, there being no pulleys between bearings?

SOLUTION.—Applying rule **11** and substituting, we have

$$H = \frac{3 \times 3 \times 3 \times 100}{70} = 38.57 \text{ H. P. Ans.}$$

If there were the usual amount of power taken off, as mentioned above, we should take $C = 95$.

Then, $$H = \frac{27 \times 100}{95} = 28.42 \text{ H. P.} \quad \text{Ans.}$$

61. To compute the number of revolutions a shaft must make to transmit a given horsepower:

Rule 12.—*The number of revolutions necessary for a given horsepower equals the product of the value of C for the given material and the number of horsepower, divided by the cube of the diameter.*

Or, $$R = \frac{C \times H}{D^3}.$$

EXAMPLE.—How many revolutions must a 3-inch wrought-iron shaft make per minute to transmit 28.42 horsepower, power being taken off at intervals between the bearings?

SOLUTION.—Applying the rule just given and substituting, we have

$$R = \frac{95 \times 28.42}{3 \times 3 \times 3} = 100 \text{ rev.} \quad \text{Ans.}$$

62. To compute the diameter of a shaft that will transmit a given horsepower, the number of revolutions the shaft makes per minute being given:

Rule 13.—*The diameter of a shaft equals the cube root of the quotient obtained by dividing the product of the value of C for the given material and the number of horsepower by the number of revolutions.*

Or, $$D = \sqrt[3]{\frac{C \times H}{R}}.$$

EXAMPLE.—What must be the diameter of a wrought-iron shaft to transmit 38.57 horsepower, the shaft to make 100 revolutions per minute, no power being taken off between bearings?

SOLUTION.—By rule **13**, we have

$$D = \sqrt[3]{\frac{70 \times 38.57}{100}} = \sqrt[3]{27} = 3 \text{ in.} \quad \text{Ans.}$$

63. As the speed of shafting is used as a multiplier in the calculations of the horsepower of shafts, it is readily seen that a shaft having a given diameter will transmit more

power in proportion as its speed is increased. Thus, a shaft that is capable of transmitting 10 horsepower when making 100 revolutions per minute will transmit 20 horsepower when making 200 revolutions per minute. We may, therefore, say *the number of horsepower transmitted by a shaft is directly proportional to the number of revolutions.*

EXAMPLES FOR PRACTICE.

1. What horsepower will a $2\frac{1}{2}$-inch wrought-iron shaft transmit when running at 110 revolutions per minute, it being used for transmission only? Ans. 24.55 H. P.

2. A 6-inch cast-iron shaft transmits 150 horsepower. How many revolutions per minute must it make, no power being taken off between bearings? Ans. $62\frac{1}{2}$ R. P. M.

3. What should be the diameter of a wrought-iron shaft to transmit 100 horsepower at 150 revolutions per minute, power being taken off between bearings? Ans. 4 in., nearly.

4. The machines driven by a certain line of wrought-iron shafting take their power from various points between the bearings, and if all were working together at their full capacity, they would require 65 horsepower to drive them. What diameter should the shaft be if it runs at 150 revolutions per minute? Ans. $3\frac{1}{2}$ in., nearly.

ELEMENTS OF ELECTRICITY AND MAGNETISM.

INTRODUCTION.

1. Electricity is the name given to the cause of all electrical phenomena. The word is derived from a Greek word meaning *amber*, that substance having been observed by the Greeks to possess peculiar properties which we now understand to be due to electricity.

Although electrical science has advanced sufficiently far to recognize the fact that the exact nature of electricity is unknown, yet recent research tends to demonstrate that all electrical phenomena are due to a peculiar strain or stress of a medium called *ether;* that when in this condition the *ether* possesses *potential energy* or *capacity for doing work*, as is manifested by attractions and repulsions, by chemical decomposition, and by luminous, heating, and various other effects.

In all probability, electricity is not a form of matter because it does not possess most of the ordinary properties of matter. Electricity, itself, is not a form of energy, though energy may be necessary to move it under certain conditions, and electricity in motion is capable of performing work.

NOTE.—This section is the same as that formerly entitled *Dynamos and Motors*, Part 1.

Electrical science is founded upon the effects produced by the action of certain forces upon matter, and all knowledge of the science is deduced from these effects. The study of the fundamental principles of electricity is an analysis of a series of experiments and the classification of the results in each particular case under general laws and rules. It is not necessary to keep in mind any hypothesis of the exact nature of electricity; its effects and the laws which govern them are quite similar to those of well-known mechanical and natural phenomena and will be best understood by comparison. The two most essential features, therefore, in acquiring a knowledge of the electrical science are: first, to learn how to develop electrical action; and, second, to determine the effects produced by it.

2. The number of processes for developing electrical action is almost innumerable, but the most important can be classified under one of the following general heads:

(*a*) By the contact of dissimilar substances.

(*b*) By chemical action.

(*c*) By the application of heat.

(*d*) By magnetic induction.

3. The presence of electricity, also, can be detected in many different ways; under certain conditions, it will

(*a*) Cause attractions and repulsions of light particles of matter, such as feathers, pith, gold-leaf, pieces of paper, etc.

(*b*) Decompose certain forms of matter into their various elements and cause other chemical changes.

(*c*) Produce motion in a freely suspended magnetic needle, such as the needle of a compass.

(*d*) Violently agitate the nervous system of all animals, causing a **shock.**

(*e*) Heat the substances through which it acts.

These are the principal effects produced by the action of electricity; others of less importance will appear from time

to time during the study of the different branches of the science.

4. Electricity may either appear to reside upon the surface of bodies as a **charge,** under *high pressure* or *tension*, or flow through their substance as a **current,** under comparatively *low pressure* or *tension*.

That branch of the science which treats of charges upon the surface of bodies is termed **electrostatics,** and the charges are said to be **static charges** of electricity.

Electrodynamics is that branch which treats of the action of *electric currents*.

STATIC CHARGES.

5. When a glass rod or a piece of amber is rubbed with silk or fur, the parts rubbed will have the property of attracting light particles of matter, such as pieces of silk, wool, feathers, gold-leaf, pith, etc., which, after momentary contact, are repelled. These attractions and repulsions are caused by a static charge of electricity residing upon the surface of those bodies. A body in this condition is said to be **electrified.**

A better experiment for demonstrating this action is to suspend a small pith-ball by a silk thread from a support or bracket, as shown in Fig. 1. If a *static charge* of electricity be developed on a *glass rod*, by rubbing it with *silk*, and the rod be brought near the pith-ball, the ball will be attracted to the rod, but, after momentary contact, will be repelled. By this contact the ball receives a charge of the same nature as that on the glass rod, and as long as the two bodies retain their charges, mutual repulsion will take place whenever they

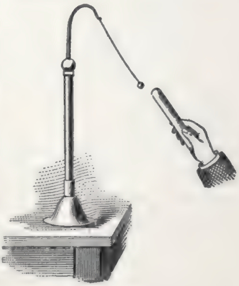


FIG. 1.

are brought near each other. If a stick of *sealing-wax*, electrified by being rubbed with fur, is approached to another pith-ball, the same results will be produced, i. e., the ball will fly towards the sealing-wax, and after contact will be repelled. But the charges respectively developed in these two cases are not in the same condition. For if, after the pith-ball in the first case had been touched with the *glass rod* and repelled, the electrified *sealing-wax* be brought in the vicinity, *attraction* would take place between the ball and the sealing-wax. Conversely, if the pith-ball be charged with the electrified sealing-wax, it will be repelled by the wax and attracted by the glass rod.

An electric charge developed upon *glass* by rubbing it with silk has been termed, for convenience, a **positive** (+) **charge,** and that developed on resinous bodies by rubbing with flannel or fur, a **negative** (–) **charge.**

Neither a positive nor a negative charge is produced alone, for there is always an equal quantity of both charges produced, one charge appearing on the body rubbed and an equal amount of the opposite charge upon the rubber.

The intensity of the charge developed by rubbing the two substances together is independent of the actual amount of friction which takes place between the bodies. For, in order to obtain the highest possible degree of electrification, it is only necessary to bring every portion of one surface into intimate contact with every particle or every portion of the other; when this is done, no extra amount of rubbing can develop any greater charge upon either substance.

6. From the foregoing experiments are derived the following laws:

When two dissimilar substances are placed in contact, one of them always assumes the positive and the other the negative condition, although the amount may sometimes be so small as to render its detection very difficult.

Electrified bodies with similar charges are mutually repellent, while electrified bodies with dissimilar charges are mutually attractive.

7. In the following list, called the **electric series,** the substances are arranged in such order that each receives a *positive* charge when rubbed or placed in contact with any of the bodies following it, and a *negative* charge when rubbed with any of those which precede it:

1. Fur.	6. Cotton.	11. Sealing-wax.
2. Flannel.	7. Silk.	12. Resins.
3. Ivory.	8. The body.	13. Sulphur.
4. Crystals.	9. Wood.	14. Gutta-percha.
5. Glass.	10. Metals.	15. Guncotton.

For example, *glass* when rubbed with *fur* receives a *negative* charge, but when rubbed with *silk* it receives a *positive* charge.

CONDUCTORS AND NON-CONDUCTORS.

8. Only that part of a dry glass rod which has been rubbed will be electrified; the other parts will produce neither repulsion nor attraction when brought near a suspended pith-ball. The same is true of a piece of sealing-wax or resin. These bodies do not readily *conduct* electricity; that is, they *oppose* or *resist* the passage of electricity through them. Therefore, it can only reside as a *charge* upon that part of their surface where it is developed. Experiments show that when a metal receives a charge at any point, the electricity immediately passes or flows through its substance to all parts. Metals, therefore, are said to be **good conductors** of electricity. Bodies have accordingly been divided into two classes, i. e., **non-conductors,** or **insulators,** or those bodies which offer a very high **resistance** to the passage of electricity, and **conductors,** or those bodies which offer a comparatively low resistance to its passage. This distinction is not absolute, for all bodies conduct electricity to some extent, while there is no known substance which does not offer some resistance to the flow of electricity.

In giving the following list and dividing the different substances into two classes, it should be understood that it is done only as a guide for the student. Between these two classes are many substances which might be included in either list, and no hard or fast line can be drawn.

Silver, Copper, Other Metals, Charcoal, Ordinary Water, The Body.	CONDUCTORS.
Paper, Oils, Porcelain, Wood, Silk, Resins, Gutta-percha, Shellac, Ebonite, Paraffin, Glass, Dry Air, etc.	NON-CONDUCTORS OR INSULATORS.

ELECTRODYNAMICS.

9. In dealing with *electric currents*, the word **potential** will be substituted for the general and vague phrase *electrical condition.*

The term *potential*, as used in electrical science, is analogous with *pressure* in gases, *head* in liquids, and *temperature* in heat.

When an electrified body *positively* charged is connected to the earth by a conductor, electricity is said to flow *from* the body *to* the earth; and, conversely, when an electrified body *negatively* charged is connected to the earth in a

similar manner, electricity is said to flow *from* the earth *to* that body. This is called the **direction of flow** of an electric current. That which determines the *direction of flow* is the relative *electrical potential*, or *pressure*, of the two charges in regard to the earth.

It is impossible to say with certainty in which direction electricity really flows, or, in other words, to declare which of two points has the higher and which the lower electrical potential, or pressure. All that can be said with certainty is that when there is a *difference of electrical potential*, or *pressure*, electricity tends to flow *from* the point of higher *to* that of the lower *potential*, or *pressure*.

For convenience, it has been arbitrarily assumed and conventionally adopted that that electrical condition called *positive* is at a higher potential, or pressure, than that called *negative*, and that electricity tends to flow *from* a *positively to* a *negatively* electrified body.

The zero or normal level of water is taken as that of the surface of the sea, and the normal pressure of air and gases as that of the atmosphere at the sea level; similarly, there is a *zero potential*, or *pressure*, *of electricity* in the earth itself. The earth may be regarded as a reservoir of electricity of infinite quantity, and its potential, or pressure, may therefore be taken as zero.

The electrical condition called *positive* is assumed to be at a higher potential, or pressure, than the earth, and that called *negative* is assumed to be at a lower potential, or pressure, than the earth.

10. It must be understood that electricity is a *condition of matter*, and not matter itself, for it possesses neither *weight* nor *dimensions*. Consequently, the statement that electricity is *flowing* through a conductor must not be taken too literally; it must not be supposed that any material substance, such as a liquid, is actually passing through the conductor in the same sense as water flows through a pipe. The statement that electricity is flowing through a conductor is only another way of expressing the fact that the conductor

and the space surrounding it are in different conditions than usual and that they possess unusual properties. The action of electricity, however, is quite similar in many respects to the flow of liquids, and the study of electric currents is much simplified by the analogy.

11. In order to produce what is called an electric current, *it is first necessary to cause a difference of electrical potential between two bodies or between two parts of the same body.*

It was stated that when two dissimilar substances are simply placed in contact, one always assumes the positive and the other the negative condition; or, in other words, *a difference of electrical potential is developed between the two bodies.*

Placing a piece of copper and zinc in contact will develop a difference of electrical potential which can easily be detected. The same results will follow if the plates are slightly separated from each other and placed in a vessel containing saline or acidulated water, leaving a small portion of one end of each plate exposed. The exposed ends of the zinc and copper are now electrified to different degrees, or, in other words, there is a *difference of electrical potential* between them, one plate being at a higher potential than the other.

When the exposed ends are connected by any conducting material, the potential between the plates tends to *equalize* and a momentary rush or discharge of electricity passes between the exposed ends through the conductor, and also between the submerged ends through the liquid. During its passage through the liquid, the electricity causes certain chemical changes to take place; these chemical changes cause in their turn a fresh *difference of potential* between the plates, which is followed immediately by another equalizing discharge, and that by a further difference, and so on. These changes follow one another with great rapidity—so rapidly, in fact, that it is impossible to distinguish them apart, and they appear absolutely *continuous.* The equalizing flow

which is constantly taking place from one plate to the other is known as a **continuous current** of electricity. Consequently, an electric current becomes continuous when the difference of potential is constantly maintained.

By the use of a very delicate instrument, the submerged end of the copper is found to be electrified with a *negative* charge, while the submerged end of the zinc is electrified with a *positive* charge. The direction of the current, therefore, will be *from* the *submerged* end of the zinc through the liquid *to* the *submerged* end of the copper, and *from* the *exposed* end of the copper *to* the *exposed* end of the zinc.

12. A **simple voltaic,** or **galvanic, cell,** Fig. 2, is an apparatus for developing a continuous current of electricity. It consists essentially of a vessel containing saline or acidulated water in which are submerged two plates of dissimilar metals, or one metal and a metalloid (as, for instance, carbon).

Electrolyte is the name given to the liquid, which, as it transmits the current, is decomposed by it.

The two dissimilar metals, when spoken of separately, are called **voltaic,** or **galvanic, elements;** and, when taken collectively, are known as a **voltaic couple.**

A **voltaic,** or **galvanic, battery** is a number of simple cells properly joined together.

Electrodes, or **poles,** of a cell or battery are metallic terminals or connectors attached to the exposed ends of the plates, and are used to connect the cell or battery to any exterior conductor or to another cell or battery.

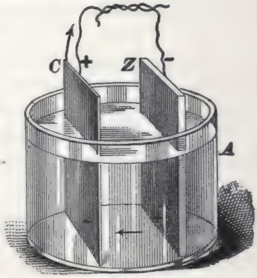


FIG. 2.

It should be remembered that the polarity of the submerged ends of the plates is always of opposite sign to that

of their electrodes. For example, in the case of the zinc-and-copper couple, the electrode fastened to the zinc would be spoken of as the *negative* electrode of the cell, while the zinc itself would be the *positive* element of the cell, its submerged end being *positive.*

In any voltaic, or galvanic, couple, the element which is acted upon by the electrolyte will always be the *positive* element and its electrode the *negative* electrode of the cell.

13. The following list of voltaic elements composes what is called the **electromotive series:**

1. Zinc.	5. Iron.	10. Silver.
2. Cadmium.	6. Nickel.	11. Gold.
3. Tin.	7. Bismuth.	12. Platinum.
4. Lead.	8. Antimony.	13. Graphite.
	9. Copper.	

Any two of these metals form a *voltaic couple* and produce a difference of potential when submerged in saline or acidulated water, the one standing first on the list being the *positive* element or plate, and the other the *negative.* For example, if *nickel* and *graphite* are used, the *nickel* will be acted upon by the liquid and will form the *positive* element; but if *nickel* and *zinc* are used, the *zinc* will be acted upon by the liquid and hence will be the *positive* element.

The difference of potential will be greater in proportion to the distance between the positions of the two substances in the list. For example, the difference of potential developed between *zinc* and *graphite* is much greater than that developed between *zinc* and *nickel;* in fact, the difference of potential developed between zinc and graphite is equal to the difference of potential developed between zinc and nickel *plus* that developed between nickel and graphite.

Electricity flowing as a *current* differs from *static charges* in three important degrees—namely, (1) its *potential* is much lower; (2) its *actual quantity* is greater; and (3) it is *continuous.*

A substance charged from a strong voltaic battery possesses the property of attracting light substances in only the slightest degree; in fact, the attractions can only be detected with the most delicate instruments. The *potential* of a current of electricity is comparatively so small that a voltaic battery composed of a large number of cells is not sufficient to produce a spark more than one or two one-hundredths of an inch long in air, whereas a small, rapidly moving leather belt will sometimes produce static sparks of more than an inch in length. The length of the spark affords a means of estimating potentials, a high potential being capable of producing a longer spark than a low potential; but the length of spark gives us no means of estimating the **current strength** or quantity of electricity flowing. The actual *quantity* of electricity is measured by the amount of water it will decompose. Gauged by this standard, the quantity of electricity produced by a voltaic cell no larger than a thimble would be found greater than that from a large, rapidly moving belt giving static sparks several inches in length.

14. There are three different methods of connecting or grouping the cells in a voltaic battery: *In series; in parallel, or multiple-arc; in multiple-series.*

Cells are connected **in series** when the positive electrode of the first cell is connected to the negative electrode of the second, and the positive electrode of the second is connected to the negative electrode of the third, and so on, as shown in the diagram, Fig. 3. In this we have adopted the usual signs for representing a cell, the short, broad line representing one of the electrodes of the cell, and the long, narrow line the other electrode. In this method of connecting or grouping cells, when the negative electrode of the first cell is connected to the positive electrode of the last by some exterior conductor, the total current produced will flow successively through each cell. This method of

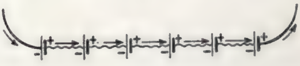


FIG. 3.

grouping is used when there is available a large number of *low*-potential cells and a *high* potential is desired, as in long telegraph lines or any other *high*-resistance circuit.

15. Cells are connected in **parallel,** or **multiple-arc,** when the positive electrodes of all the cells are connected to one main positive conductor and all the negative electrodes are connected to one main negative conductor, as shown by the diagram, Fig. 4. In parallel, or multiple-arc, grouping, only a part of the total current flowing in main conductors will pass through each cell. This method of grouping is used when it is desired to obtain a strong current from a number of cells (when the external resistance is *low*), as in electroplating.

FIG. 4.

16. Cells are connected in **multiple-series** by arranging them in several groups, each group being composed of several cells connected in series, and then connecting all the groups together in parallel, or multiple-arc, as shown in the diagram, Fig. 5. This method is used where both a higher potential and a stronger current are required than any one cell of the group will give.

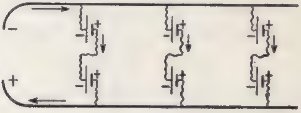


FIG. 5.

CIRCUITS.

17. A **circuit** is a path composed of a conductor or of several conductors joined together, through which an electric current flows from a given point around the conducting path back again to its starting point.

A circuit is **broken,** or **open,** when its conducting elements are disconnected in such manner as to prevent the current from flowing.

A circuit is **closed,** or **complete,** when its conducting elements are so connected as to allow the current to flow.

A circuit in which the earth, or ground, forms part of the conducting path is called an **earth,** or a **grounded,** circuit.

The **external** circuit is that part of a circuit which is outside or external to the electric source.

The **internal** circuit is that part of a circuit which is included within the electric source.

In the case of the simple voltaic cell, the *internal circuit* consists of the two metallic plates, or elements, and the electrolyte; an *external circuit* would be a wire or any conductor connecting the free ends of the electrodes.

18. Conductors are said to be connected **in series** when they are so joined together as to allow the current to pass consecutively through each. For example, Fig. 6 represents a *closed* circuit consisting of a simple voltaic cell B and four conductors a, b, c, and d connected *in series.*

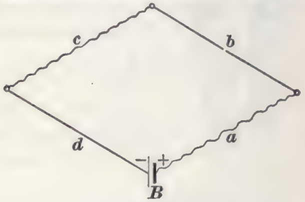


Fig. 6.

A circuit which is divided into two or more branches, each branch transmitting part of the main current, is a **derived,** or **shunt, circuit,** and the separate branches are said to be connected in **parallel,** or **multiple-arc.** An example of a *derived* circuit of two branches in *parallel* is shown in Fig. 7. The main current flows first through the conductor a, then divides between the branches c and b, and finally unites and completes the circuit through the conductor d, the two branches c and b being the conductors which are connected in *parallel,* or *multiple-arc.* The way the current divides and how the amount which will flow through the branches b and c is determined will be treated of later.

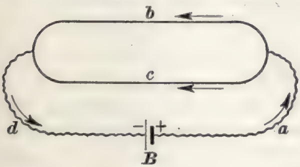


Fig. 7.

MAGNETISM.

19. Magnets are substances which have the property of attracting pieces of iron or steel, and the term **magnetism** is applied to the cause of this attraction. *Magnetism* exists in a natural state in an ore of iron, which is known in chemistry as *magnetic oxide of iron*, or *magnetite*. This magnetic ore was first found by the ancients in *Magnesia*, a city in Asia Minor; hence, substances possessing this property have been called magnets.

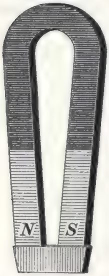


FIG. 8.

It was also discovered that when a small bar of this ore is suspended in a horizontal position by a thread, it has the property of pointing in a north and south direction. From this fact the name **lodestone**—*leading-stone*—was given to the ore.

When a bar or needle of hardened steel is rubbed with a piece of lodestone, it acquires magnetic properties similar to those of the lodestone without the latter losing any of its own force. Such bars are called **artificial** magnets.

Artificial magnets which retain their magnetism for a long time are called **permanent magnets.**

The common form of artificial, or permanent, magnet, Fig. 8, is a bar of steel bent into the shape of a *horseshoe* and then hardened and magnetized. A piece of soft iron, called an **armature,** or a **keeper,** is placed across the two free ends, which helps to prevent the steel from losing its magnetism.

20. If a bar magnet is dipped into iron filings, the filings are attracted toward the two ends and adhere there in tufts, while toward the center of the bar, half way between the two ends, there is no such tendency. (See Fig. 9.) That part of the magnet where there is no apparent magnetic attraction is called the **neutral line,** and the parts around the two ends where the attraction is greatest are called **poles.** An imaginary line drawn through

the center of the magnet from end to end, connecting the two poles together, is called the **axis of magnetism.**

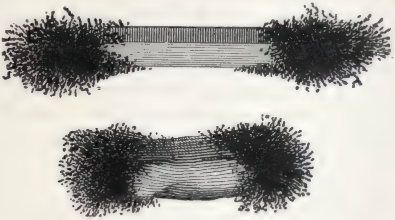


FIG. 9.

A **compass** consists of a magnetized steel needle, Fig. 10, resting upon a fine point, so as to turn freely in a horizontal plane. When not in the vicinity of other magnets or magnetized iron, the needle will always come to rest with one end pointing towards the north and the other towards the south. The end pointing northward is the **north-seeking pole,** or, simply, the *north pole*, and the opposite end is the **south-seeking** or *south pole.* This *polarity* applies as well to all magnets.

FIG. 10.

If the *north pole* of one magnet is brought near the *south pole* of another magnet, attraction takes place; but if two north poles or two south poles are brought together, they repel each other. In general, *like magnetic poles repel one another; unlike poles attract one another.*

The earth is a great magnet whose magnetic poles coincide nearly, but not quite, with the true geographical north and south poles. A freely suspended magnet, therefore, will always point in an approximately north and south direction.

It is impossible to produce a magnet with only one pole. If a long bar magnet is broken into any number of parts, each part will still be a magnet and have two poles, a north and a south one.

21. Magnetic substances are those substances which, although not in themselves magnets, that is, not possessing poles and neutral lines, are, nevertheless, capable of being attracted by a magnet. In addition to iron and its alloys, the following elements are magnetic substances: *Nickel*, *cobalt*, *manganese*, *oxygen*, *cerium*, and *chromium*. These, however, possess magnetic properties in a very inferior degree compared with iron and its alloys. All other known substances are called **non-magnetic substances.**

22. The space surrounding a magnet, in which any magnetic substance will be attracted or repelled, is called its **magnetic field,** or, simply, its **field.** Magnetic attractions and repulsions are assumed to act in a definite direction and along imaginary lines called **lines of magnetic force,** or, simply, **lines of force,** and every magnetic field is assumed to be traversed by such *lines of force*—in fact, to exist by virtue of them. Their position in any plane may be shown by placing a sheet of paper over a magnet and sprinkling fine iron filings over the paper. In the case of a bar magnet lying on its side, the iron filings will arrange themselves in curved lines extending from the north to the south pole, as shown in Fig. 11. A view of the magnetic field looking towards either pole of a bar magnet would exhibit merely radial lines, as shown by the filings in Fig. 12.

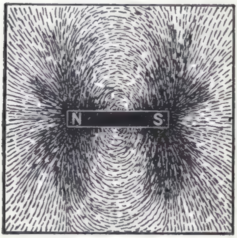


FIG. 11.

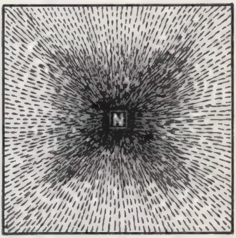


FIG. 12.

Every line of force is assumed to pass out from the north pole, make a complete circuit through the surrounding medium, and pass into the south pole, thence

through the magnet to the north pole again, as shown in Fig. 13. This is called the *direction of the lines of force*, and the path which they take is called the *magnetic circuit*.

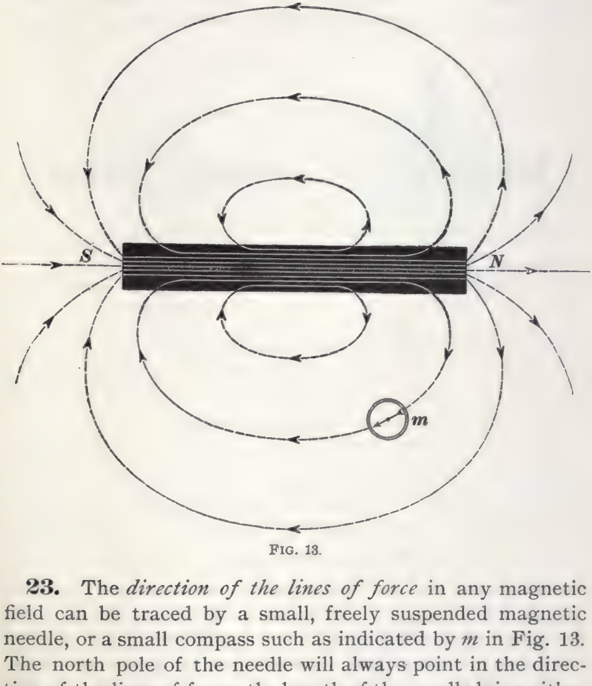


FIG. 13.

23. The *direction of the lines of force* in any magnetic field can be traced by a small, freely suspended magnetic needle, or a small compass such as indicated by *m* in Fig. 13. The north pole of the needle will always point in the direction of the lines of force, the length of the needle lying either parallel or tangent to the lines of force at that point. If the needle be moved bodily in the direction towards which the north pole points, its center or pivot will describe a path coinciding with the direction of the lines of force in that part of the magnetic field.

NOTE.—In all diagrams, the *direction of the lines of force* will be represented by arrowheads upon dotted lines.

Lines of force can never intersect one another; when two opposing magnetic fields are brought together, as indicated by the iron filings in Fig. 14 and Fig. 15, the lines of force from each will be crowded and distorted from their original direction until they coincide in direction with

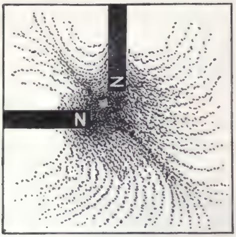


FIG. 14.

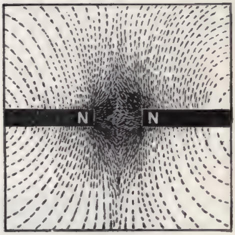


FIG. 15.

those opposing, and form a resultant field in which the direction of the lines of force will depend upon the relative strengths of the two opposing negative fields. The resulting poles thus formed are called **consequent poles.**

In every magnetic field there are certain stresses which produce a *tension* along the lines of force and a *pressure* across them; that is, they tend to *shorten* themselves from end to end and to *repel* one another as they lie side by side.

24. When a magnetic substance is brought into a magnetic field, the lines of force in that vicinity crowd together and all tend to pass through the substance. If the substance is free to move on an axis (but not bodily) towards the magnet pole, it will always come to rest with its greatest extent or length in the direction of the lines of force. The body will then become a magnet, its south pole being situated where the lines of force enter it and its north pole where they pass out. The production of magnetism in a magnetic substance in this manner is called **magnetic induction.** The production of artificial magnetism in a hardened steel needle or bar by contact with lodestone is one case of magnetic induction.

The **amount,** or **quantity, of magnetism** is expressed by the total number of lines of force contained in a magnetic circuit.

Magnetic density is the number of lines of force passing through a unit area measured perpendicularly to their direction.

ELECTROMAGNETISM.

25. If a conductor be placed parallel to the magnetic axis of a compass needle and a current be passed through the conductor in either direction, the needle will tend to place itself at right angles to the conductor, as shown by arrows

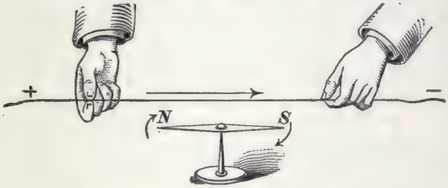


FIG. 16.

in Fig. 16; or, in general, an electric current and a magnet exert a mutual force upon each other. From the definition given in Art. **22,** the space surrounding the conductor is a *magnetic field.* If the conductor is threaded up through a piece of cardboard, and iron filings are sprinkled on the cardboard, they will arrange themselves in concentric circles around the conductor, as represented in Fig. 17. This effect will be observed throughout the entire length of the conductor and is caused entirely by the current. In fact, every conductor conveying a current of electricity can be imagined as completely surrounded by a sort of magnetic *whirl,* the

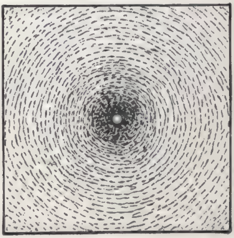


FIG. 17.

magnetic density decreasing as the distance from the current increases. (See Fig. 18.)

26. If the current in a horizontal conductor is flowing *towards* the *north*, and a compass is placed *under* the conductor, Fig. 19, the north pole of the needle will be deflected towards the *west;* by placing the compass *over* the wire, Fig. 20, the north pole of the needle will be deflected towards the *east.* By reversing the direction of the current in the conductor, the needle will point in the opposite direction in each case, respectively.

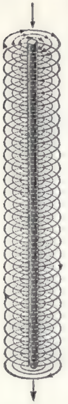


FIG. 18.

If the conductor is placed *over* the needle and then bent back *under* it, forming a loop as shown in Fig. 21, the tendency of the current in both top and bottom portions of the wire is to deflect the north pole of the needle in the same direction.

From these experiments, knowing the direction of current in the conductor, the following rule is deduced for the direction of the lines of force around the conductor:

Rule.—*If the current is flowing in the conductor away from the observer, then the direction of the lines of force*

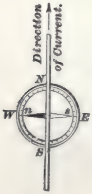


FIG. 19.

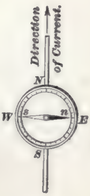


FIG. 20.

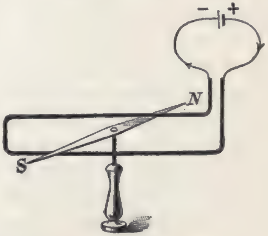


FIG. 21.

will be around the conductor in the direction of the hands of a watch.

The direction of the lines of force around a conductor is indicated in Fig. 22, where the current is assumed to be flowing downwards, that is, piercing the paper.

27. Two parallel conductors, both transmitting currents of electricity, are either mutually attractive or repellent, depending upon the relative direction of their currents. If the currents are flowing in the *same* direction in both conductors, as represented in Fig. 23, the lines of force will tend to surround both conductors and contract, thus *attracting* the conductors. If, however, the currents are flowing in opposite directions, as in Fig. 24,

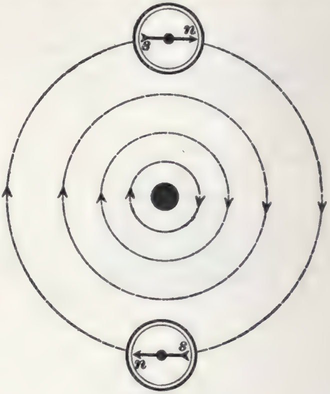


FIG. 22.


FIG. 23.


FIG. 24.

the lines of force lying between the conductors will have the same direction and therefore *repel* the conductors.

28. If the conductor carrying the current is bent into the form of a loop, as in Fig. 25, then all the lines of force

around the conductor will thread through the loop in the same direction. By bending the conductor into a long *helix* of several loops, the lines of force around each loop will coincide with those around the adjacent loops, forming several long lines of force which thread through the entire helix, entering at one end and passing out at the other. The

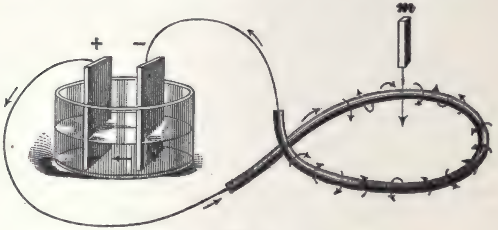


FIG. 25.

same conditions now exist in the helix as exist in a bar magnet, i. e., the lines of force *pass out* from one end and *enter* the other. In fact, the helix possesses a *north* and a *south pole*, a *neutral line*, and all the properties of attraction and repulsion of a magnet. If it is suspended in a horizontal position and free to turn, it will come to rest pointing in a north-and-south direction.

A helix made in this manner, around which a current of electricity is circulating, is called a **solenoid.**

29. The **polarity** of a solenoid, that is, the direction of the lines of force which thread through it, depends upon the direction in which the conductor is coiled and the direction of the current in the conductor.

To determine the polarity of a solenoid, knowing the direction of the current:

Rule.—*In looking at the end of the helix, if it is so wound that the current circulates around the helix in the direction of the hands of a watch, that end will be a south pole; if in the other direction, it will be a north pole.*

Fig. 26 represents a conductor coiled in a right-handed helix. If the current starts to flow from the end where the observer stands, that end will be a south pole and the observer will be looking through the helix *in the direction of the lines of force.*

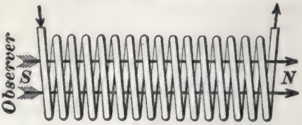


FIG. 26.

The polarity of a solenoid can be changed by reversing the direction of the current in the conductor.

30. It has been stated that when a magnetic substance is brought into a magnetic field, the lines of force in that field crowd together, and all try to pass through that substance; in fact, they will alter their circular shape and extend a considerable distance from their original position in order to pass through it. A magnetic substance, therefore, offers a better path for the lines of force than air or other non-magnetic substances.

The facility afforded by any substance to the passage through it of lines of force is called **magnetic permeability,** or, simply, **permeability.**

The *permeability* of all non-magnetic substances, such as air, copper, wood, etc. is taken as 1, or unity. The permeability of soft iron may be as high as 2,000 times that of air. If, therefore, a piece of soft iron be inserted into the magnetic circuit of a solenoid, the number of lines of force will be greatly increased, and the iron will become highly magnetized.

31. A magnet produced by inserting a magnetic substance into the magnetic circuit of a solenoid is an **electromagnet,** and the magnetic substance around which the current circulates is called the **core.** (See Fig. 27.) The solenoid is generally termed the **magnetizing coil.**

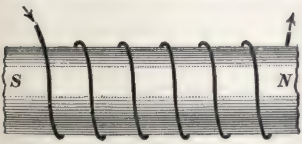


FIG. 27.

In the ordinary form of electromagnet, the magnetizing coil consists of a large number of turns of *insulated* wire, that is, wire covered with a layer or coating of some non-conducting or insulating material, usually silk or cotton; otherwise the current would take a shorter and easier circuit from one coil to the adjacent one or from the first to the last coil through the iron core without circulating around the magnet.

The simplest form of an electromagnet is the bar magnet. As usually constructed, it consists of a straight bar of iron or steel B, fitted into a *spool*, or *bobbin*, made of hard vulcanized rubber or some other inflexible insulating material. The magnetizing coil of fine insulated copper wire w is wound in layers in the bobbin, as shown by the cross-section in Fig. 28.

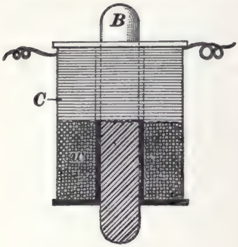


FIG. 28.

The rule for determining the polarity of a solenoid is the same for an electromagnet. It makes no difference whether the wire is wound in one layer or in any number of layers, or whether it is wound towards one end and then wound back again over the previous layer towards the other end; so long as the current circulates continually in the same direction around the core, the polarity of the magnet will remain unchanged.

32. The most convenient form of electromagnet for a great variety of uses is the *horseshoe*, or **U**-*shaped*, electromagnet, Fig. 30. It consists of a bar of iron bent into the shape of a horseshoe with straight ends and provided with two magnetizing coils, one on each end of the magnet. The two ends which are surrounded by the coils are the *cores* of the magnet, and the arc-shaped piece of iron joining them together is known as the **yoke** of the magnet. The ordinary **U**-shaped electromagnet is made in three parts, namely, two iron cores wound with the magnetizing coils, and a straight bar of iron joining the two cores together for a yoke,

as shown in Fig. 29. In looking at the free ends of the two cores, Fig. 30, the current should circulate around one core in an opposite direction to that around the other. If the current circulates around both cores in the same direction, the lines of force produced in the two cores, respectively, oppose one another, forming two like poles at their free ends and a *consequent pole* in the yoke. The total number of lines of force produced by both coils will be greatly diminished,

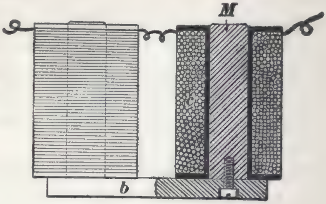


FIG. 29.

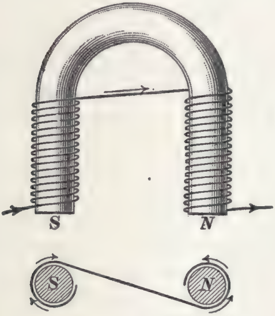


FIG. 30.

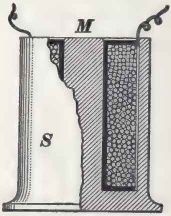


FIG. 31.

and the magnet will exhibit only a small amount of magnetic attraction.

Another common form of electromagnet is known as the **iron-clad** electromagnet. In its simplest form, Fig. 31, it contains only one magnetizing coil and one core. The core is fastened to a disk-shaped iron yoke, and the magnetic circuit is completed through an iron shell which rises up from the yoke and completely surrounds and protects the coil.

ELECTRICAL UNITS.

33. The three principal units used in practical measurements of a current of electricity are:

The **ampere,** *or the practical unit denoting the rate of flow of an electric current, or the strength of an electric current.*

The **ohm,** *or the practical unit of resistance.*

The **volt,** *or the practical unit of electromotive force, or pressure.*

Electromotive force, written E. M. F., or simply E., is the *total generated difference of potential* in any electric source or in any circuit. For example, the total difference of potential developed between the plates of a simple voltaic cell would be the *electromotive force* of that cell.

Ordinarily, the term *electromotive force* is used to express any difference of potential between two points.

The relation of these three practical units will be better understood by the analogy of the flow of water through a pipe. The force which causes the water to flow through the pipe is due to the *head*, or *pressure;* that which *resists* the flow is the friction of the water against the inside of the pipe, and the amount will vary with circumstances. The *rate of flow*, or the *current*, may be expressed in *gallons per minute*, and is a ratio between the head, or pressure, and the resistance caused by the friction of the water against the inside of the pipe. For, as the pressure, or head, *increases*, the rate of flow or current *increases* in proportion; as the resistance *increases*, the current *diminishes.*

In the case of electricity flowing through a conductor, the *electromotive force*, or *potential*, corresponds to the pressure, or head, of water, and the resistance which a conductor offers to the flow of electricity to the friction of the water against the pipe. The *strength of an electric current*, or the *rate of flow of electricity*, is also a ratio—the ratio of the electromotive force to the resistance of the conductor through which the current is flowing. This ratio as applied to electricity was first discovered by Dr. G. S. Ohm, and has since been called **Ohm's law.**

34. Ohm's Law.—*The strength of an electric current in any circuit is directly proportional to the electromotive force developed in that circuit and inversely proportional to the resistance of the circuit;* i. e., *it is equal to the electromotive force divided by the resistance.*

Ohm's law is usually expressed algebraically thus:

$$\text{Strength of current} = \frac{\text{electromotive force}}{\text{resistance}}.$$

If the electromotive force (E) is expressed in *volts* and the resistance (R) in *ohms*, the formula will give the strength of current (C) directly in *amperes;* thus, $C = \frac{E}{R}$.

Before giving examples of the application of Ohm's law, the value and significance of each unit will be treated separately.

35. The Ampere, or the Unit Strength of Current.—The strength of an electric current can be described as a *quantity* of electricity flowing continuously every second, or, in other words, it is the rate of flow of electricity, just as the current expressed in *gallons per minute* is the *rate of flow* of liquids. When one unit quantity of electricity is flowing continuously every second, then the rate of flow, or the strength of current, is *one ampere;* if two unit quantities are flowing continuously every second, then the strength of current is *two amperes*, and so on. It makes no difference in the number of amperes whether the current flows for a long period or for only a fraction of a second; if the quantity of electricity that would flow in one second is the same in both cases, then the strength of the current *in amperes* is the same.

The *international ampere* is defined as the strength of an unvarying current, which, when passed through a solution of nitrate of silver and water, deposits silver at the rate of .01725 grain per second.

Electricity possesses neither *weight* nor *extension*, and therefore an electric current cannot be measured by the usual methods adopted for measuring liquids and gases.

In liquids, the strength of the current is determined by measuring or weighing the actual quantity of the liquid which has passed between two points in a certain time and dividing the result by that time. The strength of an electric current, on the contrary, is determined indirectly by the effect it produces, and the actual quantity of electricity which has passed between two points in a certain time is afterwards calculated by multiplying the strength of the current by the time.

36. The principal effects produced by an electric current are given in Art. **3**; of these, the one most generally used for measuring is the action of the current upon a magnetic needle, as shown in Art. **25.** The instrument commonly used in laboratory practice for measuring and detecting small currents of electricity is called the **galvanometer.**

The action of the galvanometer is based upon the principle given in Art. **25,** where a magnetic needle, freely suspended in the center of a looped or coiled conductor, is deflected by a current of electricity passing around the coil, or loop. In ordinary practice, the needle is suspended either upon a pivot projecting into an agate cup fixed in the needle, or by a fiber suspension, as shown by F in Fig. 32. In the simpler forms of galvanometers, the magnetic needle itself swings over a dial graduated in degrees; in other forms, a light index needle is rigidly attached to the magnetic needle and swings over a similar dial, as indicated by I in Fig. 32; and in the more sensitive galvanometers, Fig. 33, a small reflecting mirror is attached to the fiber suspension and reflects a beam of light upon a horizontal scale situated several inches from the galvanometer.

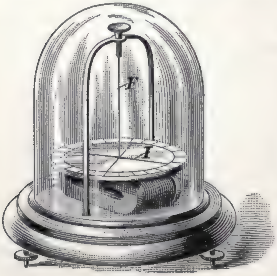


FIG. 32.

In any of these galvanometers, when no current is flowing in the coils, the needle should point in a direction parallel to the length of the coil, Fig. 34. The measuring of currents by most galvanometers depends upon the magnetic needle being held in this position by the magnetic attraction of the

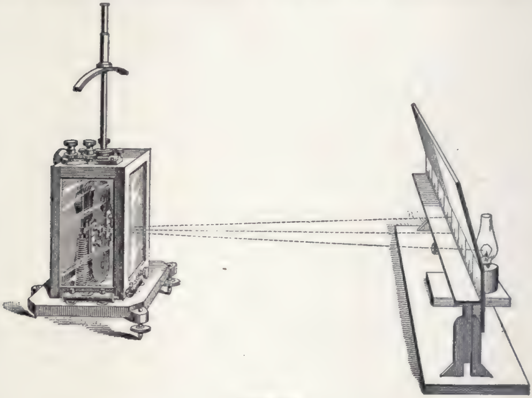


FIG. 33.

earth's magnetism or the attraction of some adjacent magnet. When a current of electricity passes around the coil, its tendency is to deflect the magnetic needle at right angles to its original position, as explained in Art. **25**, while the tendency of the earth's magnetism is to oppose the movement. The couple thereby produced will cause the needle to be deflected a certain number of degrees from its original position, depending upon the relative strengths of the two magnetic fields. The stronger the current in the coil, the greater the deflection. With a

FIG. 34.

galvanometer of standard dimensions and a magnetic field of known strength, such as the earth's magnetism at a convenient place on its surface, a strength of current can be conventionally adopted as a unit which will produce a certain deflection; all other galvanometers can be calibrated from this standard, and their dials graduated to read the strength of current directly in the conventional unit adopted.

37. Commercial and portable instruments are devised for measuring the strength of current directly in *amperes*, and are called **ampere meters,** or simply *ammeters*. The action of the current flowing through the coils in these instruments causes small magnetic needles or other coils of wire to act either against the tension of springs or against gravitational forces. The majority of ammeters are provided with an index needle which travels over a scale or dial graduated in divisions, each division representing one ampere or fractions or multiples of one ampere.

Fig. 35 shows the general form of a standard **Weston ammeter** used for commercial testing purposes. The strength of the current flowing in a circuit can be measured directly in amperes by opening the circuit at any convenient place and connecting the two ends thus formed to the binding post p and p'. The direction of the current in the circuit should be determined beforehand, so that it passes *into* the instrument by the binding post marked with the positive (+) sign; otherwise the index needle will be deflected off the scale in the wrong direction, which is liable to damage the instrument and cause error in reading when the current passes through in the proper direction.

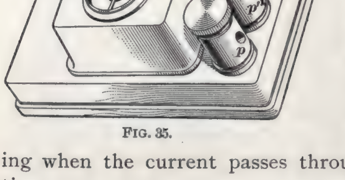


FIG. 35.

38. The Ohm, or the Unit of Resistance.—In Art. **8** it was stated that the resistance varied in different substances; that is, one substance offers a higher resistance to a current of electricity than another. Electrical resistance, therefore, can be defined as a property of matter, varying with different substances, and in virtue of which such matter opposes or resists the passage of electricity.

The resistance which all substances offer to the passage of an electric current is one of the most important quantities in electrical measurements. In the first place, it is that which determines the strength of an electric current in any circuit in which a difference of potential is constantly maintained, as shown by Ohm's law; and in the second place, the unit of resistance, the *ohm*, is the only unit in electrical measurements for which a material standard can be adopted, other quantities being measured by the effect they produce. The basis of any system of physical measurements is generally some material standard conventionally adopted as a unit, physical measurements in each system being made by comparison with the unit of that system.

The unit of electrical resistance now universally adopted is called the **international ohm.** One international ohm is the resistance offered by a column of pure mercury 106.3 centimeters in length and 1 square millimeter in sectional area at 32° F., or the temperature of melting ice. The dimensions of the column expressed in inches are as follows: length, 41.85 inches; sectional area, .00155 square inch. Hereafter the word "international" will be omitted and simply the word "ohm" used; the *international ohm*, however, as defined above, will always be implied unless otherwise stated.

39. If a given conductor offers a resistance of 2 ohms to a current of 1 ampere, it offers the same amount, no more nor less, to a current of 10 amperes. Hence, *the resistance of a given conductor at equal temperatures is always constant, irrespective of the strength of current flowing through it or the electromotive force of the current.*

40. If the length of a conductor be doubled, its resistance will be doubled; that is, the resistance of a given conductor increases as the length of the conductor increases, the resistance being directly proportional to the length of the conductor.

When it is required to find the resistance of a conductor of which the length is varied, and other conditions remain unchanged, the following formula may be used:

$$r_2 = \frac{r_1 l_2}{l_1}. \qquad \textbf{(1.)}$$

In this formula

r_1 = the original resistance;
r_2 = the required or changed resistance;
l_1 = the original length;
l_2 = the changed length.

As in all examples of proportion, the two lengths must be reduced to the same unit.

By this formula, we see that *the resistance of a conductor after its length is changed is equal to the original resistance multiplied by the changed length, and the product divided by the original length.*

EXAMPLE.—Find the resistance of 1 mile of copper wire, if the resistance of 10 feet of the same wire be .013 ohm.

SOLUTION.— $r_1 = .013$ ohm; $l_1 = 10$ feet; $l_2 = 1$ mile $= 5,280$ feet. Then, by formula **1**, the required resistance

$$r_2 = \frac{.013 \times 5,280}{10} = 6.864 \text{ ohms. Ans.}$$

41. If the sectional area of a conductor is doubled and other conditions remain unchanged, the resistance will be halved. We may, then, obtain the value of the resistance of a conductor for any change in sectional area by the following formula:

$$r_2 = \frac{r_1 a_1}{a_2}, \qquad \textbf{(2.)}$$

in which r_1 = the original resistance of the conductor;
r_2 = the changed resistance;
a_1 = the original sectional area;
a_2 = the changed sectional area.

From the relations here expressed, it will be seen that the resistance varies inversely as the sectional area; that is, *the resistance of a given conductor diminishes as its sectional area increases.*

The resistance of a conductor is independent of the *shape* of its cross-section. For example, this shape may be circular, square, rectangular, or irregular; if the sectional area be the same in all cases, the resistances will be the same, other conditions being similar.

EXAMPLE.—The resistance of a conductor whose sectional area is .025 sq. in. is .32 ohm; what would be the resistance of the conductor if its sectional area were increased to .125 sq. in. and other conditions remain unchanged?

SOLUTION.— $r_1 = .32$ ohm; $a_1 = .025$ sq. in.; and $a_2 = .125$ sq. in. Then, by formula **2**, the required resistance

$$r_2 = \frac{r_1 a_1}{a_2} = \frac{.32 \times .025}{.125} = .064 \text{ ohm.} \quad \text{Ans.}$$

EXAMPLE.—The sectional area of a certain conductor is .01 sq. in. and its resistance is 1 ohm; if its sectional area be decreased to .001 sq. in. and other conditions remain unchanged, what will be the resistance?

SOLUTION.— $r_1 = 1$ ohm; $a_1 = .01$ sq. in., and $a_2 = .001$ sq. in. By formula **2**, the resistance

$$r_2 = \frac{1 \times .01}{.001} = 10 \text{ ohms.} \quad \text{Ans.}$$

42. When comparing resistances of round copper wires the following formula is used:

$$r_2 = \frac{r_1 D^2}{d^2}, \qquad (3.)$$

in which r_1 = the original or known resistance;
r_2 = the required resistance;
D = the original diameter;
d = the changed diameter.

This formula is based on the rule that, since the sectional area of a round conductor is proportional to the square of

its diameter (sectional area = diameter^2 $\times$ .7854), *the resistance of a round conductor is inversely proportional to the square of its diameter.*

EXAMPLE.—The resistance of a round copper wire .2 in. in diameter is 45 ohms; from this calculate the resistance of a round copper wire .3 in. in diameter, other conditions remaining the same in both cases.

SOLUTION.—In this example, $r_1 = 45$ ohms; $D = .2$ inch; and $d = .3$ inch. Hence, by formula **3**, the required resistance

$$r_2 = \frac{45 \times .2^2}{.3^2} = \frac{45 \times .04}{.09} = 20 \text{ ohms. Ans.}$$

EXAMPLE.—If the resistance of a round German-silver wire $\frac{1}{8}$ in. in diameter is 12.6 ohms, what is the resistance of a round German-silver wire $\frac{1}{16}$ in. in diameter, other conditions being equal in the two cases?

SOLUTION.—In this example, $r_1 = 12.6$ ohms; $D = \frac{1}{8} = .125$ inch; and $d = \frac{1}{16} = .0625$ inch. Hence, by formula **3**,

$$r_2 = \frac{12.6 \times .125^2}{.0625^2} = 50.4 \text{ ohms. Ans.}$$

43. The resistance of two or more conductors connected in *series* (Art. **14**) is equal to the sum of their separate resistances. For example, if four conductors having separate resistances of 8, 12, 22, and 34 ohms, respectively, are connected in series, their total or joint resistance would be $8 + 12 + 22 + 34 = 76$ ohms.

44. The **microhm** is a unit of resistance devised to facilitate calculations and measurements of exceedingly small resistances, and is equal to *one millionth* $\left(\frac{1}{1,000,000}\right)$ of an *ohm*. Hence, to express the resistance in *microhms*, multiply the resistance in *ohms* by 1,000,000; and, conversely, to express the resistance in *ohms*, divide the resistance in *microhms* by 1,000,000. For example, .75 ohm $= .75 \times 1,000,000 = 750,000$ microhms; or, 750,000 microhms $= 750,000 \div 1,000,000 = .75$ ohm.

45. The **megohm** is a unit of resistance devised to facilitate calculations and measurements of exceedingly large resistances, and is equal to 1,000,000 ohms. Therefore, to express the resistance in *megohms*, divide the

resistance in *ohms* by 1,000,000; and, conversely, to express the resistance in *ohms*, multiply the resistance in *megohms* by 1,000,000. For example, 850,000 ohms $= \frac{850,000}{1,000,000} = .85$ megohm; or, .85 megohm $= .85 \times 1,000,000 = 850,000$ ohms. The megohm is used chiefly to measure the resistance of bad conductors and insulators.

46. In order to compare the resistances of different substances, the dimensions of the pieces to be measured must be equal; for, by changing its dimensions, a good conductor may be made to offer the same resistance as an inferior one. Under like conditions, annealed silver offers the least resistance of all known substances. Soft, annealed copper comes next on the list, and then follow all other metals and conductors.

The resistance of a given conductor, however, is not always constant; it changes with the temperature of the conductor. In all metals, the resistance *increases* as the temperature rises; in liquids and carbons, the resistance *decreases* as the temperature rises. The amount of variations in the resistance caused by a change in temperature of one degree is called the **temperature coefficient.** The temperature coefficients for the common metals are given in Table I for degrees Fahrenheit. These coefficients, however, only hold true for a limited change of temperature, and should not be used with extreme changes. The rules given below, making use of these coefficients, are not absolutely accurate, but enough so for practical purposes. To be strictly correct, the quantity r_1 in formula **4** below should, in each case, represent the resistance at the freezing point.

To find the resistance of a conductor after its temperature has risen, knowing its original resistance and the number of degrees rise, other conditions remaining unchanged:

Let r_1 = the original resistance;
r_2 = the resistance after a change in temperature;
k = the temperature coefficient;
t = rise or fall in temperature, degrees Fahrenheit.

Then, for a *rise* in temperature,

$$r_2 = r_1 \, (1 + t\,k). \qquad \textbf{(4.)}$$

That is, *the resistance of a conductor after its temperature has risen may be obtained by multiplying the original resistance by 1 plus the product of the number of degrees rise and the temperature coefficient.*

EXAMPLE.—The resistance of a piece of copper wire at 32° F. is 40 ohms; determine its resistance when its temperature is 52° F.

SOLUTION.— $r_1 = 40$ ohms;
$k = .002155$ (from Table I);
$t = 52 - 32 = 20$ degrees.

By formula **4,** the required resistance

$r_2 = r_1\,(1 + t\,k) = 40\,(1 + 20 \times .002155) = 40 \times 1.0431 = 41.724$ ohms. Ans.

47. To find the resistance of a conductor after its temperature has fallen, knowing its original resistance and the number of degrees fall, other conditions remaining unchanged:

For a *fall* in temperature, $r_2 = \dfrac{r_1}{1 + t\,k}$. **(5.)**

That is, *the resistance of a conductor after its temperature has fallen may be obtained by dividing the original resistance by one plus the product of the number of degrees fall and the temperature coefficient.*

EXAMPLE.—The original resistance of a piece of German-silver wire is 16 ohms; find its resistance after its temperature has fallen 22° F.

SOLUTION.— $r_1 = 16$ ohms;
$k = .000244$ (from Table I);
$t = 22°$ F.

By formula **5,** the required resistance

$$r_2 = \frac{r_1}{1 + t\,k} = \frac{16}{1 + 22 \times .000244} = \frac{16}{1.005368} = 15.9145 \text{ ohms. Ans.}$$

48. Specific resistance is the term given to the resistance of substances of unit length and unit sectional area at some standard temperature. In what follows, the specific resistance of a substance is the resistance of a piece of that

substance 1 inch in length and 1 square inch in sectional area at 32° F., that is, at the temperature of melting ice; this may also be expressed as the resistance of a cube of that substance taken between two opposing faces.

A list of the common metals is given in Table I, in the order of their relative resistances, beginning with silver, which offers the least resistance. The first column of figures gives the *specific resistance*, in microhms, of 1 cubic inch of the corresponding metal at 32° F. From formulas **1** and **2**, the resistance of any conductor of known dimensions which is made of one of the metals in the table can be determined. The second column of figures gives the relative resistance of the different metals compared with silver. For example,

TABLE I.

Name of Metal.	Resistance, Microhms per Cu. In.	Relative Resistance.	Temperature Coefficient.
Silver, annealed	.5921	1.000	.002094
Copper, annealed	.6292	1.063	.002155
Silver, hard-drawn	.6433	1.086	.002094
Copper, hard-drawn	.6433	1.086	.002155
Gold, annealed	.8102	1.369	.002028
Gold, hard-drawn	.8247	1.393	.002028
Aluminum, annealed	1.1470	1.935	
Zinc, pressed	2.2150	3.741	.002028
Platinum, annealed	3.5650	6.022	
Iron, annealed	3.8250	6.460	
Nickel, annealed	4.9070	8.285	
Tin, pressed	5.2020	8.784	.002028
Lead, pressed	7.7280	13.050	.002150
German Silver	8.2400	13.920	.000244
Antimony, pressed	13.9800	23.600	.002161
Mercury	37.1500	62.730	.000400
Bismuth, pressed	51.6500	87.230	.001967

the resistance of mercury is 62.73 times the resistance of silver, or the resistance of iron is 6.46 times the resistance of silver, and so on.

EXAMPLE.—Find the resistance in ohms of a round column of mercury 70″ high and .05″ in diameter. Ans. 1.3244 ohms.

EXAMPLE.—Find the resistance in ohms of 1 mile of square iron wire (annealed) .1″ on a side. Ans. 24.2352 ohms.

49. In a simple voltaic cell, the *internal* resistance—that is, the resistance of the two plates and the electrolyte—is of great importance, for it determines the maximum strength of current that can possibly be obtained from the cell. In the common forms of cells, the internal resistance may be excessively large, owing to the resistance of the electrolyte, the specific resistance of ordinary liquids used as electrolytes being from 1 to 20 million times that of the common metals. In liquids, as in all conductors, the resistance increases as the length of the circuit increases, and diminishes as its sectional area increases. Hence, the internal resistance of a simple voltaic cell is reduced by decreasing the distance between the plates or elements and by increasing their active surfaces. The internal resistance of the ordinary forms of cells varies from about .2 to 20 ohms.

50. For practical and commercial testing, the standard column of mercury, representing the resistance of 1 ohm, has been replaced by a coil of wire, usually a platinum-silver alloy. The coil is carefully calibrated to offer a resistance of exactly 1 ohm at some convenient temperature, and is enclosed in a metallic case, the connections to the two ends of the coils being made by two heavy terminals of copper wire passing up through the hard-rubber cover. Such coils are known as *standard*

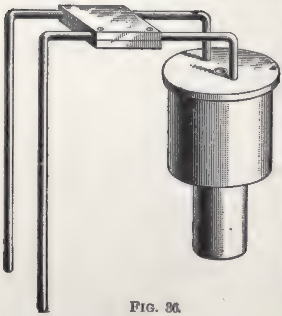


FIG. 36.

ohm coils. The commercial form of standard ohm coils is shown in Fig. 36.

51. A device called a **resistance box** or **rheostat** is largely used for reducing or controlling the strength of currents in various circuits. Such rheostats are connected directly in *series* or *shunt* with the circuit, and are termed *dead resistances.* The resistance in these rheostats is usually made adjustable; that is, the amount of resistance which they offer may be varied at the will of the operator by the use of a sliding contact or by removable plugs. Rheostats in which the amount of resistance is varied by sliding contacts are used mostly where accuracy is of less importance and where the currents are comparatively large.

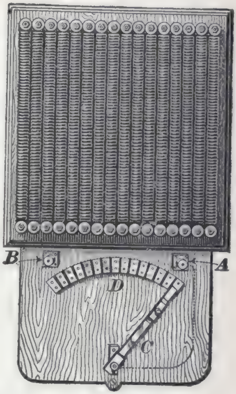


FIG. 37.

Fig. 37 shows a typical form of sliding-contact rheostat. In this particular rheostat, the coils of resistance wire are connected to a row of contact pieces D, as shown in the diagram, Fig. 38. The current enters the rheostat through the terminal A, passes through the movable arm C, and then through all the resistance coils between the contact piece on which the arm rests and the terminal B. When the arm rests upon the first contact piece, as shown by the full lines in this diagram, all the resistance is said to be *in circuit;* that is, the current passes through all the

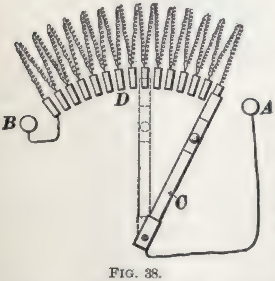


FIG. 38.

coils. By moving the arm to the left, towards the terminal B, as shown by the dotted lines, the coils connected to the contact pieces which have been passed over by the arm are said to be *cut out* of circuit, and the current passes through the remaining coils only.

52. Rheostats in which the resistance is adjusted by means of removable plugs are employed in laboratory practice, where small currents are used and where great accuracy is required. The resistance coils in these rheostats are enclosed in a wooden box, and the actual resistance of each coil is carefully determined. A resistance box offering 10,000 ohms resistance is shown in Fig. 39, the separate coils offering resistances from 1 ohm up to 5,000 ohms. The

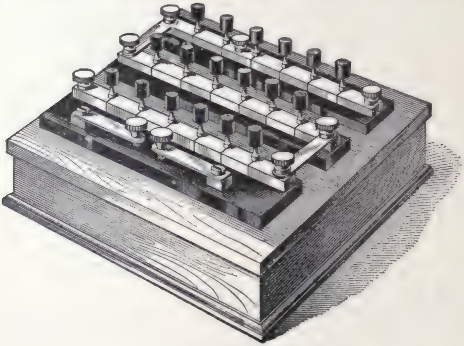


FIG. 39.

operation of adjusting the resistance by means of the removable plugs can be seen from the diagram in Fig. 40. The contact pieces a, b, c, etc. are arranged side by side on the top of the case and are separated from one another by a small air space. The ends of each contact piece are provided with a tapered recess in such a manner as to allow a metallic plug to be inserted between them and thereby connect the two together electrically. The current passes into the rheostat by the terminal A, and when all the plugs are

removed flows consecutively through all the coils *1*, *2*, *3*, *4*, *5*, and *6* to the terminal *B*. The total resistance of the rheostat can be lowered by inserting the plug *P* between the contact pieces; this operation *short-circuits*, or *cuts out*, the

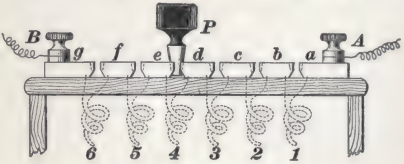


FIG. 40.

particular coil connected to the two contact pieces, or, in other words, the current, instead of flowing through the coils, passes directly from one contact piece to the other through the metallic plug.

53. Electrical resistance may be measured by an apparatus called a **Wheatstone bridge.** A bridge, when completed, ready for taking measurements, consists of three main parts: (1) an adjustable resistance box containing a number of coils, the exact resistance of each coil being known; (2) a galvanometer for detecting small currents; and (3) a battery of several cells. The coils of the resistance box are divided into three groups, two of which are called **proportional** or **balance arms,** and the third is known as the **adjustable arm.** Each proportional arm is composed of three and sometimes four coils of 1, 10, 100, and 1,000 ohms resistance, respectively. The adjustable arm contains a large number of coils ranging from .1 ohm up to 10,000 ohms.

The operation of the bridge depends upon the principle of the relative difference of potential between two points in a divided circuit of two branches. The electrical connections of the bridge are shown in the diagram, Fig. 41. *M* represents the resistance of one of the balance arms, which will be termed for convenience the *upper* balance arm; *N* represents the resistance of the other balance arm, which will be termed the *lower* balance arm; *P* represents the resistance of the

adjustable arm, and X represents an unknown resistance, the value of which is to be determined. One terminal of the detecting galvanometer G is connected at c, the junction of the upper balance arm and the unknown resistance; the other terminal is connected at d, the junction of the lower balance arm and the adjustable arm. One pole of the battery is connected at a, the junction of the two balance arms; the other pole at b, the junction of the adjustable resistance and the unknown resistance. The current from the battery divides at a, part of it flowing through resistances M and X, and the rest through N and P. When the resistances M, N, P, and X fulfil the proportion $\frac{M}{N} = \frac{X}{P}$, then the two points c and d will have the same potential, and no current will flow

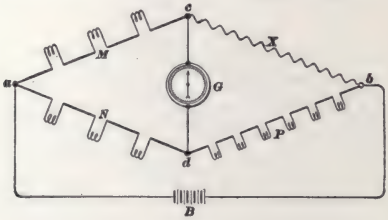


FIG. 41.

through the galvanometer G. Since the resistances of M, N, and P are known, the resistance of X will be given by the fundamental equation $X = \frac{M}{N} \times P$, when the arms are so adjusted as to cause no deflection of the galvanometer. For example, suppose that the two ends of a copper wire are connected to the terminals b and c, and after adjusting the resistance in the arm so that the galvanometer shows no deflection, the resistances of the different arms read as follows: $M = 1$ ohm, $N = 100$ ohms, and $P = 112$ ohms. Then, substituting these values in the fundamental equation gives

$$X = \frac{M}{N} \times P = \frac{1}{100} \times 112 = 1.12 \text{ ohms.}$$

54. The actual various forms of resistance boxes used with the bridges differ widely from the diagram, but all are based upon this same principle and fundamental equation. A common pattern of resistance box for this purpose is constructed similar to the adjustable rheostat, as previously described, where the adjustments are made with removable plugs. Ordinarily, the contact pieces are arranged in the shape of a letter S, and the galvanometer and battery circuits are connected as shown in Fig. 42. The position of the two balance arms and the adjustable arm can be readily seen by comparing the connections of the battery and galvanometer circuits with those in the original diagram. K and K' represent keys for opening the circuits when the

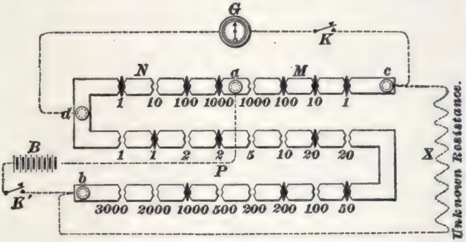


FIG. 42.

plugs are withdrawn or inserted in varying the resistance or when the bridge is not in use. In this particular case, the 1,000-ohm plug in the upper balance arm is supposed to be drawn, and therefore $M = 1,000$ ohms. In the lower balance arm the 10-ohm plug is supposed to be drawn, and therefore $N = 10$ ohms. In the adjustable arm the following plugs are supposed to be drawn: 1, 2, 5, 10, 20, 100, 200, 500, 2,000, and 3,000 ohms; therefore, the resistance P is the sum of these resistances, or 5,838 ohms. If, under these conditions, there is no deflection of the galvanometer when the two keys K and K' are pressed and both circuits are closed, the resistance of X will be 583,800 ohms; for, substituting the values of M, N, and P in the fundamental equation gives $X = \frac{M}{N} \times P = \frac{1,000}{10} \times 5,838 = 583,800$ ohms.

Fig. 43 shows a special pattern of resistance box for a Wheatstone bridge, in which the coils of the adjustable arm

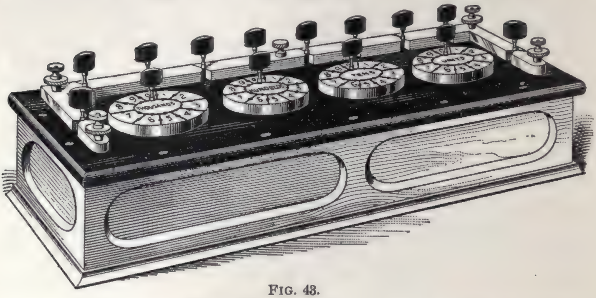


FIG. 43.

are arranged in the form of four dials. This pattern is known as the *dial* pattern, and is widely used in making resistance measurements.

EXAMPLE.—The diagram in Fig. 44 represents a particular type of Wheatstone's bridge to which a battery and galvanometer are properly connected for measuring unknown resistances. An unknown resistance x is connected to the terminals A and H; when the plugs a, e, f,

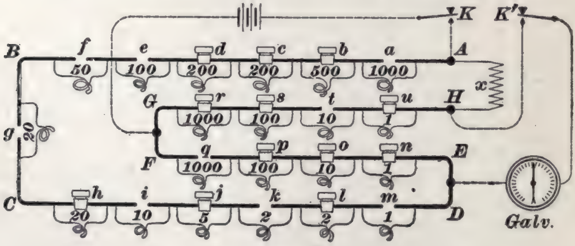


FIG. 44.

g, i, k, m, q, and t are drawn, and when both the contact keys K and K' are pressed, the galvanometer shows no deflection. Determine the resistance of x.

SOLUTION.—From the connections of the galvanometer and battery circuits, it will be seen that the resistance coils in line $G H$ represent the upper balance arm M of the bridge; that the coils in the line $E F$

represent the lower balance arm N; and that the coils in the lines $A B$ and $C D$ represent the adjustable arm P. From the fundamental equation of the Wheatstone bridge, X (the unknown resistance) = $\frac{M}{N} \times P$. In this particular case, the plug t in the upper arm is drawn; hence, $M = 10$ ohms; in the lower arm q is drawn; hence, $N = 1{,}000$ ohms; and in the adjustable arm, the plugs a, e, f, g, i, k, and m are drawn; hence, $P = 1{,}000 + 100 + 50 + 20 + 10 + 2 + 1 = 1{,}183$ ohms. Substituting these values in the fundamental equation gives $X = \frac{M}{N} \times P = \frac{10}{1{,}000} \times 1{,}183 = 11.83$ ohms. Ans.

55. The Volt, or the Practical Unit of Electromotive Force.—In mechanics, pressures of all kinds are measured by the *effects* they produce; similarly, in electrotechnics, *potential* is measured by the effect it produces.

It has been shown that electrical potential will cause an electric current to flow against the resistance of a conductor, and also how the units of resistance and current are obtained. It follows that a *unit potential* would be that electromotive force which would maintain a current of unit strength in a circuit whose resistance is unity. By definition, therefore, the *volt*, or *the practical unit of potential*, is that electromotive force which will maintain a current of *one ampere* in a circuit whose resistance is *one ohm*. With a known resistance in ohms and a known strength of current in amperes, the electromotive force in volts is determined by Ohm's law, Art. **34**, for, by transposing, $E = C R$.

This method of determining the potential of a circuit can be readily shown by the following illustration: Suppose, for example, it is desired to determine the electromotive force in volts required to drive a current of 2 amperes through a certain copper wire. In the first place, the resistance of the copper wire is found by Wheatstone's bridge as previously described. For convenience, it is assumed that its resistance is found to be 1.2 ohms. Then the electromotive force E required to drive 2 amperes through the wire will be 2.4 volts, for, by substituting, $E = C R = 2 \times 1.2 = 2.4$ volts.

The maximum difference of potential developed by any single voltaic couple placed in any electrolyte is about 2.25 volts; in the common forms of cells, the difference of potential developed averages from .75 to 1.75 volts.

56. When several cells are connected in *series*, the total electromotive force developed will be equal to the sum of the electromotive forces developed by the separate cells; or, if the cells are composed of the same voltaic elements, the total electromotive force developed will be equal to the electromotive force of one cell multiplied by the number of cells in series. For example, a battery is composed of 12 cells connected in series, and the electromotive force in each cell is 1.5 volts; the total electromotive force of the battery is, therefore, $1.5 \times 12 = 18$ volts.

Connecting cells in *parallel*, or *multiple-arc*, does not increase the electromotive force of a battery; the electromotive force will always be equal to the electromotive force of one cell, no matter how many cells are connected to the main conductors, provided, of course, that all cells develop equal electromotive forces.

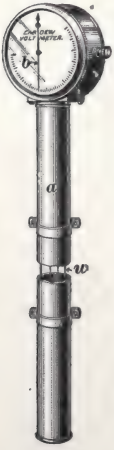


FIG. 45.

57. Measuring instruments called **voltmeters** have been devised for indicating electromotive forces and differences of potential directly in volts.

The **Cardew** voltmeter, Fig. 45, depends for its operation upon the linear expansion of a metallic wire when heated by an electric current. The expansion wire w is enclosed in a long cylindrical case a, and is attached in such a way that its expansion causes a small grooved wheel on the axis of the index needle to revolve in one direction when the wire expands, or lengthens, and in the opposite direction when the wire contracts, or shortens. The movements of this wheel cause the index b to move over the scale. Since the resistance

is nearly constant, the current that will flow is proportional to the E. M. F.; the greater the E. M. F. the more the wire will be expanded, and the greater will be the consequent deflection. The resistance of the wire, however, is so large as to permit only a weak current to pass through it when the needle is deflected over the entire scale. A Cardew voltmeter which indicates up to 100 volts has a resistance of about 500 ohms. The circular scale is divided into small divisions, each representing one volt or fractions or multiples of one volt.

58. The **Weston voltmeter,** Fig. 46, is based upon the same principles as the Weston ammeter, and in appearance is quite similar to it. Its internal resistance, as in all voltmeters, is exceedingly large; the resistance of a Weston voltmeter for indicating up to 150 volts is about 19,000 ohms, while the resistance of a Weston ammeter, measuring strengths of currents up to 15 amperes, is only .0022 ohm. It will be seen that, owing to the great resistance, the current passing through a voltmeter is exceedingly small. For example, in the instrument described above, when indicating 150 volts, the current, by Ohm's law, is only $150 \div 19{,}000 = .0079$ ampere. All voltmeters are provided with at least two terminals, or binding posts, such as p and p', Fig. 46. Connections are made by two separate conductors, called *voltmeter leads*, from these binding posts to two points between which the difference of potential, or the electromotive force, is to be measured.

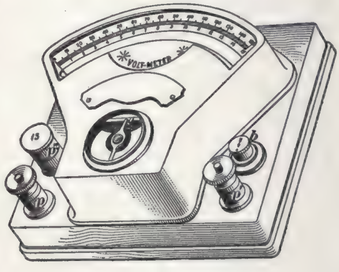


FIG. 46.

The Weston voltmeters usually have a third binding post p'', which when used with p' corresponds with a second

graduated scale situated directly under the main scale, one division of the upper scale having the value of two lower divisions. The majority of voltmeters are also provided with a contact button *b*, which when pressed closes the circuit and allows the index needle to be deflected by the current. When the pressure upon the button is relaxed, the circuit is opened, and the index needle returns to the zero mark.

59. The methods of connecting voltmeters and ammeters for measuring electromotive forces and currents of various circuits should be thoroughly understood. Suppose, for example, that the terminals of a battery composed of four cells connected in series are connected to an unknown resistance, and it is desired to know the strength of current flowing through the circuit, and also the difference of potential required to drive that current through the unknown resistance when the only instruments available are an ammeter and a voltmeter. In Fig. 47 let B represent the battery and R the unknown resistance; C, C', and C'' are three large conductors for making necessary connections. With the connections as shown, there is practically a continuous current flowing through the closed circuit, that is, from the battery through the conductors and the unknown resistance. The first step is to determine the strength of this current by the use of an ammeter. Assuming that the battery is constant, that is, that the electromotive force developed in it does not vary, then, so long as the resistance of the circuit is not altered, the strength of the current will remain unchanged and *will be the same in all parts of the circuit.* Hence, if an ammeter be inserted in any part of the

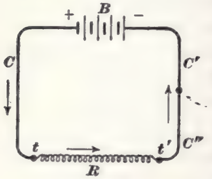


FIG. 47.

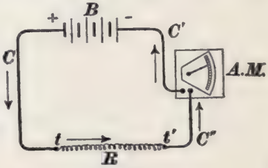


FIG. 48.

circuit, as between C' and C'', Fig. 48, it will measure the total strength of current flowing through the entire circuit. As has been stated, the internal resistance of the ammeter is so small that its insertion makes no appreciable change in the total resistance of the circuit, and therefore does not to any extent affect the current flowing. For convenience, assume that the strength of the current flowing in the circuit is found to be 1.2 amperes. The next operation is to find the electromotive force required to drive a current of 1.2 amperes through the resistance R; or, in other words, to find the difference of potential between the terminals t and t' when a current of 1.2 amperes is flowing in the circuit. This is accomplished by connecting the two terminals t and t', Fig. 49, of the unknown resistance R, to the two binding posts p and p' of the voltmeter $V\,M$ by two voltmeter leads l and l'. Any small wires of reasonable length can be used for voltmeter leads, as the current they transmit is exceedingly weak, owing to the extremely high resistance of the voltmeter. After pressing the contact button, assume the needle indicates a potential of 6 volts; this, then, is the electromotive force required to force a current of 1.2 amperes through the unknown resistance R; or, in other words, the difference of potential between the terminals t and t' is 6 volts. From these readings of the current and voltage, and by the application of Ohm's law, the resistance R of the circuit between t and t' can be determined. By algebra, Ohm's law can be transposed from the equation $C = \frac{E}{R}$ to $R = \frac{E}{C}$ and be equally true; this signifies that the resistance R of any conductor, or circuit, is equal to the electromotive force, or the difference of potential E in volts, divided by the strength of current C in amperes flowing through that

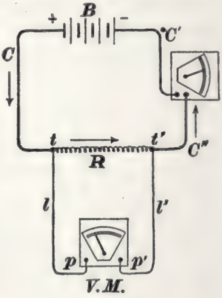


FIG. 49

circuit or conductor. In the previous case, it has been found that it requires an electromotive force of 6 volts to drive a current of 1.2 amperes through the resistance R; hence, from Ohm's law, $R = \frac{E}{C} = \frac{6}{1.2} = 5$ ohms.

APPLICATIONS OF OHM'S LAW.

TO CLOSED CIRCUITS.

60. The following facts are to be carefully noted regarding the application of Ohm's law to closed circuits:

The strength of current (C) *is the same in all parts of a closed circuit, except in the cases of derived circuits, where the sum of the currents in the separate branches is always equal to the current in the main or undivided circuit.*

The resistance (R) *is the resistance of the internal circuit plus the resistance of the external circuit.*

The electromotive force (E) *in a closed circuit is the total generated differenee of potential in that circuit.*

61. The following formula may be used to determine the strength of current in amperes flowing in a closed circuit when the electromotive force and the total resistance are known:

$$C = \frac{E}{R}, \qquad (6.)$$

where C = current in amperes;
E = electromotive force in volts;
R = resistance in ohms.

That is to say, *the strength of current in amperes is found by dividing the electromotive force in volts by the total resistance in ohms.*

EXAMPLE.—The two electrodes of a simple voltaic cell are connected by a conductor whose resistance is 1.6 ohms. If the internal resistance of the cell is 5 ohms and the total electromotive force developed is 1.75 volts, what is the strength of current flowing in the circuit ?

SOLUTION.—Let r_i = the internal resistance and r_e = the resistance of the copper wire. Then, $R = r_i + r_e = 1.6 + 5 = 6.6$ ohms, the total resistance of the circuit. Then, by formula **6**, the current

$$C = \frac{E}{R} = \frac{1.75}{6.6} = .265 \text{ ampere. Ans.}$$

62. The following formula may be used to find the total resistance in ohms of a closed circuit when the electromotive force and the strength of current are known:

$$R = \frac{E}{C}, \qquad (7.)$$

the letters having the same significance as in formula **6.** By formula **7** it will be seen that *the resistance in ohms of a closed circuit is found by dividing the electromotive force in volts by the current in amperes.*

EXAMPLE.—The total electromotive force developed in a closed circuit is 1.8 volts and the strength of the current flowing is .6 ampere; find the resistance in ohms.

SOLUTION.—By formula **7**, the resistance

$$R = \frac{E}{C} = \frac{1.8}{.6} = 3 \text{ ohms. Ans.}$$

63. The following formula may be used to find the total electromotive force in volts developed in a closed circuit when the strength of current and the total resistance are known:

$$E = C R. \qquad (8.)$$

The letters have the same meaning as in formulas **6** and **7.** We find here that *the electromotive force in volts developed in a closed circuit is obtained by multiplying together the current in amperes and the resistance in ohms.*

EXAMPLE.—The internal resistance of a closed circuit is 2 ohms and the external resistance is 3 ohms; if the current flowing is .4 ampere, what is the electromotive force developed?

SOLUTION.—Let r_i = the internal resistance and r_e = the external resistance. Then, $R = r_i + r_e = 2 + 3 = 5$ ohms. By formula **8**, the electromotive force

$$E = C R = .4 \times 5 = 2.0 \text{ volts. Ans.}$$

TO DROP, OR LOSS, OF POTENTIAL.

64. Referring again to water flowing in a pipe, it is evident that although the *quantity* of water which passes is the same at any cross-section of the pipe, the *pressure per square inch* is not the same. Even in the case of a horizontal pipe of the same diameter throughout, the water when flowing suffers a *loss* of head, or pressure. It is this difference of pressure that causes the water to flow between two points against the friction of the pipe.

This is precisely similar to a current of electricity flowing through a conductor. Though the *quantity of electricity* that flows is equal at all cross-sections, the electromotive force is by no means the same at all points along the conductor. It suffers a loss, or drop, of electrical potential in the direction in which the current is flowing, and it is this difference of electrical potential that causes the electricity to flow against the resistance of the conductor. *Ohm's law* not only gives the strength of the current in a closed circuit, but also the *difference of potential* in volts along that circuit. The difference of potential (E') in volts between any two points along a circuit is equal to the product of the strength of the current (C) in amperes and the resistance (R') in ohms of that part of the circuit between those two points, or $E' = C R'$, which is an example of the use of formula **8**. E' also represents the *loss*, or *drop*, of potential in volts between the two points. If any two of these quantities are known, the third can be readily found; for, by transposing, $C = \frac{E'}{R'}$ and $R' = \frac{E'}{C}$, as already given in formulas **6** and **7**.

EXAMPLE.—Fig. 50 represents part of a circuit in which a current of 3 amperes is flowing. The resistance from a to b is 1.5 ohms, from b to c is 2.3 ohms, and from c to d is 3.6 ohms. Find the difference of potential between a and b, b and c, c and d, and a and d.

FIG. 50.

SOLUTION.—Since, by formula **8**, $E' = C R'$, then,

the difference of potential between a and $b = 3 \times 1.5 = 4.5$ volts.
" " " " " b and $c = 3 \times 2.3 = 6.9$ "
" " " " " c and $d = 3 \times 3.6 = 10.8$ "
" " " " " a and $d = 4.5 + 6.9 + 10.8 =$
22.2 volts; or, in other words, the *loss*, or *drop*, of potential caused by a current of 3 amperes flowing between a and d is 22.2 volts.

65. In a great many cases it is desirable to have the current flow from the source a long distance to some electrical receptive device and return without causing an excessive drop, or loss, of potential in the conductors leading to and from the two places. In such circuits, the greater part of the total generated electromotive force is expended in the receptive device itself, and only a small fraction of it is lost in the rest of the circuit. Under these conditions, it is customary to decide upon a certain *drop, or loss, of potential* beforehand, and from that and the current calculate the resistance of the two conductors.

EXAMPLE.—It is desired to transmit a current of 5 amperes to an electrical device situated 500 feet from the source; the total generated E. M. F. is 120 volts, and only $\frac{1}{10}$ of this potential is to be lost in the conductors leading to and from the receptive device. (a) Find the resistance of the two conductors, and (b) find the resistance per foot of the conductors, assuming each to be 500 feet long.

SOLUTION.—(a) $\frac{1}{10}$ of 120 volts = 12 volts, which represents the *drop*, or *loss*, of potential on the two conductors. Let $E' = 12$ volts; $C = 5$ amperes; and $R' =$ the total resistance of the two conductors. Then, by formula **7**,

$$R' = \frac{E'}{C} = \frac{12}{5} = 2.4 \text{ ohms. Ans.}$$

(b) The resistance per foot of the conductor is found by formula **1.** In this case, $r_1 = 2.4$ ohms; $l_1 = 1{,}000$ feet; $l_2 = 1$ foot. Then the resistance per foot

$$r_2 = \frac{2.4 \times 1}{1{,}000} = .0024 \text{ ohm. Ans.}$$

TO VOLTAIC CELLS.

66. The difference of potential between the two electrodes of a simple voltaic cell when no current is flowing—that is, when the circuit is open—is always equal to the total

electromotive force developed within the cell; but when a current is flowing—that is, when the circuit is *closed*—a certain amount of potential is expended in forcing the current through the internal resistance of the cell itself. Hence, the difference of potential between the two electrodes when the circuit is closed is always smaller than when the circuit is open. This difference of potential between the two electrodes when the circuit is closed is sometimes called the *available* or *external* eloctromotive force, to distinguish it from the *internal* or *total generated* electromotive force.

67. To find the available electromotive force of a cell,

let E = the total generated E. M. F.;
E' = *available* E. M. F. when the circuit is closed;
C = the current flowing when the circuit is closed;
r_i = the internal resistance of the cell.

Then, the drop, or loss, of potential in the cell $= C r_i$, and the available electromotive force

$$E' = E - C r_i. \qquad \textbf{(9.)}$$

The available electromotive force of a cell is equal to the difference between the total generated electromotive force and the potential expended in forcing the current through the internal resistance of the cell when the circuit is closed. From Ohm's law, this loss, or drop, of potential in the cell itself is equal to the product of the internal resistance in ohms and the strength of the current in amperes flowing through the circuit.

EXAMPLE.—In a voltaic cell, the total generated E. M. F. is 2.2 volts and the internal resistance is .8 ohm. If a current of 1.2 amperes flows through the cell when the circuit is closed, what is the available E. M. F., or, in other words, the difference of potential between the two electrodes?

SOLUTION.—Let E' = the available E. M. F.; E = the total generated electromotive force; C = the current in amperes; and r_i = the internal resistance.

Then, by formula **9**,

$$E' = E - C r_i = 2.2 - (1.2 \times .8) = 1.24 \text{ volts. Ans.}$$

TO DERIVED CIRCUITS.

68. In treating upon derived circuits, only that part of the circuit will be considered which is divided into branches and each branch transmitting part of the total current; the rest of the circuit is assumed to be closed through some electric source, as, for instance, a voltaic battery.

Before applying Ohm's law to derived circuits, the word *conductivity* should be thoroughly understood. Conductivity can be defined as the facility with which a body transmits electricity, and is the opposite of resistance. For example, copper is of low resistance and high conductivity; mercury is of high resistance and low conductivity. In other words, conductivity is the inverse or reciprocal of resistance. There is no established unit of conductivity; it is used merely as a convenience in calculations. For example, if the resistance of a circuit is 2 ohms, its conductivity is represented by one-half; if the resistance is increased to 4 ohms, the conductivity would only be one-half as much as in the former case and would be represented by one-quarter.

The **conductivity** of any conductor is, therefore, unity divided by the *resistance* of that conductor; and, conversely, the resistance of any conductor is unity divided by its conductivity.

69. Fig. 51 represents a derived circuit of two branches. Let r_1 and r_2 = the separate resistances of the two branches; c_1 and c_2 = the separate currents in each branch, respectively; and C = the sum of the currents in the two branches; that is, the current in the main or undivided branch. Then, $c_1 + c_2 = C$, and $C - c_2 = c_1$.

When the current flows from a to b, if the resistances r_1 and r_2 are equal, the current will divide equally between the two branches; thus, if a current of 2 amperes is flowing in the main circuit, 1 ampere will flow through each branch.

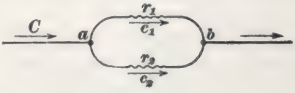


FIG. 51.

When the resistances of the two branches are unequal, the current will divide between them in inverse proportion to their respective resistances. In Fig. 51 the resistances of the two branches are r_1 and r_2. Therefore, $c_1 : c_2 :: r_2 : r_1$.

By algebra, this proportion gives the two following formulas:

For the first branch, $c_1 = \frac{C r_2}{r_1 + r_2}$. **(10.)**

That is, *of two branches in parallel, dividing from a main circuit, the current in the first branch is equal to the current in the main multiplied by the resistance of the second branch, and the product divided by the sum of the resistances of the two branches.*

For the second branch, $c_2 = \frac{C r_1}{r_1 + r_2}$. **(11.)**

Of two branches in parallel, dividing from a main circuit, the current in the second branch is equal to the current in the main multiplied by the resistance of the first branch, and the product divided by the sum of the resistances of the two branches.

EXAMPLE.—Suppose the resistance r_1 of the first branch is 2 ohms, and the resistance r_2 of the second branch is 3 ohms, find the separate currents c_1 and c_2 in the two branches, respectively, when the current C in the main or undivided branch is 60 amperes.

SOLUTION.— $r_1 = 2$ ohms, $r_2 = 3$ ohms, and $C = 60$ amperes. To find the current c_1 in the first branch, substitute these values in formula **10,** which will give

$$c_1 = \frac{C r_2}{r_1 + r_2} = \frac{60 \times 3}{2 + 3} = \frac{180}{5} = 36 \text{ amperes. Ans.}$$

To find the current c_2 in the second branch, substitute these values in formula **11,** which will give

$$c_2 = \frac{C r_1}{r_1 + r_2} = \frac{60 \times 2}{2 + 3} = \frac{120}{5} = 24 \text{ amperes. Ans,}$$

70. It is clear that two conductors in parallel will conduct an electric current more readily than one alone; that is, their *joint conductivity* is greater than either of their separate conductivities taken alone. This being the case,

their resistances must follow the inverse law—viz., the joint resistance of two conductors in parallel must be *less* than either of their separate resistances taken alone.

Rule.—*If the separate resistances of two conductors are equal, their joint resistance when connected in parallel is one-half of the resistance of either conductor.*

For example, take two conductors, the separate resistance of each being 2 ohms, and connect them in parallel; their joint resistance will then be one-half their separate resistance, or 1 ohm.

71. When the separate resistances of two conductors in parallel are unequal, the determination of their joint resistance when connected in parallel involves some calculation.

In Fig. 51, the conductivities of the branches are $\frac{1}{r_1}$ and $\frac{1}{r_2}$. Hence, their joint conductivity when connected in parallel is $\frac{1}{r_1} + \frac{1}{r_2} = \frac{r_2 + r_1}{r_1 r_2}$; now, since the resistance of any conductor is the reciprocal of its conductivity, then the *joint resistance* of the two branches in parallel is the reciprocal of their joint conductivity; or, $1 \div \frac{r_2 + r_1}{r_1 r_2} = \frac{r_1 r_2}{r_2 + r_1}$. Hence, joint resistance

$$R'' = \frac{r_1 r_2}{r_1 + r_2}. \qquad \textbf{(12.)}$$

That is, *the joint resistance of two conductors connected in parallel is equal to the product of their separate resistances divided by the sum of their separate resistances.*

Example.—In Fig. 51, given $r_1 = 2$ ohms and $r_2 = 3$ ohms; find their joint resistance in parallel.

Solution.—From formula **12,** their joint resistance

$$R'' = \frac{r_1 r_2}{r_1 + r_2} = \frac{2 \times 3}{2 + 3} = \frac{6}{5} = 1\tfrac{1}{5} \text{ ohms. Ans.}$$

72. Fig. 52 represents a divided circuit of three branches. Let r_1, r_2, and r_3 be the separate resistances of those branches, respectively. Then, $\frac{1}{r_1}$, $\frac{1}{r_2}$, and $\frac{1}{r_3}$ represent the separate conductivities of the three branches, respectively. Their joint conductivity $= \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3}$. Since the joint resistance is the reciprocal of their joint conductivity, then it is equal to

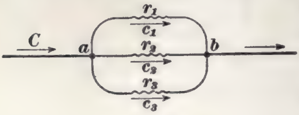


FIG. 52.

$$1 \div \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3} = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}.$$

Hence, the joint resistance of three branches in parallel

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}. \qquad \textbf{(13.)}$$

That is, *the joint resistance of three or more conductors connected in parallel is equal to the reciprocal of their joint conductivity.*

EXAMPLE.—In Fig. 52, given $r_1 = 5$ ohms; $r_2 = 10$ ohms; and $r_3 = 20$ ohms; find their joint resistance from a to b.

SOLUTION.—By formula **13**, their joint resistance

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{5 \times 10 \times 20}{10 \times 20 + 5 \times 20 + 5 \times 10} = \frac{1{,}000}{350} = \frac{20}{7}$$

$= 2\frac{6}{7}$ ohms. Ans.

73. In a derived circuit of any number of branches, the difference of potential between where the branches divide and where they unite is equal to the product of the sum of the currents in the separate branches and their joint resistance in parallel, as will be apparent from consideration of Ohm's law, Art. **34.**

For example, if the currents in the three branches, Fig. 52, are 16, 8, and 4 amperes, respectively, and the joint resistance from a to b is $2\frac{6}{7}$ ohms, then the difference of potential between a and $b = (16 + 8 + 4) \times 2\frac{6}{7} = 28 \times \frac{20}{7} = 80$ volts.

74. The separate currents in the branches of a derived circuit can be determined by finding the difference of potential between where the branches divide and where they unite, and dividing the result by the separate resistance of each branch.

For example, in Fig. 52, assume that the separate resistances of the three branches are 5, 10, and 20 ohms, respectively, and that the difference of potential between a and b is 80 volts. Then, the current in the first branch is $\frac{80}{5} = 16$ amperes; in the second, $\frac{80}{10} = 8$ amperes; and in the third, $\frac{80}{20} = 4$ amperes.

75. The separate resistances of the branches of a derived circuit can be determined by finding the difference of potential between where the branches divide and where they unite, and dividing the result by the separate currents in each branch.

For example, in Fig. 52, assume the difference of potential between a and b to be 80 volts and the currents in the separate branches to be 16, 8, and 4 amperes, respectively; then, the resistance of the first branch is $\frac{80}{16} = 5$ ohms; of the second, $\frac{80}{8} = 10$ ohms; and of the third, $\frac{80}{4} = 20$ ohms.

EXAMPLE.—Fig. 53 represents a closed circuit, part of which, from a to b, forms a derived, or shunt, circuit of three separate branches A, B, and C in parallel; r_1, r_2, and r_3 represent the separate resistance of the branches, respectively, from a to b; and R' represents the resistance of the rest of the closed circuit from b to a in the direction in which the current is supposed to be flowing, including the internal resistance of the battery K. Let $r_1 = 2$ ohms; $r_2 = 3.2$ ohms; $r_3 = 4.4$ ohms; and $R' = .8$ ohm. If a current of 2 amperes is flowing in the main, or undivided, circuit, find the total electromotive force developed in the battery K.

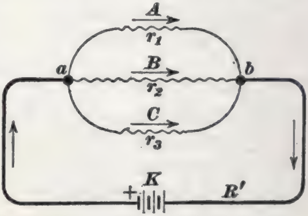


FIG. 53.

SOLUTION.—From the application of Ohm's law to closed circuits, formula **8**, $E = C R$, where E is the total electromotive force developed within the electric source; C the strength of current flowing; and

R the total resistance of the circuit through which the current passes. In this particular problem, the total resistance of the closed circuit will be the *joint* resistance of the three branches in parallel, plus the resistance R' of the rest of the circuit. Hence, first find the joint resistance of the three branches A, B, and C in parallel from a to b. By formula **13,** the joint resistance of three conductors in parallel is $\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}$, where r_1, r_2, and r_3 represent the separate resistances of the three conductors. Substituting gives

$$\frac{2 \times 3.2 \times 4.4}{3.2 \times 4.4 + 2 \times 4.4 + 2 \times 3.2} = \frac{28.16}{14.08 + 8.8 + 6.4} = \frac{28.16}{29.28} = .9617 \text{ ohm},$$

the joint resistance of the three branches A, B, and C in parallel from a to b. The total resistance of the closed circuit is, therefore, $.9617 + .8 = 1.7617$ ohms, and $E = C \times R = 2 \times 1.7617 = 3.5234$ volts. Ans.

ELECTRICAL QUANTITY.

76. The rate of flow of liquids is expressed in units of quantity per second or minute, and similarly the strength of an electric current can be defined as a quantity of electricity flowing per second. The practical unit of *electrical quantity* is called the **coulomb.**

The coulomb is such a quantity of electricity as will pass in one second through a circuit in which the strength of current is one ampere.

As stated previously, the *quantity* of electricity is calculated from the strength of current; it cannot be actually measured. For example, suppose the strength of current in a closed circuit to be 10 amperes, as measured by an ammeter; if such a current flows for only one second, the quantity of electricity which has passed around the circuit is 10 coulombs; but if the current flows for two seconds, the quantity of electricity would be 20 coulombs.

Hence, to calculate the quantity of electricity which has passed in a circuit in a certain time when the strength of the current in amperes is known:

Let Q = the quantity of electricity in coulombs, C the strength of current in amperes, and t the time in seconds.

Then, $$Q = C t. \qquad \textbf{(14.)}$$

If any two of these quantities are known, the third can be readily found. By transposition, $C = \frac{Q}{t}$ and $t = \frac{Q}{C}$.

Therefore, to obtain the quantity of current which has passed through a circuit in a given time, *multiply the strength of current in amperes by the time in seconds.*

EXAMPLE.—Find the quantity of electricity in coulombs that flows around in a closed circuit in $1\frac{1}{2}$ hours when the strength of current is 12 amperes.

SOLUTION.—Reducing the time to seconds gives $1.5 \times 60 \times 60 = 5{,}400$ seconds; hence, $t = 5{,}400$ seconds and $C = 12$ amperes. Then from formula **14**,

$$Q = C t = 12 \times 5{,}400 = 64{,}800 \text{ coulombs. Ans.}$$

ELECTRICAL WORK.

77. When an electric current flows from a higher to a lower potential, *electrical energy* is expended and *work* is done by the current. The principle of the *conservation of energy* teaches that energy can never be destroyed; it follows, therefore, that if energy has to be expended in forcing a quantity of electricity against a certain amount of resistance, the equivalent of that energy must be transformed into some other form. This other form is usually *heat;* that is, when a quantity of electricity flows against the resistance of a conductor, a certain amount of *electrical* energy is transformed into *heat* energy.

The actual amount of heat developed is an exact equivalent of the *work done* in overcoming the resistance of the conductor, and varies directly as that resistance. For example, take two wires, the resistance of one being twice that of the other, and send currents of equal strengths through each. The amount of heat developed in the wire of higher resistance will be twice that developed in the wire offering the lower resistance.

The unit used to express the amount of mechanical work done is known as the *foot-pound.* The work done in raising any mass through any height is found by multiplying

the weight of the body lifted by the vertical height through which it is raised; similarly, the practical unit of *electrical work* is that amount accomplished when a unit quantity of electricity, *one coulomb*, flows between potentials differing by *one volt*.

The unit of electrical work is, therefore, the *volt-coulomb*, and is called the **joule**.

1 *joule* = .7373 foot-pound.

78. By means of the following formulas, we may find directly the amount of electrical work accomplished in *joules* during a given time in any circuit:

Let J = electrical work in joules;
C = current in amperes;
t = time in seconds during which the current flows;
E = potential, or E. M. F., of circuit;
R = resistance of circuit.

When the current and electromotive force are known,

$$J = CEt. \qquad \textbf{(15.)}$$

When the current and resistance are known,

$$J = C^2 Rt. \qquad \textbf{(16.)}$$

When the resistance and electromotive force are known,

$$J = \frac{E^2 t}{R}. \qquad \textbf{(17.)}$$

To determine, therefore, the electrical work done in a given time, *multiply the quantity of electricity in coulombs which has passed in the circuit during that time by the loss, or drop, of potential as measured directly, or as computed from the values of the current and resistance.*

EXAMPLE.—Find the amount of work done in joules when a current of 15 amperes flows for ½ an hour against a resistance of 2 ohms.

SOLUTION.—Reducing the time to seconds gives $30 \times 60 = 1{,}800$ seconds $= t$. The current $= C = 15$ amperes, and the resistance $=$ 2 ohms $= R$. Then, by formula **16,** the electrical work done

$$J = 15 \times 15 \times 2 \times 1{,}800 = 810{,}000 \text{ joules. Ans.}$$

79. When the work in joules is known, the work in foot-pounds

$$F.\ P. = .7373\, J. \qquad \textbf{(18.)}$$

That is, *the equivalent work done in foot-pounds is obtained by multiplying the number of joules by .7373.*

EXAMPLE.—Express the work done in foot-pounds in a circuit when a current of 8 amperes flows for 2 hours between potentials differing by 10 volts.

SOLUTION.—Reducing the time to seconds gives $2 \times 60 \times 60 = 7{,}200$ seconds $= t$. The current $= 8$ amperes $= C$, and the electromotive force $= 10$ volts $= E$. Then, by formula **15,** the electrical work done $= J = 8 \times 10 \times 7{,}200 = 576{,}000$ joules. Expressed in foot-pounds, this will be, by formula **18,**

$$F.\ P. = .7373 \times 576{,}000 = 424{,}684.8 \text{ foot-pounds. Ans.}$$

ELECTRICAL POWER.

80. Power, or *rate of doing work*, is found by dividing the amount of work done by the time required to do it. In mechanics, the unit of power is called the **horsepower;** in electrotechnics, the unit of power is the **watt.** It is found by dividing the amount of electrical work done by the time required to do it.

Let E = the electromotive force in volts; J, the electrical work in joules; t, the time in seconds; C, the current in amperes; and W, the power in watts.

By formula **15,** the amount of electrical work $J = C\,E\,t$.

Then, $$W = \frac{C\,E\,t}{t} = C\,E. \qquad \textbf{(19.)}$$

The power in watts is equal to the strength of current in amperes multiplied by the electromotive force in volts.

EXAMPLE.—What is the power in watts developed in a closed circuit in which a current of 12 amperes is flowing between potentials differing by 25 volts?

SOLUTION.— $E = 25$ volts and $C = 12$ amperes. Hence, by formula **19,**

$$W = C\,E = 12 \times 25 = 300 \text{ watts. Ans.}$$

By taking into consideration the resistance of the circuit, the equation for determining the power in watts may be expressed in two other ways:

By derivation from formula **16**,

$$W = \frac{C^2 R t}{t} = C^2 R. \qquad \textbf{(20.)}$$

That is, *the power in watts is equal to the strength of current in amperes squared, multiplied by the resistance in ohms.*

EXAMPLE.—Find the power in watts in a closed circuit in which a current of 30 amperes is flowing against a resistance of 3 ohms.

SOLUTION.— $C = 30$ and $R = 3$. Hence, by formula **20**,

$$W = C^2 R = 30^2 \times 3 = 2{,}700 \text{ watts. Ans.}$$

By derivation from formula **17**,

$$W = \frac{E^2 t}{R t} = \frac{E^2}{R}. \qquad \textbf{(21.)}$$

That is, *the power in watts is the quotient arising from dividing the electromotive force in volts squared by the resistance in ohms.*

EXAMPLE.—The drop of potential in a closed circuit when a current is flowing is 20 volts and the resistance is 10 ohms; what is the power in watts expended?

SOLUTION.— $E = 20$ volts and $R = 10$ ohms. Hence, by formula **21**,

$$W = \frac{E^2}{R} = \frac{20^2}{10} = 40 \text{ watts. Ans.}$$

81. One watt equals $\frac{1}{746}$ of a horsepower; or, 1 horsepower equals 746 watts.

If H. P. = horsepower,

$$\text{H. P.} = \frac{W}{746}. \qquad \textbf{(22.)}$$

That is, *to express the rate of doing electrical work in horsepower units, find the number of watts and divide the result by 746.*

The horsepower may also be expressed by three other equations, by expressing the watts in terms of electromotive force, current, and resistance, as obtained from formulas **19, 20, 21,** viz.:

$$\text{H. P.} = \frac{E\,C}{746};\ \text{H. P.} = \frac{C^2\,R}{746};\ \text{and H. P.} = \frac{E^2}{746\,R}.$$

EXAMPLE.—Given, current = 50 amperes and electromotive force = 250 volts; express the power directly in horsepower units.

SOLUTION.— $E = 250$ volts; $C = 50$ amperes; hence,

$$\text{H. P.} = \frac{E\,C}{746} = \frac{250 \times 50}{746} = 16.756 \text{ horsepower. Ans.}$$

EXAMPLE.—Given, strength of current = 25 amperes and resistance = 14.92 ohms; express the power directly in horsepower units.

SOLUTION.— $C = 25$ amperes; $R = 14.92$ ohms; hence,

$$\text{H. P.} = \frac{C^2\,R}{746} = \frac{25^2 \times 14.92}{746} = 12.5 \text{ horsepower. Ans.}$$

EXAMPLE.—Given, electromotive force = 110 volts and resistance = 4 ohms; express the power directly in horsepower units.

SOLUTION.— $E = 110$ volts; $R = 4$ ohms; hence,

$$\text{H. P.} = \frac{E^2}{746\,R} = \frac{110^2}{746 \times 4} = 4.055 \text{ horsepower. Ans.}$$

82. To express the power in watts when the horsepower is known, use the following formula:

$$W = \text{H. P.} \times 746. \qquad \textbf{(23.)}$$

That is to say, *the power in watts is found by multiplying the horsepower by 746.*

EXAMPLE.—Express the equivalent of 4.35 horsepower in watts.

SOLUTION.— H. P. = 4.35; by formula **23,** the electrical power $W = 4.35 \times 746 = 3{,}245.1$ watts. Ans.

83. The watt is too small a unit for convenient use in expressing the output of large dynamos, so the **kilowatt** is generally used. One kilowatt is equal to 1,000 watts or about $1\frac{1}{3}$ horsepower. For example, if a dynamo were rated at 75 kilowatts, it would have an output of 75,000 watts or

roughly about 100 horsepower. The **kilowatt-hour** is a unit of *work* commonly used in electrical work. It is the amount of *work* done when 1 kilowatt is expended for 1 hour, or $\frac{1}{2}$ kilowatt for 2 hours, etc. The kilowatt-hours are, therefore, found by multiplying the average number of kilowatts by the average number of hours during which the kilowatts were expended. Since 1 kilowatt = 1,000 watts, 1 kilowatt-hour = 1,000 watt-hours. Now 1 watt expended for 1 second is equal to 1 joule; hence, 1 kilowatt-hour $= 1{,}000 \times 3{,}600 = 3{,}600{,}000$ joules, or $3{,}600{,}000 \times .7373 = 2{,}654{,}280$ foot-pounds. The kilowatt-hour represents a definite amount of work, whereas the kilowatt expresses the rate at which work is done, and is, therefore, a unit of power.

HEAT AND STEAM.

HEAT.

NATURE OF HEAT.

1. All modern scientists and investigators agree that **heat** is a *form of energy*. It is conceived to be a motion of the molecules composing matter. All matter is composed of molecules, which, according to the generally accepted theory, are not in a state of rest, but are moving or vibrating back and forth with a greater or less velocity. It is this movement of the molecules that is generally believed to cause the sensations of warmth and cold; if the motion is slow, the body feels cold; whereas, if the motion is rapid, the body feels warm. Since a body in motion has kinetic energy and since the molecules composing matter are supposed to be in motion, each molecule possesses kinetic energy; hence, we can conceive heat to be a form of energy.

2. Temperature is a term used to indicate how hot or cold a body is; i. e., to indicate the velocity of the vibration of the molecules of a body. A body having a high temperature is said to be hot; a body having a low temperature is said to be cold. When a body, as, for example, an iron bar, receives heat from any source, its temperature rises; on the other hand, when a body loses heat, its temperature falls.

The temperature is *not* a measure of the *quantity* of heat a body possesses. *Temperature* may be considered to be a measure of the velocity with which the molecules of a body vibrate to and fro, while the *quantity of heat* may be considered to be the total energy of the molecules composing the body. A small iron rod may be heated to whiteness and yet possess a very small quantity of heat. Its temperature is very high, but this simply indicates that the molecules of the rod are vibrating with an extremely high velocity. An iron ball 1 foot in diameter and an iron ball 1 inch in diameter may have exactly the same *temperature*, but the larger ball would have by far the greater quantity of heat.

3. The Thermometer.—Temperature is measured by an instrument called the **thermometer,** which is so familiar as to scarcely need description. It consists of a thin glass tube, at one end of which is a bulb filled with mercury. Upon being heated, the mercury expands in proportion to the rise of temperature. Thermometers are graduated in different ways. In the Fahrenheit thermometer, which is the one generally used in this country, the point where the mercury stands when the instrument is placed in melting ice is marked 32°. The point indicated by the mercury when the thermometer is placed in water boiling in the open air at the level of the sea is marked 212°. The tube between these two points is divided into 180 equal parts, called **degrees.**

4. Effects of Heat.—Suppose we take a vessel filled with some substance, say water. Let the vessel be a cylinder fitted with a piston, as shown in Fig. 1. The water is, say, at the freezing point, and the millions of molecules composing the water are moving to and fro with a comparatively small velocity. Place the vessel over a fire or furnace. Heat is communicated to the molecules of water, and they begin to move faster and faster. That is, their kinetic energy increases, and if a thermometer is inserted in the vessel, it will be found that the temperature of the water rises. Consequently, one effect of heat is to raise the

temperature of the body to which it is applied. But, after reaching a certain temperature, the molecules of the water not only move faster, but they move farther from each other, and their paths are longer. It is plain that if the molecules are farther apart than they were originally, the whole body of them must take up more space. In other words, after reaching a certain temperature, the water expands as heat is added. Hence, another effect of heat is to cause bodies to expand. Common examples of the expansion of bodies by heat are seen in the setting of tires, the expansion of the rails of a railway in summer, etc.

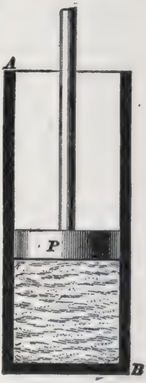


FIG. 1.

5. The heat supplied to the vessel of water has so far done three things: (1) It has raised the temperature of the water and thus has increased the kinetic energy of the molecules. Let the amount of heat expended for this purpose be denoted by S. (2) A certain quantity of heat has been used in expanding the water, that is, in pushing the molecules farther apart against the force of cohesion. Denote the amount of heat so expended by I. (3) Since the water expands, it must raise the piston P against the pressure of the atmosphere, and consequently more heat must be used to expand the water than would be required if there were no pressure on the upper side of the piston. Call this extra quantity of heat W.

If we denote by Q the total quantity of heat given to the vessel of water, we have

$$Q = S + I + W.$$

Ordinarily, the greater part of the heat given to a body is spent in raising its temperature, and but little is used in expanding the body. That is, the quantity S is nearly equal to the quantity Q, while the quantities I and W are extremely small.

6. Suppose that the piston in Fig. 1 is removed from the cylinder, so that the water will be in contact with the atmosphere, and that a thermometer is inserted. As the water becomes more and more heated, the temperature indicated by the thermometer will rise until it reaches 212°. So far, most of the heat has been used to raise the temperature of the water. But now, no matter how much heat is added to the water, the mercury stands at 212°, and cannot be made to rise higher. This is the reason: When the temperature reaches 212° the molecules of water have been set into such rapid motion that the force of cohesion is no longer able to hold them and they tend to separate. In other words, the water changes to a gas (steam), and all the heat is being used to effect this change. The temperature of the steam will remain at 212° until all water is changed to steam; then, if more heat is applied, the temperature of the steam will begin to rise.

7. Again, suppose we take a block of ice at a temperature of, say, 14° and heat it. If a thermometer is placed in contact with the ice, the mercury will rise until it reaches 32° and will then remain stationary. As soon as this temperature is reached, the ice begins to melt or change to water, and the heat, instead of raising the temperature farther, is all used to effect this change of state. Here, then, is another effect produced by heat. It will change a solid to a liquid or a liquid to a gas.

8. Summary.—By a study of the results of applying heat to the substances that have just been considered, we see that the effects of heat most commonly observed are: (1) It increases the rate of motion of the molecules, an effect that is indicated by an increase in temperature. (2) It increases the lengths of the paths of, and the distance between, the molecules, thus causing the body to expand and fill a greater space. (3) It overcomes the attractive forces that tend to hold the molecules of a substance together, and thus changes it from a solid to a liquid or from a

liquid to a gas, according to the state it was in when the heat was applied.

The second statement of the summary, while generally true, is subject to exceptions, the most notable one of which is water, which in rising from a temperature of 32° Fahrenheit to a temperature of 39.2° contracts instead of expands.

9. Latent Heat.—The heat that is expended in changing a body from the solid to the liquid state or from the liquid to the gaseous state is called **latent heat.** The portion of the heat applied that raises temperature and that, therefore, affects the thermometer is sometimes called **sensible heat.**

10. Measurement of Heat.—Since heat is not a substance, it cannot be measured directly in pounds or quarts, but, like force, it may be measured by the effects it produces. Suppose that a certain quantity of heat raises the temperature of 1 pound of water from 52° to 53° Fahrenheit. It will take the same quantity of heat to raise that pound from 53° to 54°, and therefore it will take nearly double that quantity to raise the temperature of 1 pound of water from 52° to 54°. The unit quantity of heat is the quantity required to raise the temperature of a pound of water from 62° to 63°. This unit is called the **British thermal unit,** and is commonly abbreviated to B. T. U.

11. For temperatures above 63°, it takes slightly more than 1 B. T. U. to produce a change of 1 degree in 1 pound of water—the difference increasing the farther the temperature is from 63°. For temperatures below 62°, it takes slightly less than 1 B. T. U. to produce a change of 1 degree in 1 pound of water; and, as before, the difference is greater the farther the temperature is from 62°. Thus it will take more heat to raise the temperature of 1 pound of water from 75° to 76° than it will to raise it from 74° to 75°. Likewise, it will take less heat to raise the temperature of 1 pound of water from 42° to 43° than to raise it from 43° to

44°. However, the difference between the actual B. T. U., as defined above, and the quantity of heat required to change the temperature of 1 pound of water 1 degree for any other temperature is so small that, for all ordinary purposes, it may be assumed that it takes 1 B. T. U. to produce a change of 1 degree in the temperature of 1 pound of water for all temperatures that we are likely to meet in practice.

12. Relation Between Heat and Work.—Suppose that, in the experiment shown in Fig. 1, the piston had been allowed to remain in the cylinder while the water was being changed to steam. Steam at 212° occupies nearly 1,700 times the space that the water originally occupied. Hence, the piston would be lifted in the cylinder to give room for the steam that was being formed. But to raise the piston requires work. Here, then, is an example of work being performed by heat. On the other hand, work will produce heat. If two blocks of wood are rubbed briskly together, they will become warm, and may even ignite. The work done in overcoming friction causes the journals and bearings of fast-running machines to heat. A small iron rod may be heated to redness by pounding it on an anvil.

13. Since work may be changed into heat and heat into work, it seems probable that there is some fixed ratio between the unit of heat (B. T. U.) and the unit of work, the foot-pound. By a series of careful experiments, Dr. Joule, of England, discovered this ratio.

He found that 1 B. T. U. is equivalent to 772 foot-pounds; later and more careful experiments show that 778 foot-pounds is more nearly correct. This number, 778 foot-pounds, is called the **mechanical equivalent** of 1 B. T. U.

We have, then, the following important law: *Heat may be changed to work or work to heat; 778 foot-pounds of work are required to produce 1 B. T. U.; and, conversely, the expenditure of 1 B. T. U. produces 778 foot-pounds of work.*

EXAMPLE 1.—The burning of 1 pound of coal gives out sufficient heat to raise 14,000 pounds of water from 62° to 63°. If all this heat is utilized, how high will it lift a weight of 700 pounds?

SOLUTION.—Since 1 B. T. U. raises the temperature of 1 pound of water from 62° to 63°, it requires 14,000 B. T. U. to raise 14,000 pounds of water from 62° to 63°; hence, the burning of 1 pound of coal gives out 14,000 B. T. U. One B. T. U. is equivalent to 778 foot-pounds; hence, 14,000 B. T. U. are equivalent to $14,000 \times 778 = 10,892,000$ foot-pounds. Then, the height to which the weight can be raised is $10,892,000 \div 700 = 15,560$ ft. Ans.

EXAMPLE 2.—A cannon ball weighing 60 pounds moves with a velocity of 1,300 feet per second. Suppose the ball were suddenly stopped and all its kinetic energy changed into heat, how many B. T. U. would be developed? If all this heat were applied to 100 pounds of water at a temperature of 60°, to what temperature would the water be raised?

SOLUTION.—The kinetic energy of the cannon ball is $\frac{W v^2}{64.32} = \frac{60 \times 1,300^2}{64.32} = 1,576,492$ foot-pounds. But 778 foot-pounds = 1 B.T.U. Therefore, the number of B. T. U. developed is $1,576,492 \div 778 = 2,026.3$ B. T. U. Since 1 B. T. U. raises the temperature of 1 pound of water 1 degree, it will take 100 B. T. U. to raise 100 pounds of water 1 degree. Hence, 2,026.3 B. T. U. will raise 100 pounds of water $2,026.3 \div 100 = 20.26$ degrees, and the final temperature of the water will be $60° + 20.26° = 80.26°$. Ans.

14. Specific Heat.—One B. T. U. raises the temperature of 1 pound of water 1 degree; will it have the same effect on a pound of mercury? Heat two 1-pound iron balls to the temperature of boiling water, 212°; having now the same weights and temperatures, each ball has the same quantity of heat. Place one of these balls in a vessel, into which slowly pour enough water having a temperature of 60° so that the iron will be cooled to 70° while the water is heated to the same temperature. Now place the other hot ball in another vessel, into which pour mercury having a temperature of 60°, until the iron and mercury reach a common temperature of 70°. In each case the hot ball will have been cooled from 212° to 70°, and therefore each will have given up the same quantity of heat. When, however, we consider the effects produced by the heat, we find that what has been given off by one ball has raised nearly 1.62 pounds of water 10 degrees; that given off by the other ball,

which, it will be remembered, is the same amount as in the first case, has raised 48.5 pounds of mercury (or nearly 30 times more mercury than water) the same number of degrees. It is plain, therefore, that to raise 1 pound of mercury from 62° to 63° requires $\frac{1}{30}$ the heat necessary to raise 1 pound of water from 62° to 63°. Hence, we say the specific heat of the mercury is $\frac{1}{30}$ or .0333.

15. *The* **specific heat** *of a body is the ratio between the quantity of heat required to warm that body 1 degree and the quantity of heat required to warm an equal weight of water 1 degree.*

EXAMPLE 1.—It is found that to raise the temperature of 20 pounds of iron from 62° to 63° requires 2.276 B. T. U. What is the specific heat of iron?

SOLUTION.—To raise 20 pounds of water from 62° to 63° requires 20 B. T. U. The specific heat of the iron is, according to the above definition, the ratio between the quantities of heat required to raise the temperature of the iron and the water, respectively, through 1 degree, that is, it is the ratio $2.276 : 20 = 2.276 \div 20 = .1138$. Ans.

EXAMPLE 2.—The specific heat of silver is .057. How many B. T. U. are required to raise 22 pounds of silver from 50° to 60°?

SOLUTION.—To raise the temperature of 1 pound of water 1 degree requires 1 B. T. U. Since the specific heat of silver is .057, only .057 B. T. U. is required to raise 1 pound of silver 1 degree. Hence, to raise 22 pounds of silver 10 degrees must require $.057 \times 22 \times 10 = 12.54$ B. T. U. Ans.

Rule 1.—*To find the number of B. T. U. required to raise the temperature of a body a given number of degrees, multiply the specific heat of the body by its weight in pounds and by the number of degrees.*

Denote the number of B. T. U. required by U; the specific heat by c; the weight by W; and let t and t_1, respectively, be the temperatures before and after the heat is applied.

Then, $$U = c\,W(t_1 - t).$$

The specific heat of some of the more common substances is given in the following table:

SPECIFIC HEAT OF SUBSTANCES.

Substance.	Specific Heat.	Substance.	Specific Heat.
Water	1.0000	Ice	.5040
Sulphur	.2026	Steam (superheated)	.4805
Iron	.1138	Air	.2375
Copper	.0951	Oxygen	.2175
Silver	.0570	Hydrogen	3.4090
Tin	.0562	Carbon monoxide	.2479
Mercury	.0333	Carbon dioxide	.2170
Lead	.0314	Nitrogen	.2438

16. Latent Heat of Fusion.—This term is applied to the quantity of heat required to change a pound of a given substance from the solid to the liquid state. The only case of interest to the engineer is the heat required to change 1 pound of ice to water. Careful experiments have shown that about 144 B. T. U. are required to change 1 pound of ice at 32° to water at 32°. Hence, the latent heat of water is 144 B. T. U.

17. The **latent heat of steam** is the quantity of heat required to change 1 pound of water at a given temperature into steam at the same temperature. Experiment has shown that at a temperature of 212° this quantity of heat is about 966 B. T. U. This means that the heat required to change 1 pound of water at 212° to steam is 966 times as great as the heat required to raise the temperature of a pound of water from 62° to 63°. The latent heat of steam is different for different temperatures, as will be seen by referring to the Steam Tables.

EXAMPLE.—How many B. T. U. are required to change 5 pounds of ice at 15° into steam at 212°?

SOLUTION.—To raise the temperature of the ice from 15° to 32° (the melting temperature) requires, according to rule **1**,

$$.504 \times 5 \times (32 - 15) = 42.84 \text{ B. T. U.}$$

To change the ice to water requires 144 B. T. U. for each pound, or $144 \times 5 = 720$ B. T. U. To raise the water from 32° to 212° requires, from rule **1**,

$$1 \times 5 \times (212 - 32) = 5 \times 180 = 900 \text{ B. T. U.}$$

Finally, to change the water to steam requires 966 B. T. U. per pound, or $966 \times 5 = 4{,}830$ B. T. U. Therefore, in all, $42.84 + 720 + 900 + 4{,}830 = 6{,}492.84$ B. T. U. are required. Ans.

Expressed in foot-pounds, the work required to effect the above change would be $6{,}492.84 \times 778 = 5{,}051{,}429.5$ foot-pounds, or work enough to lift a weight of 1,000 pounds nearly a mile.

18. Since a pound of ice requires 144 B. T. U. to change it to water, it follows that when a pound of water at 32° changes to ice (freezes), 144 B. T. U. are given out in the process. Similarly, the condensation of a pound of steam into water at 212° liberates 966 B. T. U. This principle is applied in heating buildings by steam. The steam passes through the radiators and condenses. The latent heat thus set free warms the building.

19. Temperatures of Mixtures.—It is often desirable to calculate the final temperature of a mixture of different substances at different temperatures. The following law is to be observed in such cases: *The quantity of heat in a mixture is the same as the quantity of heat contained in the substances before being combined.* If two substances of different temperatures are placed together, they both finally attain the same temperature; the heat lost by one in coming from a higher to a lower temperature is gained by the other in passing from a lower to a higher temperature.

Rule 2.—*To find the temperature of a mixture of several substances, multiply together the weight, specific heat, and temperature of each substance separately and add the products. Next multiply together the weight and specific heat of each of the substances separately and add these products. Divide the former sum by the latter. The result will be the temperature of the mixture.*

EXAMPLE.— 15 pounds of water at 42° and 30 pounds of mercury at 70° are placed in the same vessel, and a ball of lead weighing 19 pounds and having a temperature of 110° is immersed in the mixture. What is the final temperature of the contents?

SOLUTION.—Applying rule **2**, the product of the weight, specific heat, and temperature of the water is $15 \times 1 \times 42 = 630$; of the mercury, $30 \times .0333 \times 70 = 69.93$; of the lead, $19 \times .0314 \times 110 = 65.626$; and the sum is $630 + 69.93 + 65.626 = 765.556$.

The product of the weight and specific heat of the water is $15 \times 1 = 15$; of the mercury, $30 \times .0333 = .999$; of the lead, $19 \times .0314 = .5966$; and the sum is $15 + .999 + .5966 = 16.5956$. Then, the temperature of the mixture is $765.556 \div 16.5956 = 46.13°$. Ans.

20. A particularly important case is the mixture of steam and water. Let W and t_1 represent the weight and temperature of the steam and let w and t represent the weight and temperature of the water. Let T represent the final temperature of the mixture and L the latent heat of the steam at the given temperature. Then, the temperature of a mixture of steam and water may be found by means of the following rule:

Rule 3.—*When steam and water are mixed, the steam condenses. To find the final temperature of the mixture, add the latent heat and the temperature of the steam and multiply this sum by the weight of the steam. To this product add the product of the weight and temperature of the water and divide the sum so obtained by the sum of the weights of the steam and water. The quotient will be the temperature of the mixture.*

Or, $$T = \frac{W(L + t_1) + w\,t}{W + w}.$$

In the case of a mixture of water and steam, it is to be observed that the temperature of the mixture can never be more than the temperature of the steam, no matter whether the mixture takes place in an open or a closed vessel. The reason for this is to be found in the fact that after the steam has heated the water to its own temperature, there can be no further giving up of heat, since in order that heat

may pass from one body to another, there must be a difference in the temperature of the two bodies, the heat passing from the hotter body to the colder one. The only way in which heat can be made to pass from a colder to a hotter body or from one body to another one at an equal temperaature is by the expenditure of work; but as this condition does not ordinarily exist in the case of steam and water being mixed, it follows that the temperature of the mixture can never exceed that of the steam.

Whenever rule **3** gives a temperature in excess of that of the steam, it is an indication that the quantity of water mixed with the steam is not sufficient to condense all the steam, and consequently the given temperature of the steam (in case of a closed vessel) or the temperature of the steam corresponding to the existing atmospheric pressure (in case of an open vessel) should be substituted for the temperature given by rule **3.**

EXAMPLE.—If 8 pounds of steam at 212° are run into a barrel containing 300 pounds of water at 43°, what will be the final temperature?

SOLUTION.—The weight of the steam $W = 8$ pounds; the latent heat $L = 966$ B. T. U.; the temperature $t_1 = 212°$. The weight of the water $w = 300$ pounds; the temperature $t = 43°$. Hence, from rule **3,** the final temperature is

$$\frac{8 \times (966 + 212) + 300 \times 43}{8 + 300} = 72\tfrac{1}{2}°. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

1. A body weighing 143 pounds falls 62 feet. If the energy of the body at the end of the fall is changed into heat, how many B. T. U. will be developed? Ans. 11.39 B. T. U.

2. An expenditure of 210 B. T. U. per minute will develop how many horsepower? Ans. 4.95 H. P.

3. If $\frac{1}{7}$ the total heat in the coal is used in doing work, how many pounds of coal must be burned per hour to run a 40-horsepower engine? Each pound of the coal gives out 13,500 B. T. U.

Ans. 52.8 lb.

4. A bar of iron weighing 20 pounds and having a temperature of 350° is plunged into a tank containing 130 pounds of water at 55°. To what temperature will the water be raised? Ans. 60°.

5. How many pounds of ice at 32° can be melted by 3 pounds of steam at 212°? Ans. 23.875 lb.

SUGGESTION.—Each pound of ice requires 144 B. T. U. to melt it; each pound of steam in changing to water at 32° gives up 966 + (212 − 32) = 1,146 B. T. U.

6. How many B. T. U. are required to raise the temperature of 26 pounds of copper from 57° to 93°? Ans. 89.01 B. T. U.

7. Four pounds of a certain substance at a temperature of 212° are mixed with 1 pound of water at 50°; the specific heat of the substance being .03125, what is the resulting temperature of the mixture?

Ans. 68°.

8. How many B. T. U. are required to change 13 pounds of water at 59° into steam at 212°? Ans. 14,547 B. T. U.

9. Twenty pounds of steam at 212° are run into a tank containing 340 pounds of water at 42°. What will be the final temperature of the mixture? Ans. 105.11°.

STEAM.

PROPERTIES OF STEAM.

21. Steam is *water vapor;* that is, it is *water* changed into a *gaseous state* by the application of *heat.*

The process of changing water (or other liquid) into vapor by means of heat is called **evaporation** or **vaporization.**

When an open vessel containing water is placed in contact with fire, the air contained in the water is first driven off and escapes from the surface. The water in contact with the part of the vessel nearest the fire first receives the heat and expands. Its specific gravity is reduced; that is, it becomes lighter than the cooler water above it, and it rises to the surface, cooler water taking its place. In this manner the water keeps up a circulation until it reaches a temperature of 212°. At this stage the molecules nearest the fire attain such a velocity of vibration that they rise through the water above them, overcome the pressure of the air, and escape in the form of a gas. When this occurs the water boils.

22. Pressure and Temperature.—It is plain that if the pressure on the surface of the water is increased, it will take more work to force the molecules to the surface; that is, more heat must be given to the water to make it boil, and therefore the boiling point will be raised. We have seen that water exposed to the atmospheric pressure of 14.7 pounds per square inch boils at a temperature of 212°. If the pressure is increased to, say, 32 pounds per square inch, the water will not boil until it reaches a temperature of 254°. On the other hand, if the pressure is lowered to 6 pounds per square inch, the water boils at 170°. Hence, we have the following law:

An increase of pressure on the surface of a liquid raises the boiling point; a decrease of pressure lowers the boiling point.

23. Saturated Steam.—Steam in contact with water is called **saturated steam.** This is the condition of steam in a boiler. Steam at a given pressure is also said to be saturated when its temperature is the same as the temperature at which water boils when subjected to the same pressure; this is true even though the steam is entirely separated from water. According to the law just given, the temperature of saturated steam depends only on the pressure. When the steam in a boiler shows a gauge pressure of 60 pounds, its temperature *must be* 307°. A thermometer placed in a boiler could be used to tell the pressure of the steam. It would be as accurate, though not as convenient, as a steam gauge.

The reader is cautioned against the idea that saturated steam necessarily implies "wet" steam. It may be perfectly free from water particles, but it is saturated if it is in any way in contact with water, so that the pressure and temperature are mutually dependent. In the steam boiler, for example, the space above the water, if viewed through a glass-covered opening, appears to be perfectly transparent, as though filled with air, provided the boiler is not working. This shows that the steam that fills this space is perfectly "dry." When, however, the boiler is steaming rapidly, the violent

ebullition may throw up a certain amount of water in the form of a spray, that mingles with the steam, giving it the misty appearance shown in the exhaust from an engine. Steam in this condition is "*wet*" *saturated steam.*

24. In physics the word saturation, whence the term *saturated steam* has been derived, has a meaning somewhat different from that commonly assigned to it. It there means the filling of a space with vapor to that point where condensation begins. Then, we may say that saturated steam is steam subjected to a pressure at which condensation is about to begin; that is, the slightest abstraction of heat or the slightest increase in pressure will cause part of it to condense, and if the steam be separated from the water, the slightest addition of heat will cause it to become superheated.

25. Superheated Steam.—Steam separated from water may be heated like air or any other gas until its temperature is higher than the boiling point corresponding to its pressure.

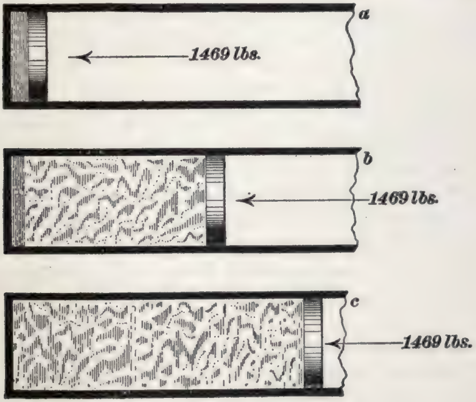


FIG. 2.

To illustrate, put a little water in a cylinder open to the atmosphere, as shown in Fig. 2 at *a*. Suppose that the area

of the cylinder is 100 square inches; then, the pressure of the atmosphere upon the piston is $14.69 \times 100 = 1,469$ pounds. The number 14.69 is a little more exact than 14.7.

When a part of the water is changed to steam, as at *b*, Fig. 2, the steam is in a saturated state, and at this pressure its temperature cannot be higher than 212°. When, however, the water is all changed to steam, as at *c*, Fig. 2, any farther addition of heat raises its temperature, while the pressure remains at 14.69 pounds per square inch. Steam in this latter condition is known as superheated steam.

The specific heat of superheated steam at constant pressure is .4805, or, say, .48 for ordinary purposes; that is, .48 of 1 B. T. U. will raise the temperature of 1 pound of superheated steam 1 degree. The temperature of saturated steam cannot be raised if the pressure remains constant. All the heat is expended in changing water to steam, and until all the water is vaporized the temperature remains constant.

26. If the steam in the cylinder has been subjected to a uniform pressure greater than that of the atmosphere, its temperature will be correspondingly higher; but as long as any water remains, any addition of heat will merely change more of the water to steam without increasing the temperature. As soon, however, as the last drop of water is gone, the effect of adding more heat will be to increase the temperature. We may, therefore, define **superheated steam** as steam separated from water and heated so as to give it a temperature higher than the boiling point corresponding to its pressure.

STEAM TABLES.

PROPERTIES OF SATURATED STEAM.

27. Whenever the pressure of saturated steam is changed, there are other properties that change with it. These properties are the following:

1. The temperature of the steam, or, what is the same thing, the boiling point.

2. The number of B. T. U. required to raise 1 pound of water from 32° (freezing) to the boiling point corresponding to the given pressure. This is called the **heat of the liquid.**

3. The number of B. T. U. required to change 1 pound of water at the boiling temperature into steam at the same temperature. This is called the **latent heat of vaporization,** or simply **latent heat.**

4. The number of heat units required to change 1 pound of water at 32° to steam of the required temperature and pressure. This is called the **total heat of vaporization,** or simply **total heat.**

It is plain that the total heat is the sum of the heat of the liquid and the latent heat. That is, total heat = heat of liquid + latent heat.

5. The **specific volume** of the steam at the given pressure; that is, the number of cubic feet occupied by 1 pound of steam of the given pressure.

6. The **density** of the steam; that is, the weight of 1 cubic foot of the steam at the given pressure.

28. All the above properties vary with the pressure of the steam. For example, if steam is at atmospheric pressure, the temperature is at 212°; the heat of the liquid is 180.531 B. T. U.; the latent heat, 966.069 B. T. U.; the total heat, 1,146.6 B. T. U. A pound of steam at this pressure occupies 26.37 cubic feet and 1 cubic foot of the steam weighs about .037928 pound. When the pressure is 70 pounds per square inch above vacuum, the temperature is 302.774°; the heat of the liquid is 272.657 B. T. U.; the latent heat is 901.629 B. T. U.; the total heat, 1,174.286 B. T. U. A pound of the steam occupies 6.076 cubic feet, and 1 cubic foot of the steam weighs .164584 pound.

These properties have been determined by direct experiment for all ordinary steam pressures. They are given in the Table of the Properties of Saturated Steam.

29. Explanation of Table.—Column 1 gives the pressures from 1 to 300 pounds. These pressures are above vacuum. The steam gauges fitted on steam boilers register the pressure above the atmosphere. That is, if the steam is at atmospheric pressure, 14.7 pounds per square inch, the gauge registers 0. Consequently, the atmospheric pressure must be added to the reading of the gauge to obtain the pressure above vacuum. In using the table, care must be taken *not* to use the gauge pressures without first adding 14.7 pounds per square inch for places at sea level. As the pressure of the atmosphere varies with the altitude, the true atmospheric pressure can always be obtained by consulting the barometer. The barometric reading should be reduced to pounds pressure per square inch and added to the gauge pressure in order to obtain the correct absolute pressure for the time and place. However, in nearly all engineering calculations, it is customary to take the pressure of the atmosphere as 14.7 pounds per square inch, which value will hereafter be used unless stated otherwise.

Pressures registered above vacuum are called **absolute pressures.** The pressures given in column 1 are *absolute.*

Column 2 gives the temperature of the steam when at the pressure shown in column 1.

Column 3 gives the **heat of the liquid.** It will be noticed that the values in column 3 may be obtained, approximately, by subtracting 32° from the temperature in column 2. If the specific heat of water were exactly 1.00, it would, of course, take exactly $212 - 32 = 180$ B. T. U. to raise 1 pound of water from 32° to 212°. But experiment shows that the specific heat of water is slightly greater than 1.00 when the temperature of the water is above 62°, and it therefore takes 180.531 B. T. U. to raise 1 pound of water from 32° to 212°.

Column 4 gives the **latent heat of vaporization,** which is seen to decrease slightly as the pressure increases.

Column 5 gives the **total heat of vaporization.** The values in column 5 may be obtained by adding together the corresponding values in columns 3 and 4.

Column 6 gives the weight of 1 cubic foot of steam in

pounds. As would be expected, as the pressure increases, the steam becomes denser and weighs more per cubic foot.

Column 7 gives the number of cubic feet occupied by 1 pound of steam at the given pressure. It will be noticed that the corresponding values of columns 6 and 7 multiplied together always produce 1. That is, for 31.3 pounds pressure gauge, $.11088 \times 9.018 = 1.000$, nearly.

Column 8 gives the ratio of the volume of 1 pound of steam at the given pressure and the volume of 1 pound of water at 39.1°. The values in column 8 may be obtained by dividing 62.425, the weight of 1 cubic foot of water at 39.1°, by the numbers in column 6.

30. Directions for Using Steam Table.—Manifestly it would be impossible to compile Steam Tables that would include the values corresponding to all pressures. The table here given covers the range of pressures likely to be met in practice. For values that are not given in the table, though within its range, a method of calculation, known as **interpolation,** may be used. In finding the values of t, q, L, etc. for an absolute pressure of, say, 76.35 pounds per square inch, we must make use of the two values of t, q, L, etc., and also of the two values of p given in the table that are nearest to 76.35 pounds; that is, the nearest given value of p that is less than 76.35 pounds and that which is greater than 76.35 pounds. In the present case, these two values are 76 pounds and 78 pounds. In like manner, for any other given value of p that, though within the range of the table, is not given, we must make use of the two values of p between which it is included.

31. The method of using the tables is illustrated by the following examples and their accompanying solutions.

What are the values of t, q, L, H, V, and W for steam whose gauge pressure is 61.65 pounds per square inch?

The pressures in the Steam Table are absolute pressures; hence, when the gauge pressure of steam is given, 14.7 must be added to it in order to use it in connection with the table.

Therefore,

$$p = 61.65 + 14.7 = 76.35 \text{ pounds per square inch.}$$

Turning to the tables, we find that this pressure lies between the two given values 76 and 78.

(*a*) To find t (temperature):

$$\begin{array}{lrlr} \text{For } p = & 78 & \text{pounds, } t = & 310.123^\circ \\ \text{For } p = & \underline{76} & \text{pounds, } t = & \underline{308.344^\circ} \\ \text{Difference,} & 2 & \text{pounds,} & 1.779^\circ \end{array}$$

For a difference in pressure of 2 pounds, we have a difference in temperature of 1.779°. A difference in pressure of 1 pound would, therefore, give a difference in temperature of $\frac{1.779}{2} = .8895°$. The actual difference in pressure for this case is $76.35 - 76 = .35$ lb. Hence, the actual difference in temperature would be $.35 \times .8895 = .311325°$, say, .311°. This means that if the pressure of saturated steam is changed from 76 pounds per square inch to 76.35 pounds per square inch, its temperature is raised through .311°. Its temperature at 76 pounds pressure is 308.344°, consequently, its temperature at 76.35 pounds pressure is $308.344 + .311 = 308.655°$. Ans.

(*b*) To find q (sensible heat):

$$\begin{array}{lrlr} \text{For } p = & 78 & \text{pounds, } q = & 280.170 \text{ B. T. U.} \\ \text{For } p = & \underline{76} & \text{pounds, } q = & \underline{278.350} \text{ B. T. U.} \\ \text{Difference,} & 2 & \text{pounds,} & 1.820 \text{ B. T. U.} \end{array}$$

For a difference in 1 pound pressure there will be a difference in q of $\frac{1.820}{2} = .91$ B. T. U.; for a difference in .35 pound pressure there is a difference in q of $.91 \times .35 = .3185$ B. T. U. Hence, the sensible heat q, corresponding to a pressure of 76.35 pounds per square inch, is $278.350 + .3185 = 278.6685$ B. T. U. Ans.

(*c*) To find L (latent heat):

$$\begin{array}{lll} \text{For } p = 76 \text{ pounds,} & L = 897.635 & \text{B. T. U.} \\ \text{For } p = \underline{78} \text{ pounds,} & L = \underline{896.359} & \text{B. T. U.} \\ \text{Difference, } 2 \text{ pounds,} & \quad 1.276 & \text{B. T. U.} \end{array}$$

For a difference of 1 pound pressure, the difference in latent heat is $\frac{1.276}{2} = .638$ B. T. U.

$$.638 \times .35 = .2233 \text{ B. T. U.}$$

By comparing p and L in the table, it will be seen that as p increases L decreases. In the present case, therefore, we must subtract from the value of L corresponding to a pressure of 76 pounds the difference due to the actual difference in pressures.

That is,

$$\text{For } p = 76.35 \text{ pounds, } L = 897.635 - .2233$$
$$= 897.4117 \text{ B. T. U. Ans.}$$

(*d*) To find H (total heat):

$$\begin{array}{lll} \text{For } p = 78 \text{ pounds,} & H = 1{,}176.529 & \text{B. T. U.} \\ \text{For } p = \underline{76} \text{ pounds,} & H = \underline{1{,}175.985} & \text{B. T. U.} \\ \text{Difference, } 2 \text{ pounds,} & \quad .544 & \text{B. T. U.} \end{array}$$

As above, $\frac{.544}{2} = .272$; $.272 \times .35 = .0952$ B. T. U.

Hence,

$$\text{For } p = 76.35 \text{ pounds, } H = 1{,}175.985 + .0952$$
$$= 1{,}176.0802 \text{ B. T. U. Ans.}$$

(*e*) To find W (weight per cubic foot):

$$\begin{array}{lll} \text{For } p = 78 \text{ pounds (pressure),} & W = .182229 & \text{pound (weight).} \\ \text{For } p = \underline{76} \text{ pounds (pressure),} & W = \underline{.177825} & \text{pound (weight).} \\ \text{Difference, } 2 \text{ pounds (pressure),} & \quad .004404 & \text{pound (weight).} \end{array}$$

Proceeding as before,

$$\frac{.004404}{2} = .002202; \quad .002202 \times .35 = .0007707.$$

Hence,

$$\text{For } p = 76.35 \text{ pounds, } W = .177825 + .0007707$$
$$= .1785957 \text{ pound. Ans.}$$

(f) To find V (volume of 1 pound):

$$\begin{array}{lll} & \text{For } p = 76 \text{ pounds,} & V = 5.624 \text{ cubic feet.} \\ & \text{For } p = \underline{78} \text{ pounds,} & V = \underline{5.488} \text{ cubic feet.} \\ \text{Difference,} & \quad 2 \text{ pounds,} & \quad .136 \text{ cubic foot.} \end{array}$$

As before,

$$\frac{.136}{2} = .068; \; .068 \times .35 = .0238 \text{ cubic foot.}$$

Here V decreases as p increases. Hence, as in (c), the difference must be subtracted from the value of V corresponding to 76 pounds, and we have

$$\text{For } p = 76.35 \text{ pounds, } V = 5.624 - .0238$$
$$= 5.6002 \text{ cubic feet. Ans.}$$

Example 1.—Calculate the heat required to change 5 pounds of water at 32° into steam at 92 pounds pressure above vacuum.

Solution.—From column 5, the total heat of 1 pound at 92 pounds pressure is 1,180.045 B. T. U.

$$1{,}180.045 \times 5 = 5{,}900.225 \text{ B. T. U. Ans.}$$

Example 2.—How many heat units are required to raise $8\frac{1}{2}$ pounds of water from 32° to 250° F.?

Solution.—Looking in column 3, the heat of the liquid of 1 pound at 250.293° is 219.261 B. T. U. $219.261 - .293 = 218.968$ B. T. U. = heat of liquid for 250°. Then, for $8\frac{1}{2}$ pounds it is $218.968 \times 8\frac{1}{2} = 1{,}861.228$ B. T. U. Ans.

Example 3.—How many foot-pounds of work will it require to change 60 pounds of boiling water at 80 pounds pressure, absolute, into steam of the same pressure?

Solution.—Looking under column 4, the latent heat of vaporization is 895.108; that is, it takes 895.108 B. T. U. to change 1 pound of water at 80 pounds pressure into steam of the same pressure. Therefore, it takes $895.108 \times 60 = 53{,}706.48$ B. T. U. to perform the same operation upon 60 pounds of water.

$$53{,}706.48 \times 778 = 41{,}783{,}641.44 \text{ ft.-lb. Ans.}$$

EXAMPLE 4.—Find the volume occupied by 14 pounds of steam at 30 pounds *gauge* pressure.

SOLUTION.— 30 pounds gauge pressure $= 30 + 14.7 = 44.7$ absolute pressure. The nearest pressure in the table is 44 pounds, and the volume of a pound of steam at that pressure is 9.403 cubic feet. The volume of a pound at 46 pounds pressure is 9.018 cubic feet. $9.403 - 9.018 = .385$ cubic foot, the difference in volume for a difference in pressure of 2 pounds. $\frac{.385}{2} = .1925$ cubic foot, the difference in volume for a difference in pressure of 1 pound. $.1925 \times .7 = .135$ cubic foot, the difference in volume for a difference in pressure of .7 pound. Therefore, $9.403 - .135 = 9.268$ cubic feet is the volume of 1 pound of steam at 44.7 pounds pressure. The .135 cubic foot is subtracted from 9.403 cubic feet, since the volume is less for a pressure of 44.7 pounds than for 44 pounds.

$$9.268 \times 14 = 129.752 \text{ cu. ft.} \quad \text{Ans.}$$

EXAMPLE 5.—Find the weight of 40 cubic feet of steam at a temperature of 254° F.

SOLUTION.—The weight of 1 cubic foot of steam at 254.002°, from the table, is .078839 pound. Neglecting the .002°, the weight of 40 cubic feet is, therefore,

$$.078839 \times 40 = 3.15356 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 6.—How many pounds of steam at 64 pounds pressure, absolute, are required to raise the temperature of 300 pounds of water from 40° to 130° F., the water and steam being mixed together?

SOLUTION.—The number of heat units required to raise 1 pound from 40° to 130° is $130 - 40 = 90$ B. T. U. (Actually a little more than 90 would be required, but the above is near enough for all practical purposes.) Then, to raise 300 pounds from 40° to 130° requires $90 \times 300 = 27{,}000$ B. T. U. This quantity of heat must necessarily come from the steam. Now, 1 pound of steam at 64 pounds pressure gives up, in condensing, its latent heat of vaporization, or 905.9 B. T. U. But in addition to its latent heat, each pound of steam on condensing must give up an additional amount of heat in falling to 130°. Since the original temperature of the steam was 296.805° F. (see table), each pound gives up by its fall of temperature $296.805 - 130 = 166.805$ B. T. U. Therefore, each pound of the steam gives up a total of

$$905.9 + 166.805 = 1{,}072.705 \text{ B. T. U.}$$

It will, therefore, take $\frac{27{,}000}{1{,}072.705} = 25.17$ lb. of steam to accomplish the desired result. Ans.

EXAMPLES FOR PRACTICE.

1. How many foot-pounds of work are required to change 42 pounds of water at the temperature corresponding to a pressure of 88 pounds, absolute, into steam at a temperature corresponding to a pressure of 105 pounds, absolute? Ans. 29,208,194.15 lb.

2. How many B. T. U. are required to convert 25 pounds of water at 32° into 109.6 cubic feet of steam? Ans. 29,541.1 B. T. U.

3. Find the number of heat units required to change 11 pounds of water at 32° into steam at 100 pounds absolute pressure. Ans. 13,000.526 B. T. U.

4. Find the weight of 712 cubic feet of steam at a pressure of 33 pounds, gauge. Ans. 81.689 lb.

5. How many pounds of steam at 47.3 pounds pressure, gauge, are required to raise 120 pounds of water from 55° to 160° at atmospheric pressure? Ans. 12.091 lb.

6. Find the volume of 19 pounds of steam at a pressure of 62 pounds, gauge. Ans. 105.952 cu. ft.

THE PROPERTIES OF SATURATED STEAM.

Pressure above Vacuum in Pounds per Square Inch.	Temperature, Fahrenheit Degrees.	Quantities of Heat in British Thermal Units. Required to Raise Temperature of the Water from 32° to $t°$.	Total Latent Heat at Pressure P.	Total Heat above 32°.	Weight of a Cubic Foot of Steam in Pounds.	Volume. Of a Pound of Steam in Cubic Feet.	Ratio of Vol. of Steam to Vol. of Eq. Weight of Dist. Water at Temp. of Maximum Density.
1	2	3	4	5	6	7	8
p	t	q	L	H	W	V	R
1	102.018	70.040	1043.015	1113.055	.003027	330.4	20623
2	126.302	94.368	1026.094	1120.462	.005818	171.9	10730
3	141.654	109.764	1015.380	1125.144	.008522	117.3	7325
4	153.122	121.271	1007.370	1128.641	.011172	89.51	5588
5	162.370	130.563	1000.899	1131.462	.013781	72.56	4530
6	170.173	138.401	995.441	1133.842	.016357	61.14	3816
7	176.945	145.213	990.695	1135.908	.018908	52.89	3302
8	182.952	151.255	986.485	1137.740	.021436	46.65	2912
9	188.357	156.699	982.690	1139.389	.023944	41.77	2607
10	193.284	161.660	979.232	1140.892	.026437	37.83	2361
11	197.814	166.225	976.050	1142.275	.028911	34.59	2159
12	202.012	170.457	973.098	1143.555	.031376	31.87	1990
13	205.929	174.402	970.346	1144.748	.033828	29.56	1845
14	209.604	178.112	967.757	1145.869	.036265	27.58	1721
14.69	212.000	180.531	966.069	1146.600	.037928	26.37	1646
15	213.067	181.608	965.318	1146.926	.038688	25.85	1614
16	216.347	184.919	963.007	1147.926	.041109	24.33	1519
17	219.452	188.056	960.818	1148.874	.043519	22.98	1434
18	222.424	191.058	958.721	1149.779	.045920	21.78	1359
19	225.255	193.918	956.725	1150.643	.048312	20.70	1292

1	2	3	4	5	6	7	8
p	*t*	*q*	*L*	*H*	*W*	*V*	*R*
20	227.964	196.655	954.814	1151.469	.050696	19.73	1231.0
22	233.069	201.817	951.209	1153.026	.055446	18.04	1126.0
24	237.803	206.610	947.861	1154.471	.060171	16.62	1038.0
26	242.225	211.089	944.730	1155.819	.064870	15.42	962.3
28	246.376	215.293	941.791	1157.084	.069545	14.38	897.6
30	250.293	219.261	939.019	1158.280	.074201	13.48	841.3
32	254.002	223.021	936.389	1159.410	.078839	12.68	791.8
34	257.523	226.594	933.891	1160.485	.083461	11.98	748.0
36	260.883	230.001	931.508	1161.509	.088067	11.36	708.8
38	264.093	233.261	929.227	1162.488	.092657	10.79	673.7
40	267.168	236.386	927.040	1163.426	.097231	10.28	642.0
42	270.122	239.389	924.940	1164.329	.101794	9.826	613.3
44	272.965	242.275	922.919	1165.194	.106345	9.403	587.0
46	275.704	245.061	920.968	1166.029	.110884	9.018	563.0
48	278.348	247.752	919.084	1166.836	.115411	8.665	540.9
50	280.904	250.355	917.260	1167.615	.119927	8.338	520.5
52	283.381	252.875	915.494	1168.369	.124433	8.037	501.7
54	285.781	255.321	913.781	1169.102	.128928	7.756	484.2
56	288.111	257.695	912.118	1169.813	.133414	7.496	467.9
58	290.374	260.002	910.501	1170.503	.137892	7.252	452.7
60	292.575	262.248	908.928	1171.176	.142362	7.024	438.5
62	294.717	264.433	907.396	1171.829	.146824	6.811	425.2
64	296.805	266.566	905.900	1172.466	.151277	6.610	412.6
66	298.842	268.644	904.443	1173.087	.155721	6.422	400.8
68	300.831	270.674	903.020	1173.694	.160157	6.244	389.8
70	302.774	272.657	901.629	1174.286	.164584	6.076	379.3
72	304.669	274.597	900.269	1174.866	.169003	5.917	369.4
74	306.526	276.493	898.938	1175.431	.173417	5.767	360.0
76	308.344	278.350	897.635	1175.985	.177825	5.624	351.1
78	310.123	280.170	896.359	1176.529	.182229	5.488	342.6
80	311.866	281.952	895.108	1177.060	.186627	5.358	334.5
82	313.576	283.701	893.879	1177.580	.191017	5.235	326.8
84	315.250	285.414	892.677	1178.091	.195401	5.118	319.5
86	316.893	287.096	891.496	1178.592	.199781	5.006	312.5
88	318.510	288.750	890.335	1179.085	.204155	4.898	305.8

1	2	3	4	5	6	7	8
p	t	q	L	H	W	V	R
90	320.094	290.373	889.196	1179.569	.208525	4.796	299.4
92	321.653	291.970	888.075	1180.045	.212892	4.697	293.2
94	323.183	293.539	886.972	1180.511	.217253	4.603	287.3
96	324.688	295.083	885.887	1180.970	.221604	4.513	281.7
98	326.169	296.601	884.821	1181.422	.225950	4.426	276.3
100	327.625	298.093	883.773	1181.866	.230293	4.342	271.1
105	331.169	301.731	881.214	1182.945	.241139	4.147	258.9
110	334.582	305.242	878.744	1183.986	.251947	3.969	247.8
115	337.874	308.621	876.371	1184.992	.262732	3.806	237.6
120	341.058	311.885	874.076	1185.961	.273500	3.656	228.3
125	344.136	315.051	871.848	1186.899	.284243	3.518	219.6
130	347.121	318.121	869.688	1187.809	.294961	3.390	211.6
135	350.015	321.105	867.590	1188.695	.305659	3.272	204.2
140	352.827	324.003	865.552	1189.555	.316338	3.161	197.3
145	355.562	326.823	863.567	1190.390	.326998	3.058	190.9
150	358.223	329.566	861.634	1191.200	.337643	2.962	184.9
160	363.346	334.850	857.912	1192.762	.358886	2.786	173.9
170	368.226	339.892	854.359	1194.251	.380071	2.631	164.3
180	372.886	344.708	850.963	1195.671	.401201	2.493	155.6
190	377.352	349.329	847.703	1197.032	.422280	2.368	147.8
200	381.636	353.766	844.573	1198.339	.443310	2.256	140.8
210	385.759	358.041	841.556	1199.597	.464295	2.154	134.5
220	389.736	362.168	838.642	1200.810	.485237	2.061	128.7
230	393.575	366.152	835.828	1201.980	.506139	1.976	123.3
240	397.285	370.008	833.103	1203.111	.527003	1.898	118.5
250	400.883	373.750	830.459	1204.209	.547831	1.825	114.0
260	404.370	377.377	827.896	1205.273	.568626	1.759	109.8
270	407.755	380.905	825.401	1206.306	.589390	1.697	105.9
280	411.048	384.337	822.973	1207.310	.610124	1.639	102.3
290	414.250	387.677	820.609	1208.286	.630829	1.585	99.0
300	417.371	390.933	818.305	1209.238	.651506	1.535	95.8

A SERIES OF QUESTIONS AND EXAMPLES

Relating to the Subjects
Treated of in This Volume

It will be noticed that the Examination Questions that follow have been divided into sections, which have been given the same numbers as the Instruction Papers to which they refer. No attempt should be made to answer any of the questions or to solve any of the examples until that portion of the text having the same section number as the section in which the questions or examples occur has been carefully studied.

ARITHMETIC.

(PART 1.)

EXAMINATION QUESTIONS.

(1) What is arithmetic?

(2) What is a number?

(3) What is the difference between a concrete number and an abstract number?

(4) Define notation and numeration.

(5) Write each of the following numbers in words: 980; 605; 28,284; 9,006,042; 850,317,002; 700,004.

(6) Represent in figures the following expressions:

Seven thousand six hundred. Eighty-one thousand four hundred two. Five million four thousand seven. One hundred eight million ten thousand one. Eighteen million six. Thirty thousand ten.

(7) What is the sum of 3,290, 504, 865,403, 2,074, 81, and 7? Ans. 871,359.

(8) 709 + 8,304,725 + 391 + 100,302 + 300 + 909 = what? Ans. 8,407,336.

(9) During a 12-hour test of a steam engine the counter showed the number of revolutions per hour to have been as follows: 12,600, 12,444, 12,467, 12,528, 12,468, 12,590, 12,610, 12,589, 12,576, 12,558, 12,546, 12,532. How many revolutions were made during the test? Ans. 150,508 rev.

(10) Find the difference between the following: (*a*) 50,962 and 3,338; (*b*) 10,001 and 15,339.

Ans. { (*a*) 47,624. (*b*) 5,338.

(11) (*a*) $70,968 - 32,975 = ?$ (*b*) $100,000 - 98,735 = ?$

Ans. { (*a*) 37,993. (*b*) 1,265.

(12) On a certain morning 7,240 gallons of water were drawn from an engine-room tank and 4,780 gallons were pumped in. In the afternoon 7,633 gallons were drawn out and 8,675 gallons pumped in. How many gallons remained in the tank at night, if it contained 3,040 gallons at the beginning of the day? Ans. 1,622 gal.

(13) Find the product of the following: (*a*) $526,387 \times 7$; (*b*) $700,298 \times 17$; (*c*) $217 \times 103 \times 67$.

Ans. { (*a*) 3,684,709. (*b*) 11,905,066. (*c*) 1,497,517.

(14) If your watch ticks once every second, how many times will it tick in one week? Ans. 604,800.

(15) An engine and boiler in a manufactory are worth $3,246. The building is worth three times as much, plus $1,200, and the tools are worth twice as much as the building, plus $1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?

Ans. { (*a*) $34,689. (*b*) $37,935.

(16) Divide the following:

(*a*) 962,842 by 84; (*b*) 39,728 by 63; (*c*) 29,714 by 108; (*d*) 406,089 by 135.

Ans. { (*a*) 11,462.4047. (*b*) 630.603. (*c*) 275.1296. (*d*) 3,008.0666.

(17) Suppose that in one hour 10 pounds of coal are burned per square foot of grate area in a certain boiler,

and that 9 pounds of water are evaporated per pound of coal burned. If the grate area is 30 square feet, how many pounds of water would be evaporated in a day of 10 hours?

Ans. 27,000 lb.

(18) If a mechanic receives $1,500 a year for his labor and his expenses are $968 per year, in what time can he save enough to buy 28 acres of land at $133 an acre?

Ans. 7 years.

(19) Solve the following by cancelation:

(*a*) $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6)$.

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Ans. $\begin{cases} (a) & 9\frac{1}{7}. \\ (b) & 32. \end{cases}$

(20) A freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week? Ans. 849 miles.

(21) If the driving wheel of a locomotive is 16 feet in circumference, how many revolutions will it make in going from Philadelphia to Pittsburg, the distance between them being 354 miles, and there being 5,280 feet in 1 mile?

Ans. 116,820 rev.

ARITHMETIC.

(PART 2.)

EXAMINATION QUESTIONS.

(1) What is a fraction?

(2) What are the terms of a fraction?

(3) What does the denominator show?

(4) What does the numerator show?

(5) Is $\frac{13}{8}$ a proper or an improper fraction, and why?

(6) Write three mixed numbers.

(7) Reduce the following fractions to their lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{8}{32}$, $\frac{32}{64}$. Ans. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$.

(8) Reduce 6 to an improper fraction whose denominator is 4. Ans. $\frac{24}{4}$.

(9) Reduce $7\frac{7}{8}$, $13\frac{5}{16}$, and $10\frac{3}{4}$ to improper fractions.
Ans. $\frac{63}{8}$, $\frac{213}{16}$, $\frac{43}{4}$.

(10) How many feet does the piston of a steam engine pass over in a week of 6 days, running $8\frac{1}{2}$ hours a day, if the length of the stroke of the engine is $1\frac{1}{2}$ feet and the number of revolutions per minute is 160?
Ans. 1,468,800 ft.

(11) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = ?$ Ans. 1.

(12) $\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ Ans. $\frac{15}{16}$.

(13) $42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Ans. $83\frac{1}{16}$.

(14) An iron plate is divided into four sections; the first contains $29\frac{3}{4}$ square inches; the second, $50\frac{5}{8}$ square inches; the third, 41 square inches; and the fourth, $69\frac{3}{16}$ square inches. How many square inches are in the plate?

Ans. $190\frac{9}{16}$ sq. in.

(15) What is the difference between $\frac{7}{8}$ and $\frac{7}{16}$? 13 and $7\frac{7}{16}$? $312\frac{9}{16}$ and $229\frac{5}{32}$?

Ans. $\frac{7}{16}$; $5\frac{9}{16}$; $83\frac{13}{32}$.

(16) Solve the following:

(*a*) $35 \div \frac{5}{16}$; (*b*) $\frac{9}{16} \div 3$; (*c*) $\frac{17}{2} \div 9$; (*d*) $\frac{113}{64} \div \frac{7}{16}$; (*e*) $15\frac{3}{4} \div 4\frac{3}{8}$.

Ans. $\begin{cases} (a) & 112. \\ (b) & \frac{3}{16}. \\ (c) & \frac{17}{18}. \\ (d) & 4\frac{1}{28}. \\ (e) & 3\frac{3}{5}. \end{cases}$

(17) The numerator of a fraction is 28 and the value of the fraction is $\frac{7}{8}$; what is the denominator?

Ans. 32.

(18) Four bolts are required, $2\frac{1}{2}$, $6\frac{7}{8}$, $3\frac{1}{16}$, and 4 inches long. How long a piece of iron will be required from which to cut them, allowing $\frac{7}{16}$ of an inch to each bolt for cutting off and finishing?

Ans. $18\frac{3}{16}$ in.

ARITHMETIC.

(PART 3.)

EXAMINATION QUESTIONS.

(1) State the difference between a common fraction and a decimal fraction.

(2) Reduce the following fractions to equivalent decimals: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{32}$, $\frac{65}{100}$, and $\frac{125}{1000}$.

Ans. $\begin{cases} .5. \\ .875. \\ .15625. \\ .65. \\ .125. \end{cases}$

(3) Write out in words the following numbers: .08, .131, .0001, .000027, .0108, and 93.0101.

(4) What is the sum of .125, .7, .089, .4005, .9, and .000027? Ans. 2.214527.

(5) Add 17 thousandths, 2 tenths, and 47 millionths.
Ans. .217047.

(6) Work out the following examples:

(*a*) $709.63 - .8514$; (*b*) $81.963 - 1.7$; (*c*) $18 - .18$; (*d*) $1 - .001$; (*e*) $872.1 - (.8721 + .008)$; (*f*) $(5.028 + .0073) - (6.704 - 2.38)$.

Ans. $\begin{cases} (a)\ 708.7786. \\ (b)\ 80.263. \\ (c)\ 17.82. \\ (d)\ .999. \\ (e)\ 871.2199. \\ (f)\ .7113. \end{cases}$

(7) The cost of the coal consumed under a nest of steam boilers during a week's run was as follows: Monday, \$15.83; Tuesday, \$14.70; Wednesday, \$14.28; Thursday, \$13.87; Friday, \$14.98; Saturday, \$12.65. What was the cost of the week's supply of coal? Ans. \$86.31.

(8) It is desired to increase the capacity of an electric-light plant to 1,500 horsepower by adding a new engine. If the indicated horsepower of the engines already in use is $482\frac{4}{5}$, $316\frac{1}{3}$, and $390\frac{3}{4}$, what power must the new engine develop? Ans. $310.11\frac{7}{8}$ H. P.

(9) The inside diameter of a 6-inch steam pipe is 6.06 inches, and the outside diameter is 6.62 inches. How thick is the pipe? Ans. .28 in.

(10) Find the products of the following expressions: (*a*) $.013 \times .107$; (*b*) $203 \times 2.03 \times .203$; (*c*) $(2.7 \times 31.85) \times (3.16 - .316)$; (*d*) $(107.8 + 6.541 - 31.96) \times 1.742$.

Ans. (*a*) .001391. (*b*) 83.65427. (*c*) 244.56978. (*d*) 143.507702.

(11) How many square feet of heating surface are in the tubes of a boiler having sixty 3-inch tubes, each $15\frac{1}{2}$ feet long, if the heating surface of each tube per foot in length is .728 square foot? Ans. 677.04 sq. ft.

(12) Find the values of the following expressions:

(*a*) $\dfrac{7}{\frac{3}{16}}$; (*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}}$; (*c*) $\dfrac{1.25 \times 20 \times 3}{\dfrac{87 + 88}{459 + 32}}$.

Ans. (*a*) $37\frac{1}{3}$. (*b*) .75. (*c*) $210\frac{3}{7}$.

(13) The distance around a cylindrical boiler is 166.85 inches. If there are 72 rivets in one of the circular seams, find what the pitch (distance between the centers of any two rivets) of the rivets is. Ans. 2.317+ in.

(14) A keg of $\frac{7}{8}'' \times 2\frac{3}{4}''$ boiler rivets weighs 100 pounds and contains 133 rivets. What is the weight of each rivet? Ans. .75+ lb.

(15) How many inches in .875 of a foot ? Ans. $10\frac{1}{2}$ in.

(16) What decimal part of a foot is $\frac{3}{16}$ of an inch ?
Ans. .015625 ft.

(17) In a steam-engine test of an hour's duration, the horsepower developed was found to be as follows at 10-minute intervals: 48.63, 45.7, 46.32, 47.9, 48.74, 48.38, 48.59. What was the average horsepower ? Ans. 47.75+ H. P.

ARITHMETIC.

(PART 4.)

EXAMINATION QUESTIONS.

(1) What is 25% of 8,428 lb.? Ans. 2,107 lb.

(2) What is $\frac{1}{2}$% of \$35,000? Ans. \$175.

(3) What per cent. of 50 is 2? Ans. 4%.

(4) What per cent. of 10 is 10? Ans. 100%.

(5) The coal consumption of a steam plant is 5,500 lb. per day when the condenser is not running, or an increase of 15% over the consumption when the condenser is used. How many pounds are used per day when the condenser is running? Ans. 4,782.61 lb.

(6) An engineer receives a salary of \$950. He pays 24% of it for board, 12$\frac{1}{2}$% of it for clothing, and 17% of it for other expenses. How much of it does he save a year?

Ans. \$441.75.

(7) If 37$\frac{1}{2}$% of a number is 961.38, what is the number?

Ans. 2,563.68.

(8) The speed of an engine running unloaded was 1$\frac{1}{2}$% greater than when running loaded. If it made 298 revolutions per minute with the load, what was its speed running unloaded? Ans. 302.47 rev. per min.

(9) Reduce 4 yd. 2 ft. 10 in. to inches. Ans. 178 in.

(10) Reduce 3,722 in. to higher denominations.

Ans. 103 yd. 1 ft. 2 in.

(11) Reduce 764,325 cu. in. to cubic yards.

Ans. 16 cu. yd. 10 cu. ft. 549 cu. in.

(12) A carload of coal weighed 16 T. 8 cwt. 75 lb. How many pounds did this amount to? Ans. 32,875 lb.

(13) Reduce 25,396 lb. to higher denominations.

Ans. 12 T. 13 cwt. 96 lb.

(14) What is the sum of 2 yd. 2 ft. 3 in., 4 yd. 1 ft. 9 in., and 2 ft. 7 in.? Ans. 8 yd. 7 in.

(15) From a barrel of machine oil is sold at one time 10 gal. 2 qt. 1 pt. and at another time 16 gal. 3 qt. How much remained? Ans. 4 gal. 1 pt.

(16) If 1 iron rail is 17 ft. 3 in. long, how long would 51 such rails be if placed end to end? Ans. 879 ft. 9 in.

(17) Multiply 3 qt. 1 pt. by 4.7. Ans. 32.9 pt.

(18) A main line shaft is composed of four lengths each 15 ft. 5 in. long; one length 14 ft. 8 in. long; and one length 8 ft. 10 in. long. There are six hangers spaced equally distant apart, one being placed at each extremity of the shaft, 8 in. from the ends. What is the distance between the hangers? Ans. 16 ft. $9\frac{1}{5}$ in.

(19) If the length of a boiler shell is 18 ft. $11\frac{1}{4}$ in., how many rivets should there be in one of the longitudinal seams if it is a single-riveted seam, supposing the rivets to be $1\frac{1}{4}$ in. between centers and the two end rivets to be $1\frac{1}{8}$ in. from each end of the boiler? Ans. 181 rivets.

ARITHMETIC.

(PART 5.)

EXAMINATION QUESTIONS.

(1) What is the square of 108? Ans. 11,664.

(2) Find the fifth power of 9. Ans. 59,049.

(3) What is the value of $.0133^3$? Ans. .000002352637.

(4) Extract the square root of 90. Ans. 9.4868.

(5) What is the cube root of 92,416? Ans. 45.211.

(6) Find the value of $\sqrt{502,681}$. Ans. 709.

(7) What is the value of $\sqrt[3]{\frac{3}{8}}$? Ans. .72112.

(8) Extract the square root of .3364. Ans. .58.

(9) Find the square root of .7854. Ans. .88623.

(10) What number multiplied by itself equals 114.9184? Ans. 10.72.

(11) Extract the square root of 3,486,784. Ans. 1,867.3.

(12) Find the square root of .00041209. Ans. .0203.

Find the value of x in the following:

(13) $11.7 : 13 :: 20 : x$. Ans. 22.22+.

(14) (*a*) $20 + 7 : 10 + 8 :: 3 : x$; (*b*) $12^2 : 100^2 :: 4 : x$.

Ans. $\begin{cases} (a) & 2. \\ (b) & 277.7+ \end{cases}$

(15) (*a*) $\frac{4}{x} = \frac{7}{21}$; (*b*) $\frac{x}{24} = \frac{8}{16}$; (*c*) $\frac{2}{10} = \frac{x}{100}$; (*d*) $\frac{15}{45} = \frac{60}{x}$; (*e*) $\frac{10}{150} = \frac{x}{600}$.

Ans. $\begin{cases} (a) & 12. \\ (b) & 12. \\ (c) & 20. \\ (d) & 180. \\ (e) & 40. \end{cases}$

(16) If a piece of 2-inch shafting $3\frac{1}{2}$ ft. long weighs 37.45 lb., how much would a piece $6\frac{3}{4}$ ft. long weigh?

Ans. 72.225 lb.

(17) If a railway train runs 444 miles in 8 hr. 40 min., in what time can it run 1,060 miles at the same rate of speed?

Ans. 20 hr. 41.44 min.

(18) If a pump discharging 135 gal. per min. fills a tank in 38 min., how long would it take a pump discharging 85 gal. per min. to fill it?

Ans. $60\frac{6}{17}$ min.

(19) The distances around the driving wheels of two locomotives are 12.56 ft. and 15.7 ft., respectively. How many times will the larger turn while the smaller turns 520 times?

Ans. 416 times.

(20) If a cistern 28 ft. long, 12 ft. wide, 10 ft. deep holds 798 bbl. of water, how many barrels of water will a cistern hold that is 20 ft. long, 17 ft. wide, and 6 ft. deep?

Ans. $484\frac{1}{2}$ bbl.

MENSURATION AND USE OF LETTERS IN FORMULAS.

EXAMINATION QUESTIONS.

$A = 5 \qquad h = 200$

$B = 10 \qquad x = 12$

$i = 3.5 \qquad D = 120$

Work out the solutions to the following formulas, using the above values for the letters:

(1) $C = \dfrac{D - x}{B + i}$. Ans. $C = 8$.

(2) $Q = \dfrac{A\,h + D}{2\,x + 6} + D$. Ans. $Q = 157\tfrac{1}{3}$.

(3) $v = \sqrt{\dfrac{A\,D}{i\,B + 1.5}}$. Ans. $v = 4.05+$.

(4) $g = \dfrac{(B - A)^2 - \sqrt{h + 2\,B + A}}{A^3 - (1 + D)}$. Ans. $g = 2\tfrac{1}{2}$.

(5) If one of the angles formed by one straight line meeting another straight line equals 152° 3′, what is the other angle equal to? Ans. 27° 57′.

(6) Draw an obtuse angle, a right angle, and an acute angle. State the name of each angle by using letters to designate them.

(7) Draw a rhombus and then draw a rectangle having the same area.

(8) A sheet of zinc measures $11\frac{1}{2}$ inches by $2\frac{1}{2}$ feet. How many square inches does it contain? Ans. 345 sq. in.

(9) How many boards 16 feet long and 5 inches wide would be required to lay a floor measuring 15 ft. × 24 ft.?

Ans. 54 boards.

(10) The accompanying figure shows the floor plan of an electric-light station. From the dimensions given, calculate the number of square feet of unoccupied floor space.

Ans. 2,059.08 sq. ft.

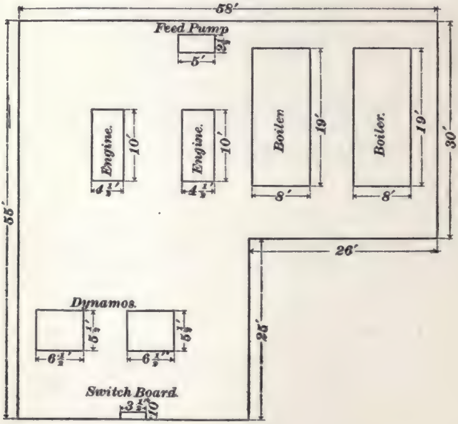


FIG. I.

(11) A triangle has three equal angles; what is it called?

(12) If a triangle has two equal angles, what kind of a triangle is it?

(13) In a triangle $A\ B\ C$, angle $A = 23°$ and $B = 32°\ 32'$; what does angle C equal? Ans. $C = 124°\ 28'$.

(14) In the figure, if $AD = 10$ inches, $AB = 24$ inches, and $BC = 13\frac{1}{2}$ inches, how long is DE, DE being parallel to BC? Ans. $DE = 5.625$ in.

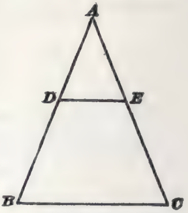


FIG. II.

(15) An engine room is 52 feet long and 39 feet wide. How many feet is it from one corner to a diagonally opposite one, measured in a straight line? Ans. 65 ft.

(16) It is required to make a miter-box in which to cut molding to fit around an octagon post. At what angle with the side of the box should the saw run? Ans. $67\frac{1}{2}°$.

(17) If the distance between two opposite corners of a hexagonal nut is 2 inches, what is the distance between two opposite sides? Ans. 1.732+ in.

(18) In the accompanying figure, if the distance BI is 6 inches and HK 18 inches, what is the diameter of the circle? Ans. 19.5 in.

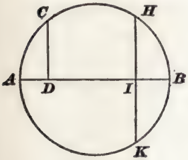


FIG. III.

(19) How many revolutions will a 72-inch locomotive driver make in going 1 mile? Ans. 280.112 revolutions.

(20) A pipe has an internal diameter of 6.06 inches; what is the area of a circle having this diameter? Ans. 28.8427 sq. in.

(21) How long must the arc of a circle be to contain 12°, supposing the radius of the circle to be 6 inches? Ans. 1.25664 in.

(22) What is the area of the sector of a circle 15 inches in diameter, the angle between the two radii forming the sector being $12\frac{1}{2}°$? Ans. 6.1359 sq. in.

(23) (*a*) What would be the length of the side of a square metal plate having an area of 103.8691 square inches?

(*b*) What would be the diameter of a round plate having this area? (*c*) How much shorter is the circumference of the round plate than the perimeter of the square plate?

Ans. $\begin{cases} (a) & 10.1916 \text{ in.} \\ (b) & 11\frac{1}{2} \text{ in.} \\ (c) & 4.638 \text{ in.} \end{cases}$

(24) Find the area in square feet of the entire surface of a hexagonal column 12 feet long, each edge of the ends of the column being 4 inches long. Ans. 24.5774 sq. ft.

(25) Find the cubical contents of the above column in cubic inches. Ans. 5,985.9648 cu. in.

(26) Compute the weight per foot of an iron boiler tube 4 inches outside diameter and 3.73 inches inside diameter, the weight of the iron being taken at .28 pound per cubic inch. Ans. $5\frac{1}{2}$ lb.

(27) The dimensions of a return-tubular boiler are as follows: Diameter, 60 inches; length between heads, 16 feet; outside diameter of tubes, $3\frac{1}{2}$ inches; number of tubes, 64; distance of mean water-line from top of boiler, 18 inches. (*a*) Compute the steam space of the boiler in cubic feet. (*b*) Determine the number of gallons of water that will be required to fill the boiler up to the mean water level.

Ans. $\begin{cases} (a) & 79.2 \text{ cu. ft.} \\ (b) & 1{,}246 \text{ gal., nearly.} \end{cases}$

(28) The length of the circumference of the base of a cone is 18.8496 inches and its slant height is 10 inches. Find the area of the entire surface of the cone.

Ans. 122.5224 sq. in.

(29) If the altitude of the above cone were 9 inches, what would be its volume? Ans. 84.8232 cu. in.

(30) A square vat is 11 feet deep, 15 feet square at the top, and 12 feet square at the bottom. How many gallons will it hold? Ans. 15,058.29 gal.

(31) How many pails of water would be required to fill the vat, the pail having the following dimensions: Depth,

11 inches; diameter at the top, 12 inches; diameter at the bottom, 9 inches? Ans. 3,627.28.

(32) Find (*a*) the area of the surface, and (*b*) the cubical contents of a ball $22\frac{1}{2}$ inches in diameter.

Ans. $\begin{cases} (a) & 1{,}590.435 \text{ sq. in.} \\ (b) & 5{,}964.1313 \text{ cu. in.} \end{cases}$

(33) (*a*) What is the volume and area of a cylindrical ring whose outside diameter is 16 inches and inside diameter 13 inches? (*b*) If made of cast iron, what is its weight? Take the weight of 1 cubic inch of cast iron as .261 pound.

Ans. Weight = 21 lb.

PRINCIPLES OF MECHANICS.

EXAMINATION QUESTIONS.

(1) Define mechanics.

(2) Define (*a*) matter; (*b*) molecule; (*c*) atom.

(3) In what three states does matter exist?

(4) Explain clearly the distinction between general properties of matter and special properties of matter.

(5) Define (*a*) motion; (*b*) velocity; (*c*) uniform velocity; (*d*) variable velocity.

(6) Define (*a*) acceleration; (*b*) retardation; (*c*) average velocity.

(7) An ocean steamer made a run of 3,240 miles in 6 days and 16 hours. What was its average speed in miles per hour? Ans. $20\frac{1}{4}$ mi.

(8) How long would it take to make a tour around the world when traveling at an average speed of 3,000 feet per minute, assuming the distance around the world to be 25,000 miles? Ans. 30 da. 13 hr. 20 min.

(9) How do we recognize the existence of a force?

(10) What conditions are necessary to compare the relative effects of different forces on different bodies?

(11) State Newton's Laws of Motion.

(12) What do you understand by the term "inertia"?

(13) Define (*a*) dynamics; (*b*) statics.

(14) Why is not the weight of a given body the same at every point on the surface of the earth?

(15) Determine the mass of a body that weighs 346 pounds at a place where g is equal to 32.174. Ans. 10.75+.

(16) How does the position of a body above or below the surface of the earth affect its weight?

(17) A locomotive weighing 30 tons has to overcome a constant force of 15 pounds per ton when it is in motion. What total force must the locomotive exert so that its speed may increase at the rate of 3 feet per second?

Ans. 6,047 lb.

(18) What will be the momentum of the locomotive in the preceding example when it has attained a velocity of 1 mile per hour? Ans. 2,736.3 lb.

(19) Define (*a*) work; (*b*) power; (*c*) energy.

(20) What horsepower is required to raise a body weighing 66,000 pounds through a distance of 80 feet in $\frac{1}{2}$ hour?

Ans. $5\frac{1}{3}$ H. P.

(21) A body weighing 6,432 pounds is moving with a constant velocity of 60 feet per second. What horsepower will be required to bring the body to rest in 3 minutes?

Ans. $3\frac{7}{11}$ H. P.

(22) What is the tangential pressure on the crank of an engine when the crank is on either dead center?

(23) What is friction?

(24) A body weighing 5,000 pounds rests on a horizontal surface. In order to slide the body along the surface, a horizontal force of 300 pounds must be exerted. What is the coefficient of friction in this particular case? Ans. .06.

(25) A crosshead weighing 1,000 pounds and having bronze shoes slides on a slightly greased, horizontal wrought-iron guide. If the contact area of the bronze shoe is 100 square inches, what will be (*a*) the total friction and (*b*) the friction per square inch of contact surface?

Ans. $\begin{cases} (a) & 160 \text{ lb.} \\ (b) & 1.6 \text{ lb.} \end{cases}$

(26) What is the center of gravity of a body?

(27) Explain the method of finding the common center of gravity of several bodies whose weights and the distances between whose centers of gravity are known.

(28) Give a practical method of determining the center of gravity of a solid body.

(29) Define (*a*) centrifugal force; (*b*) centripetal force.

(30) What is the relation between the centrifugal and centripetal forces of a revolving body?

(31) A body weighing 10 pounds revolves at a speed of 60 revolutions per minute about a point 6 feet from its center of gravity. What is the centrifugal force tending to pull the body from the point about which it revolves?

Ans. 73.44 lb.

(32) Name the three states of equilibrium and give examples of each.

(33) In the case of a body at rest, what are the conditions of the forces acting on that body?

(34) How is the condition of equilibrium of the forces acting on a body affected by the position of its "line of direction" with respect to the base?

MACHINE ELEMENTS.

EXAMINATION QUESTIONS.

(1) Define (*a*) lever; (*b*) weight arm; (*c*) force arm; (*d*) fulcrum.

(2) What is the condition necessary for the equilibrium of the lever?

(3) State the general rule that expresses the relation existing between the weight, the force, and the distances through which they move.

(4) What must be the length of the weight arm in order that a force of 12 pounds at a distance of 20 inches from the fulcrum will raise a weight of 100 pounds at the end of the weight arm? Ans. 2.4 in.

(5) Into what two classes may pulleys be divided in reference to their construction?

(6) What advantages have split pulleys over solid pulleys?

(7) Explain how a crowned pulley tends to prevent the slipping off of the belt.

(8) What is meant by "balancing pulleys," and how is it accomplished?

(9) Define the terms "driver" and "driven" as applied to pulleys.

(10) Find what diameter driver will be required which, when running at 600 revolutions per minute, will cause the driven, whose diameter is 6 inches, to run at 1,800 revolutions per minute. Ans. 18 in.

(11) What must be the speed of a driver 12 inches in diameter in order that the driven, whose diameter is 5 inches, may make 1,600 revolutions per minute?

Ans. $666\frac{2}{3}$ rev. per min.

(12) The distance between the centers of two pulleys, whose diameters are 6 feet and 2 feet, respectively, is 40 feet. What is the required length of an open belt? Ans. 93 ft.

(13) A single belt running at 1,650 feet per minute is used to transmit 40 horsepower. If the arc of contact on the small pulley is 120°, how wide should the belt be?

Ans. 28 in.

(14) What horsepower will be transmitted by the belt in question 13 if the velocity is reduced to 1,200 feet per minute? Ans. 29.3 H. P.

(15) If, in question 13, a double belt were substituted for the single belt, how wide should it be? Ans. 20 in.

(16) Which side of a leather belt should be in contact with the pulley face, and why?

(17) Should rosin be used on a belt to prevent slipping?

(18) Give the causes of flapping belts and their remedies.

(19) State the different methods used in joining belts. Which of these makes the best joint?

(20) What precaution should be observed when using rubber belts?

(21) Define the terms "driver" and "follower" as applied to a train of gear-wheels.

(22) In Fig. 19, if the diameter of A is 90 inches, that of F 30 inches, and if B has 12 teeth, C 30 teeth, D 20 teeth, and E 36 teeth, find the weight that a force of 50 pounds at P can raise. Ans. 675 lb.

(23) What is (*a*) circular pitch? (*b*) diametral pitch?

(24) What are the most common forms of teeth used in ordinary practice?

(25) What advantages have involute teeth over epicycloidal teeth?

(26) Find the pitch diameter of a gear-wheel having 60 teeth and a circular pitch of 1.152 inches. Ans. 22 in.

(27) What is the circular pitch of a gear-wheel 30 inches in diameter having 60 teeth? Ans. 1.57 in.

(28) What is the over-all diameter of a gear-wheel having 80 teeth with a diametral pitch of 8? Ans. $10\frac{1}{4}$ in.

(29) Find the number of teeth in a gear-wheel whose outside diameter is $7\frac{1}{2}$ inches and whose diametral pitch is 8. Ans. 58 teeth.

(30) What distinguishes a fixed pulley from a movable pulley?

(31) In a set of pulleys there are fourteen parts of the rope supporting the load. Neglecting friction losses, what weight can a force of 150 pounds raise, when applied to the free end of the rope? Ans. 2,100 lb.

(32) How does friction affect the force required to raise a given weight by means of a rope or chain and pulleys?

(33) In a block and tackle having eight parts of the rope supporting the load, what probable actual force is required to raise 1,500 pounds? Ans. 312.5 lb.

(34) In what respect is the Weston differential pulley block better than the ordinary block and tackle?

(35) An inclined plane is 70 feet long and 12 feet high. What force acting parallel to the plane will be required to sustain a weight of 600 pounds on the plane? Ans. 103 lb.

(36) What weight will a force of 36 pounds acting parallel to the base of the plane be able to sustain on an inclined plane having a base 50 feet long and a height of 15 feet?

Ans. 120 lb.

(37) What is the probable actual weight that can be raised by means of a screw jack that has a screw $2\frac{1}{2}$ inches in diameter with 6 threads to the inch if a force of 50 pounds is applied at the end of a lever 20 inches from the shaft?

Ans. 2,356.2 lb.

(38) Define (*a*) velocity ratio; (*b*) efficiency.

(39) What is the efficiency of the screw jack of question 37?

Ans. $6\frac{1}{4}$ per cent.

MECHANICS OF FLUIDS.

EXAMINATION QUESTIONS.

(1) Define hydrostatics.

(2) How can it be proved that liquids transmit pressure in all directions and with the same intensity?

(3) State Pascal's law.

(4) In what direction does the pressure due to the weight of a body of water act in a vessel in which water is contained?

(5) What is the pressure on the bottom of a vessel if the base is a circle 3 inches in diameter, the height 8 inches, and the vessel is completely filled with water? Ans. 2.045 lb.

(6) State the law for the upward pressure of a liquid on a horizontal surface submerged in the liquid.

(7) Why is it that in several pipes that communicate with one another and that differ in shape and size, water will stand at the same height?

(8) Why is it that water issuing from a hose connected to a hydrant cannot be made to spout up to a level with the surface of the water in the reservoir that supplies the hydrant?

(9) Define specific gravity.

(10) Calculate the weight of a block of aluminum whose volume is 100 cubic feet. Ans. 15,605 lb.

(11) State the principle of Archimedes.

(12) Explain how, by the application of the principle of Archimedes, the volume of an irregularly shaped body may be accurately determined.

(13) (*a*) What are hydrometers? (*b*) Into what two classes may hydrometers be divided?

(14) A single-cylinder pump feeds a boiler through a delivery pipe 1 inch in actual diameter. The piston speed is such as to give a velocity of flow of 400 feet per minute. How many gallons of water can be pumped into the boiler in 1 hour? Ans. 979.2 gal.

(15) What should be the commercial size of a delivery pipe from a duplex pump to deliver 936 gallons of water per hour? Ans. 1 in.

(16) Why is it important to have as few bends as possible in the suction pipe leading to a pump?

(17) What simple experiment proves that gases tend to expand and increase their volume?

(18) How high a column of mercury will the pressure of the atmosphere support?

(19) How can the degree of the vacuum in a vessel be determined?

(20) How high a column of a liquid whose specific gravity is 2.5 will the atmospheric pressure support? Ans. 163.2 in.

(21) How is the pressure of the atmosphere measured?

(22) Why does the pressure of the atmosphere decrease as the elevation above sea level increases?

(23) In what respect is the action of the pressure of the atmosphere similar to that of the pressure of a liquid?

(24) How does the tension of a gas change with the change in volume under constant temperature?

(25) Define (*a*) gauge pressure; (*b*) absolute pressure.

(26) A metallic tube closed at one end is fitted with an air-tight movable piston. When the piston is at the open end of the tube, the pressure of the air within the tube is equal to 14.7 pounds per square inch. What will be the pressure of the air between the piston and the closed end of the tube after the former has moved toward the latter a distance equal to $\frac{4}{5}$ the length of the tube, the temperature remaining constant? Ans. 73.5 lb. per sq. in.

(27) Suppose that the piston in the tube mentioned in question 26 had been moved toward the closed end until the pressure had reached 147 pounds per square inch. What fraction of the original volume would the compressed air have occupied? Ans. $\frac{1}{10}$.

(28) In pneumatics, what is an air pump?

(29) Can a perfect vacuum be produced with the air pump?

(30) Explain the operation of the dashpot of a Corliss engine.

(31) Explain the principle of operating the siphon.

(32) Suppose that two vessels, in one of which the water stands at a higher level than in the other, are connected by a siphon and water is siphoned from the vessel in which it stands at the higher level into the other vessel. What will happen when the water in each vessel reaches the same level?

(33) In practice, how far above the water level in the vessel from which water is siphoned can the highest point of the siphon be carried?

(34) (*a*) What is a pump? (*b*) Into how many types may pumps be divided in reference to their mode of action? (*c*) Name these types.

(35) Explain the principle on which the suction pump acts.

(36) What advantage has a lifting pump over a suction pump?

(37) In what respect does a force pump differ from a lifting pump?

(38) What is the difference between a single-acting and a double-acting force pump?

STRENGTH OF MATERIALS.

EXAMINATION QUESTIONS.

(1) (*a*) Define stress. (*b*) Name the various kinds of stresses to which a body can be subjected.

(2) A weight of 8,000 pounds rests on the top of a cubical block of wood the area of each face of which is 20 square inches. What is the unit stress, in pounds per square inch, to which the block is subjected? Ans. 400 lb. per sq. in.

(3) Define (*a*) strain; (*b*) elasticity; (*c*) elastic limit.

(4) Taking the ultimate tensile strength of wrought iron as 55,000 pounds per square inch, what tensile force will rupture a wrought-iron bar whose cross-sectional area is 4 square inches? Ans. 220,000 lb.

(5) How does annealing improve old chains?

(6) Give a practical method of determining the true condition of a given rope, supposing that its outer surface appears to be in good condition.

(7) (*a*) On what does the safe lifting load of a sling depend? (*b*) Explain how the style of attachment to the hook of the tackle block affects the amount of load to be raised.

(8) (*a*) For what is manila rope chiefly used? (*b*) What is the object of lubricating the fibers of a rope?

(9) In general, how should the size of the pulley compare with the size of manila rope used for transmitting power ?

(10) What is the greatest load to which an iron-wire rope $1\frac{1}{2}$ inches in circumference should be subjected ?

Ans. 1,350 lb.

(11) What should be the circumference of a steel-wire rope under a maximum working load of 16,000 pounds ?

Ans. 4 in., nearly.

(12) How does the strength of a column having both ends flat compare with that of columns both of whose ends are not flat but which in every other respect are similar to the first column ?

(13) Give practical examples of the three different classes into which columns are divided with respect to the condition of their ends.

(14) What is the safe steady working load that a cast-iron column, having fixed ends, 14 inches in diameter and 16 feet high can sustain ? Ans. 1,291,480 lb.

(15) Would you consider a steel piston rod 6 inches in diameter of sufficient size for a 40-inch cylinder using steam at 110 pounds pressure ? If so, why ?

(16) A solid yellow-pine beam 14 inches square rests on two supports 12 feet apart. What steady safe load will the beam support at its middle point ? Ans. 18,547 lb.

(17) An oak beam 4 inches wide and 6 inches deep is subjected to a sudden shearing stress of 10,000 pounds across the grain. Is this a safe load for the given conditions ?

(18) What should be the area of a wrought-iron beam to safely support a sudden shearing stress of 400,000 pounds ?

Ans. 91 sq. in., nearly.

(19) What do you understand by double shear ?

(20) What is the use of countershafts ?

(21) What is the distinction between cold-rolled shafting and bright shafting ?

(22) How is bright shafting designated commercially with respect to size?

(23) Why is it good practice to place pulleys for transmitting or receiving power as near the bearings of the shaft as possible?

(24) What horsepower can be transmitted from a steel shaft 8 inches in diameter by means of pulleys between the bearings when running at a speed of 80 revolutions per minute? Ans. 482 H. P.

(25) What must be the speed of a 4-inch shaft of cold-rolled iron, having no pulleys between bearings, to transmit 75 horsepower? Ans. 76 rev.

(26). Find the diameter of a wrought-iron shaft running at 300 revolutions per minute to transmit 84 horsepower by means of pulleys between its bearings. Ans. 3 in.

(27) How does a change in the speed of a shaft affect the amount of power transmitted?

(28) Does a high tensile strength in metals necessarily imply an ability on the part of the metal to safely resist repeated applications of sudden stresses?

(29) What is the chief advantage of a steel rope over an iron rope?

ELEMENTS OF ELECTRICITY AND MAGNETISM.

EXAMINATION QUESTIONS.

(1) Fig. I represents a helix of wire around which an electric current is supposed to be circulating in the direction indicated by the arrows. Which of the two ends, *a* or *b*, is the north pole of the solenoid? Why?

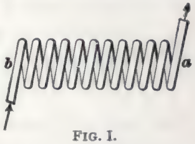


Fig. I.

(2) What will be the sign of the static charge developed (*a*) on a glass rod when rubbed with fur? (*b*) on a piece of sealing-wax when rubbed with silk?

(3) The separate resistances of two branches *A* and *B* of a derived, or shunt, circuit are 16.2 and 14.1 ohms, respectively. If the sum of the currents in the two branches is 6.37 amperes, what is the current in each branch?

Ans. { 2.9643 amperes in branch *A*.
3.4057 amperes in branch *B*.

(4) In a closed circuit, the resistance between two points is 2.3 ohms. (*a*) What current flowing between these points will cause a difference of potential of 58.4 volts?

ELEMENTS OF ELECTRICITY AND MAGNETISM.

EXAMINATION QUESTIONS.

(1) Fig. I represents a helix of wire around which an electric current is supposed to be circulating in the direction indicated by the arrows. Which of the two ends, *a* or *b*, is the north pole of the solenoid? Why?

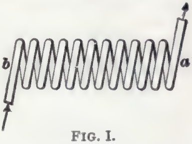


FIG. I.

(2) What will be the sign of the static charge developed (*a*) on a glass rod when rubbed with fur? (*b*) on a piece of sealing-wax when rubbed with silk?

(3) The separate resistances of two branches *A* and *B* of a derived, or shunt, circuit are 16.2 and 14.1 ohms, respectively. If the sum of the currents in the two branches is 6.37 amperes, what is the current in each branch?

Ans. { 2.9643 amperes in branch *A*.
3.4057 amperes in branch *B*.

(4) In a closed circuit, the resistance between two points is 2.3 ohms. (*a*) What current flowing between these points will cause a difference of potential of 58.4 volts?

(*b*) What is the power in watts dissipated between these two points? (*c*) Give its equivalent in horsepower.

Ans. $\begin{cases} (a) & 25.3913 \text{ amperes.} \\ (b) & 1{,}482.8521 \text{ watts.} \\ (c) & 1.9877 \text{ horsepower.} \end{cases}$

(5) Fig. II represents a closed circuit consisting of a voltaic battery B and two conductors X and Y connected in series. The internal resistance of the battery is 17.2 ohms, and the separate resistances of the conductors X and Y are, respectively, 8.2 and 11.3 ohms. What is the total E. M. F. in volts generated by the battery if a current of .75 ampere flows through the circuit? Find the difference of potential in volts between a and b, between b and c, and between c and a.

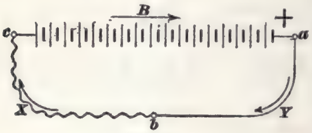


FIG. II.

Ans. $\begin{cases} \text{Total E. M. F. developed by battery} = 27.525 \text{ volts.} \\ \text{Difference of potential between } a \text{ and } b = 8.475 \text{ volts.} \\ \text{Difference of potential between } b \text{ and } c = 6.15 \text{ volts.} \\ \text{Difference of potential between } c \text{ and } a = 14.625 \text{ volts.} \end{cases}$

(6) A voltaic battery whose internal resistance is 36.2 ohms is connected to a copper wire having a resistance of 21.7 ohms. What is the total electromotive force in volts generated in the battery if a current of .127 ampere flows through the circuit? Ans. 7.3533 volts.

(7) How many coulombs of electricity pass through a circuit in $2\frac{1}{4}$ hours when the strength of current is 8.32 amperes? Ans. 67,392 coulombs.

(8) Given, electromotive force = 112.5 volts and strength of current = 12.2 amperes; find the power in watts.

Ans. 1,372.5 watts.

(9) The resistance of a copper wire is 43.2 ohms at 60° F.; find its resistance at 85° F. Ans. 45.5274 ohms.

(10) The separate resistances of three conductors A, B, and C are, respectively, 37, 45, and 72 ohms. What is their joint resistance when connected in parallel?

Ans. 15.8383 ohms.

(11) The separate resistances of four conductors A, B, C, and D are, respectively, 3, 19, 72, and 111 ohms; find their joint resistance when connected in series. Ans. 205 ohms.

(12) How much energy in *joules* is expended in a closed circuit during $1\frac{1}{4}$ hours in which the current is maintained at 14.2 amperes, the resistance of the circuit being 8 ohms?

Ans. 7,259,040 joules.

(13) In Fig. III, the difference of potential between a and b is 11.6 volts. If the strength of the current in branch A is 6.7 amperes and the strength of the current in B is 4.9 amperes, what is the separate resistance of each branch?

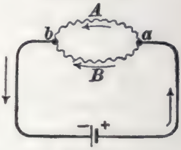


FIG. III.

Ans. $\begin{cases} \text{The separate resistance of branch } A = 1.7313 \text{ ohms.} \\ \text{The separate resistance of branch } B = 2.3673 \text{ ohms.} \end{cases}$

(14) The E. M. F. of a battery is 22.4 volts and its internal resistance is 13.4 ohms. What is the resistance of an external conductor connected to the battery when the current flowing in the circuit is .43 ampere?

Ans. 38.693 ohms.

(15) What must have been the strength of current in amperes in a closed circuit through which 368,422 coulombs of electricity passed in $4\frac{1}{2}$ hours? Ans. 22.7421 amperes.

(16) Find the work done in foot-pounds when a current of 2.4 amperes flows against a resistance of 45 ohms for 50 minutes. Ans. 573,324.48 foot-pounds.

(17) Given, the electromotive force = 525 volts and strength of current = 12.5 amperes; express the number of horsepower. Ans. 8.7969 horsepower.

(18) (*a*) How many watts are dissipated by a current of 110 amperes flowing against a resistance of 4.2 ohms? (*b*) Give its equivalent in horsepower.

Ans. { (*a*) 50,820 watts.
(*b*) 68.1233 horsepower.

(19) If the resistance of 1,000 feet of round copper wire .1 inch in diameter is 1 ohm, find the resistance of 2,000 feet of square copper wire .1 inch on a side. Ans. 1.5708 ohms.

(20) Find the equivalent of 54,200 watts in horsepower.
Ans. 72.6541 horsepower.

(21) The specific resistance of mercury is 37.15 microhms per cubic inch; find the resistance in ohms of a round column of mercury 72.3 inches high and .04 inch in diameter, at 32° F. Ans. 2.1368 ohms.

(22) The total E. M. F. developed within a battery is 45 volts and the internal resistance of the battery is 33 ohms; find the strength of current flowing when the battery is connected in circuit with a resistance of 30 ohms.

Ans. .7143 ampere.

(23) An E. M. F. of 510 volts is consumed in an electric receptive device and a current of 24.3 amperes is flowing in the circuit; calculate the power in watts supplied to the receptive device. Ans. 12,393 watts.

(24) A battery of twenty-four cells is arranged in multiple-series as shown in Fig. IV. There are four groups of six cells each, connected in series, and the four groups are connected in multiple, or parallel, to two main conductors *c* and *c'*. If the E. M. F. developed by each cell is 1.5 volts, what will be the E. M. F. indicated by the voltmeter *V M* when its binding posts are connected to the main conductors *c* and *c'*, as shown in the figure?

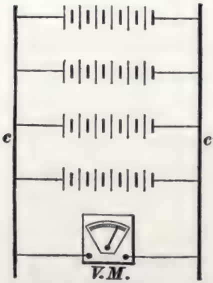


FIG. IV.

(25) The available E. M. F. developed by an electric source is 250 volts and a current of 65.7 amperes is flowing from it; determine its output in horsepower.

Ans. 22.0174 horsepower.

(26) Fig. V represents a horseshoe electromagnet M around which is wound an insulated conductor $c\ c'\ c''$. If a current circulates through the conductor in the direction as indicated by the arrows, which of the two ends, a or b, is the south pole of the magnet?

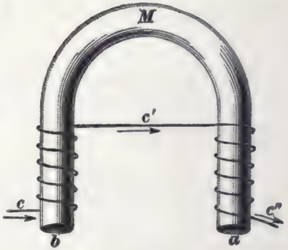


FIG. V.

(27) Give the names of all the known magnetic substances.

(28) A compass C is placed between the north and the south poles of two magnets, as shown in Fig. VI. Toward which pole will the north pole of the compass needle tend to point, and why?

FIG. VI.

(29) A compass C is placed alongside a bar magnet opposite the neutral line, as shown in Fig. VII. Toward which pole of the magnet will the south pole of the compass needle tend to point, and why?

FIG. VII.

(30) In an electromagnet, Fig. VIII, the coil of wire is wound around an iron core in a right-handed spiral. Through which end, a or b, of the wire must the current enter in order to produce the polarity as represented in the figure? Why?

FIG. VIII.

(31) The resistance of a platinum wire 112 feet 6 inches long is 100.8 ohms; calculate the resistance of 11.7 inches of the same wire, other conditions remaining unchanged.

Ans. .8736 ohm.

(32) The resistance of a German-silver wire is 91.8 ohms at 45° F.; calculate its resistance when its temperature is 72° F., other conditions remaining unchanged.

Ans. 92.4048 ohms.

(33) If the resistance of a copper wire is .144 ohm at 87° F., what is its resistance at 41° F., other conditions remaining unchanged? Ans. .131 ohm.

(34) The diagram, Fig. IX, represents a particular pattern of resistance box for a Wheatstone bridge, with battery and galvanometer circuits properly connected for taking resistance measurements. An unknown resistance X is connected

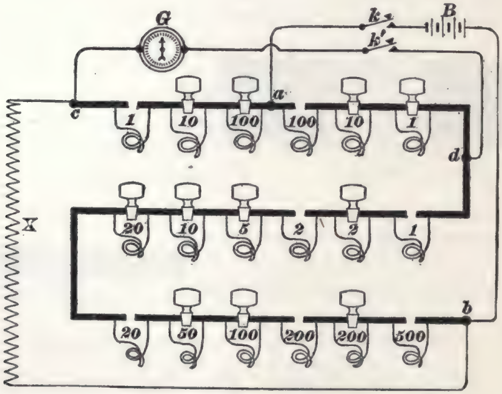


FIG. IX.

to the terminals c and b. After adjusting the resistances of the same by withdrawing the plugs, as represented by the open spaces between the contacts, the galvanometer shows

no deflection when the keys k and k' are pressed and the battery and galvanometer circuits are closed. Under these conditions, what is the resistance of X? Ans. 7.23 ohms.

(35) The total E. M. F. developed in a closed circuit is 36 volts, the internal resistance is 18 ohms, and the external resistance is 24 ohms; determine the strength of current in amperes flowing in the circuit. Ans. .8571 ampere.

(36) A current of 2.7 amperes is flowing in a closed circuit. If the total E. M. F. developed in the circuit is 12.6 volts, what is the total resistance of the circuit?

Ans. 4.6667 ohms.

(37) The external resistance of a closed circuit is 31.5 ohms and the internal is 11 ohms. If a current of .8 ampere is flowing through the circuit, what is the total E. M. F. in volts developed? Ans. 34 volts.

(38) The total E. M. F. developed in an electric source is 250 volts. If 10 per cent. of this E. M. F. is required to transmit a current of 80 amperes to and from a receptive device situated 600 feet from the source, (*a*) what is the total resistance of the two conductors, and (*b*) what is their resistance per foot, considering each to be 600 feet long?

Ans. $\begin{cases} (a) & .3125 \text{ ohm.} \\ (b) & .00026 \text{ ohm per foot.} \end{cases}$

(39) The internal resistance of a battery is 8.1 ohms and the total E. M. F. developed in it is 24 volts. What is the available or external E. M. F. of the battery when the circuit is completed by an external conductor offering a resistance of 15.9 ohms? Ans. 15.9 volts.

(40) The separate resistances of two branches A and B of a derived circuit are 1.2 and 2.2 ohms, respectively. If the sum of the currents in the two branches is 45 amperes, what is the current in each branch?

Ans. $\begin{cases} \text{The current in branch } A \text{ is 29.1176 amperes.} \\ \text{The current in branch } B \text{ is 15.8824 amperes.} \end{cases}$

(41) (*a*) What is a kilowatt? (*b*) If a machine has a capacity of 150 kilowatts, what is its capacity in horsepower?
Ans. (*b*) 201.072 horsepower.

(42) A 250-volt dynamo has a capacity of 300 kilowatts, what will be its full-load current? Ans. 1,200 amperes.

(43) (*a*) What is a kilowatt-hour? (*b*) If an average of 40 amperes flows in a 110-volt circuit for 9 hours and 30 minutes, what is the amount of work accomplished, expressed in kilowatt-hours? Ans. (*b*) 41.8 kilowatt-hours.

HEAT AND STEAM.

EXAMINATION QUESTIONS.

(1) What are the effects of heat on a substance when applied to it?

(2) What do you understand by the temperature of a body?

(3) (*a*) Define the term British thermal unit. (*b*) Give the mechanical equivalent of one unit.

(4) Give an instance of work being changed to heat.

(5) Define (*a*) latent heat of fusion; (*b*) latent heat of steam; (*c*) specific heat.

(6) It requires 1.77 B. T. U. to raise the temperature of 10 pounds of glass from 74° to 75°. What is the specific heat of glass? Ans. .177.

(7) One thousand pounds of steam at 212° is conveyed by pipes through a building; all the steam is condensed and the water of condensation returns to the boiler at a temperature of 192°. If the heat given off is applied to a quantity of water, how many pounds will it raise from 32° to 112°? Ans. 12,325 lb.

(8) If we place 10 pounds of water at a temperature of 90°, 15 pounds of mercury at 60°, and 20 pounds of alcohol at 40°, together in a vessel, what will be the temperature of the mixture? Ans. 62.66°.

NOTE.—The specific heat of alcohol is .60.

(9) If a number of substances of different temperatures are mixed together, how does the quantity of heat in the mixture compare with the quantity of heat contained in all the substances before mixing?

(10) If 20 pounds of steam at 212° are mixed with 400 pounds of water at 55°, what will be the temperature of the mixture? Ans. 108.47°

(11) How does change of pressure on the surface of a liquid affect its boiling point?

(12) Define (*a*) saturated steam; (*b*) superheated steam.

(13) Define (*a*) heat of the liquid; (*b*) latent heat: (*c*) total heat; (*d*) specific volume; (*e*) density.

(14) How many B. T. U. are required to change 10 pounds of ice at 25° into steam at 212°? Ans. 12,935.28 B. T. U.

(15) What is the difference between gauge pressures and absolute pressures, and which are given in the Steam Tables?

(16) How many foot-pounds of work will it require to change 40 pounds of water at 90 pounds gauge pressure into steam at the same pressure? Ans. 27,428,172 ft.-lb.

A KEY TO ALL THE QUESTIONS AND EXAMPLES

Contained in the Examination Questions Included in this Volume

The Keys that follow have been divided into sections corresponding to the Examination Questions to which they refer, and have been given corresponding section numbers. The answers and solutions have been numbered to correspond with the questions. When the answer to a question involves a repetition of statements given in the Instruction Paper, the reader has been referred to a numbered article, the reading of which will enable him to answer the question himself.

To be of the greatest benefit, the Keys should be used sparingly. They should be used much in the same manner as a pupil would go to a teacher for instruction with regard to answering some example he was unable to solve. If used in this manner, the Keys will be of great help and assistance to the student, and will be a source of encouragement to him in studying the various papers composing the course.

ARITHMETIC.

(PART 1.)

---

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.

605 = Six hundred five.

28,284 = Twenty-eight thousand two hundred eighty-four.

9,006,042 = Nine million six thousand forty-two.

850,317,002 = Eight hundred fifty million three hundred seventeen thousand two.

700,004 = Seven hundred thousand four.

(6) Seven thousand six hundred = 7,600.

Eighty-one thousand four hundred two = 81,402.

Five million four thousand seven = 5,004,007.

One hundred and eight million ten thousand one = 108,-010,001.

Eighteen million six = 18,000,006.

Thirty thousand ten = 30,010.

(7) In adding whole numbers, place the numbers to be added directly under each other, so that the extreme right-hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column and reserve 1 ten for the column of tens, $8 + 7 + 9 + 1 = 25$ tens, or 2 hundreds and 5 tens. Place 5 tens under the tens column and reserve 2 hundreds for the hundreds column. $4 + 5 + 2 + 2 = 13$ hundreds, or 1 thousand and 3 hundreds. Place 3 hundreds under the hundreds column and reserve the 1 thousand for the thousands column. $2 + 5 + 3 + 1 = 11$ thousands, or 1 ten-thousand and 1 thousand. Place the 1 thousand in the column of thousands and reserve the 1 ten-thousand for the column of ten-thousands. $6 + 1 = 7$ ten-thousands. Place this 7 ten-thousands in the ten-thousands column. There is but one figure, 8, in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

$$\begin{array}{r} 3290 \\ 504 \\ 865403 \\ 2074 \\ 81 \\ 7 \\ \hline 871359 \end{array} \text{ Ans.}$$

A simpler (though less scientific) explanation of the same problem is the following: $7 + 1 + 4 + 3 + 4 + 0 = 19$; write the 9 and reserve the 1. $8 + 7 + 0 + 0 + 9 + 1$ reserved $= 25$; write the 5 and reserve the 2. $0 + 4 + 5 + 2 + 2$ reserved $= 13$; write the 3 and reserve the 1. $2 + 5 + 3 + 1$ reserved $= 11$; write the 1 and reserve 1. $6 + 1$ reserved $= 7$; write the 7. Bring down the 8 to its place in the sum.

(8)

$$\begin{array}{r} 709 \\ 8304725 \\ 391 \\ 100302 \\ 300 \\ 909 \\ \hline 8407336 \end{array} \text{ Ans.}$$

(9) The steam engine, during the 12-hour test, showed that the number of revolutions made were 150,508, since 12,600 + 12,444 + 12,467 + 12,528 + 12,468 + 12,590 + 12,610 + 12,589 + 12,576 + 12,558 + 12,546 + 12,532 = 150,508 rev.

$$
\begin{array}{rl}
12600 & \text{revolutions.} \\
12444 & \text{revolutions.} \\
12467 & \text{revolutions.} \\
12528 & \text{revolutions.} \\
12468 & \text{revolutions.} \\
12590 & \text{revolutions.} \\
12610 & \text{revolutions.} \\
12589 & \text{revolutions.} \\
12576 & \text{revolutions.} \\
12558 & \text{revolutions.} \\
12546 & \text{revolutions.} \\
12532 & \text{revolutions.} \\
\hline
150508 & \text{revolutions. Ans.}
\end{array}
$$

(10) In subtracting whole numbers, place the subtrahend, or smaller number, under the minuend, or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We cannot subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. 1 *ten* = 10 *units*, so we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We cannot subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the

$$
(a)\quad
\begin{array}{r}
50962 \\
3338 \\
\hline
47624
\end{array}
\quad \text{Ans.}
$$

ten-thousands column in the minuend is brought down into the same column in the remainder, because 0 from 4 leaves 4.

$$(b)\quad \begin{array}{r} 15339 \\ 10001 \\ \hline 5338 \end{array} \text{ Ans.}$$

(11) $$(a)\quad \begin{array}{r} 70968 \\ 32975 \\ \hline 37993 \end{array} \text{ Ans.} \qquad (b)\quad \begin{array}{r} 100000 \\ 98735 \\ \hline 1265 \end{array} \text{ Ans.}$$

(12)

3040 = No. of gallons in the tank at the beginning of the day.
4780 = No. of gallons pumped in during the morning.
7820 = No. of gallons in the tank after 4,780 gallons were added.
7240 = No. of gallons drawn out during the morning.
580 = No. of gallons in the tank at the beginning of the afternoon.
8675 = No. of gallons pumped in during the afternoon.
9255 = No. of gallons in the tank after 8,675 gallons were added.
7633 = No. of gallons drawn out during the afternoon.
1622 = No. of gallons remaining in the tank at night. Ans.

(13) In the multiplication of whole numbers, place the multiplier under the multiplicand and multiply each term of the multiplicand by each term of the multiplier, writing the right-hand figure of each product obtained under the term of the multiplier which produces it.

$$(a)\quad \begin{array}{r} 526387 \\ 7 \\ \hline 3684709 \end{array} \text{ Ans.}$$

7 times 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 + 4 tens reserved = 60 tens, or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve the 2 thousands. 7 × 6 thousands =

42 thousands; 42 + 2 thousands reserved = 44 thousands, or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten-thousands. 7 × 2 ten-thousands = 14 ten-thousands; 14 + 4 ten-thousands reserved = 18 ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7 × 5 hundred-thousands = 35 hundred-thousands; 35 + 1 hundred-thousand reserved = 36 hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

$$
\begin{array}{lr}
(b) & 700298 \\
 & 17 \\
\hline
 & 4902086 \\
 & 700298 \\
\hline
 & 11905066 \text{ Ans.}
\end{array}
$$

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, 7 × 700,298 and 10 × 700,298. The actual operation is performed as follows:

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we have 1 ten times 8 units = 8 tens and 0 units. We do not write the 0 units, but write the 8 tens under the 8 tens in the first partial product above. We next multiply 1 ten and 9 tens = 90 tens = 9 hundreds + 0 tens; write the 9 hundreds under the 0 hundreds

of the first partial product above. Again, 1 ten times 2 hundreds = 2 thousands; write the 2 thousands under the 2 thousands above. 1 ten times 0 thousands = 0 ten-thousands; write the 0 in the ten-thousands place. 1 ten times 0 ten-thousands = 0 hundred-thousands; write the 0 in the hundred-thousands place. 1 ten times 7 hundred-thousands = 7 millions; write the 7 in the millions place. This completes the second partial product. Add the two partial products; their sum equals the entire product.

(*c*) Multiply any two of the numbers together, and multiply their product by the third number.

```
      217
      103
      ---
      651
     000
    217
    -----
    22351
       67
   ------
   156457
  134106
  -------
  1497517  Ans.
```

(14) If your watch ticks every second, to find how many times it ticks in 1 week, it is necessary to find the number of seconds in 1 week.

```
      60 seconds = 1 minute.
      60 minutes = 1 hour.
    ----
    3600 seconds = 1 hour.
      24 hours = 1 day.
   -----
   14400
   7200
   -----
   86400 seconds = 1 day.
       7 days = 1 week.
  ------
  604800 seconds in 1 week, or the number of times that
         your watch ticks in a week.  Ans.
```

(15) (*a*) If an engine and boiler are worth \$3,246, and the building is worth three times as much, plus \$1,200, then the building is worth

$$
\begin{array}{rr}
 & 3246 \\
 & 3 \text{ times} \\
\hline
 & 9738 \\
\text{plus} & 1200 \\
\hline
 & \$10938 = \text{value of building.}
\end{array}
$$

If the tools are worth twice as much as the building, plus $1,875, then the tools are worth

$$
\begin{array}{rr}
 & 10938 \\
 & 2 \\
\hline
 & 21876 \\
\text{plus} & 1875 \\
\hline
 & \$23751 = \text{value of tools.}
\end{array}
$$

$$
\begin{array}{lr}
\text{Value of building} = & 10938 \\
\text{Value of tools} = & 23751 \\
\hline
 & \$34689 = \text{value of the building and tools. Ans.}
\end{array}
$$

(*b*) Value of engine and boiler = 3246
Value of building and tools = 34689

$$\$37935 = \text{value of the whole plant. Ans.}$$

(16) (*a*) 84) 962842.0000 (11462.4047+ Ans.

$$
\begin{array}{r}
84 \\
\hline
122 \\
84 \\
\hline
388 \\
336 \\
\hline
524 \\
504 \\
\hline
202 \\
168 \\
\hline
340 \\
336 \\
\hline
400 \\
336 \\
\hline
640 \\
588 \\
\hline
52
\end{array}
$$

84 is contained once in 96. Place 1 as the first figure in the quotient and multiply the divisor 84 by it. Subtract the product, which is 84, from 96, leaving a remainder of 12. Bring down the next figure in the dividend, which is 2, and annex it to 12, making a new dividend of 122.

84 is contained in 122 once. Place 1 as the second figure in the quotient and multiply the divisor 84 by it. Subtract the product (84) from 122, leaving a remainder of 38. Bring down the next figure in the dividend, which is 8, and annex it to 38, making a new dividend of 388.

84 is contained in 388 4 times. Place 4 as the third figure in the quotient and multiply the divisor 84 by it. The product is 336. Subtract the product from 388, leaving a remainder of 52. Bring down the next figure, which is 4, and annex it to 52, making a new dividend of 524.

84 is contained in 524 6 times. Place 6 as the fourth figure in the quotient. Multiply the divisor 84 by it and subtract the product (504) from 524, leaving a remainder of 20. Bring down the next figure, which is 2, and annex it to 20, making a new dividend of 202.

84 is contained in 202 2 times. Place 2 as the fifth figure in the quotient. Multiply the divisor 84 by it and subtract the product (168) from 202, leaving a remainder of 34. If it is desired to carry the quotient to 4 decimal places, annex 4 ciphers to the dividend and continue in the same way. In the quotient point off as many decimal places as there are ciphers annexed, or, in this case, 4 decimal places.

```
(b)   63)39728.000(630.603+  Ans.
         378
         ---
          192
          189
          ----
           380*
           378
           ----
             200
             189
             ---
              11
```

*63 is not contained in 38, so we place a cipher in the quotient and bring down the next figure in the dividend, which is a cipher that has been annexed, making a new dividend of 380. 63 is contained in 380 6 times. $6 \times 63 = 378$. Subtracting 378 from 380 leaves 2. Bringing down the next figure in the dividend, we have 20 for a new dividend. 63 is not contained in 20, so we place a cipher in the quotient and bring down the next cipher in the dividend, making a new dividend of 200. 63 is contained 3 times in 200.

```
(c) 108)29714.0000(275.1296 Ans.
        216
        ---
         811
         756
         ---
          554
          540
          ---
           140
           108
           ---
            320
            216
            ---
            1040
             972
             ---
              680
              648
              ---
               32
```

```
(d) 135)406089.0000(3008.0666
        405                     Ans.
        ---
          1089*
          1080
          ----
             900
             810
             ---
              900
              810
              ---
               900
               810
               ---
                90
```

*135 is not contained in 10, so we place a cipher as the second figure in the quotient. Bringing down the next figure, 8, and annexing it to 10, we have a new dividend of 108. 135 is not contained in 108, so we place a cipher as the third figure in the quotient and bring down the next figure in the dividend, or 9, and annex it to 108, making a new dividend of 1,089. 135 is contained in 1,089 8 times. Write 8 as the fourth figure in the quotient. Multiply 135 by 8 and subtract the product (1,080) from 1,089, which leaves a remainder of 9. Bring down the next figure in the dividend, which is a cipher that has been annexed, and annex

it to the remainder 9, making a new dividend of 90. As 135 is not contained in 90, we place a 0 in the quotient and bring down another cipher from the dividend, making a new dividend of 900. 135 is contained in 900 6 times. Write 6 as the next figure in the quotient, multiply 135 by 6, and subtract the product (810) from 900, which leaves a remainder of 90. Bring down the next figure (0) in the dividend and annex it to the remainder 90, making a new dividend of 900. 135 is contained in 900 6 times. Place 6 as the next figure in the quotient and multiply the divisor by it. It is plain that each succeeding figure of the quotient will be 6. Point off *four* decimal places in the quotient, since four ciphers were annexed.

(17) If in 1 hour 10 pounds of coal are burned for every square foot of grate area and 9 pounds of water are evaporated for every pound of coal burned, then in 1 hour there would be 9×10 or 90 pounds of water evaporated for 1 sq. ft. of grate area; and since the grate area is 30 sq. ft., the amount of water evaporated would be $30 \times 90 = 2,700$ lb. Since 2,700 lb. of water are evaporated in 1 hour, in a day of 10 hours there would be $10 \times 2,700$ lb., or 27,000 lb. of water evaporated.

(18) If a mechanic earns \$1,500 a year and his expenses are \$968 per year, then he would save \$1500 — \$968, or \$532 per year.

$$\begin{array}{r} 1\,5\,0\,0 \\ 9\,6\,8 \\ \hline \$\,5\,3\,2 \end{array}$$

If he saves \$532 in 1 year, to save \$3,724 it would take as many years as \$532 is contained times in \$3,724, or 7 years.

$$\begin{array}{l} 5\,3\,2\ )\ 3\,7\,2\,4\ (\ 7 \text{ years. Ans.} \\ \underline{3\,7\,2\,4} \end{array}$$

(19) (*a*) $(72 \times 48 \times 28 \times 5) \div (84 \times 15 \times 7 \times 6)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{72 \times 48 \times 28 \times 5}{84 \times 15 \times 7 \times 6} = ?$$

The 5 in the numerator and 15 in the denominator are both *divisible* by 5, since 5 divided by 5 equals 1 and 15 divided by 5 equals 3. *Cross off* the 5; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \cancel{5}}{84 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

72 in the numerator and 84 in the denominator are *divisible* by 12, since 72 divided by 12 equals 6 and 84 divided by 12 equals 7. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 84 and write the 7 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

Again, 28 in the numerator and 7 in the denominator are *divisible* by 7, since 28 divided by 7 equals 4 and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, *cross off* the 7. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{\cancel{7}}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \cancel{7} \times 6} =$$

Again, 48 in the numerator and 6 in the denominator are *divisible* by 6, since 48 divided by 6 equals 8 and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, *cross off* the 6. Thus,

$$\frac{\overset{6}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \cancel{7} \times \cancel{6}} =$$

Again, 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2 and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, *cross off* the 3. Thus,

$$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{\cancel{3}}{\cancel{15}} \times \cancel{7} \times \cancel{6}} =$$

Since there are *no two remaining numbers* (one in the numerator and one in the denominator) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the numerator together and divide their *product* by the *product* of all the *uncanceled* numbers in the denominator. The *result* will be the *quotient.* The *product* of all the *uncanceled numbers* in the numerator equals $2 \times 8 \times 4 = 64$; the product of all the *uncanceled* numbers in the denominator equals 7.

Hence, $$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{\cancel{3}}{\cancel{15}} \times \cancel{7} \times \cancel{6}} = \frac{2 \times 8 \times 4}{7} = \frac{64}{7} = 9\tfrac{1}{7}.\quad \text{Ans.}$$

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the numerator and 70 in the denominator are both *divisible* by 10, since 50 divided by 10 equals 5 and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

Also, 80 in the numerator and 20 in the denominator are *divisible* by 20, since 80 divided by 20 equals 4 and 20 divided by 20 equals 1. *Cross off* the 80 and write the 4 *over* it; also, cross off the 20. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times \cancel{20}} =$$

Again, 16 in the numerator and 24 in the denominator are *divisible* by 8, since 16 divided by 8 equals 2 and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also, cross off the 24 and write the 3 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times 50 \times \underset{3}{\cancel{24}} \times \cancel{20}} =$$

Again, 60 in the numerator and 50 in the denominator are *divisible* by 10, since 60 divided by 10 equals 6 and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \cancel{20}} =$$

The 14 in the numerator and 7 in the denominator are *divisible* by 7, since 14 divided by 7 equals 2 and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\cancel{7}}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \cancel{20}} =$$

The 5 in the numerator and 5 in the denominator are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the 5 of the *dividend;* also, cross off the 5 of the *divisor*. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{\cancel{5}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\cancel{7}}{\cancel{70}} \times \underset{\cancel{5}}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \cancel{20}} =$$

The 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\cancel{5}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\cancel{7}}{\cancel{70}} \times \underset{\cancel{5}}{\cancel{50}} \times \underset{\cancel{3}}{\cancel{24}} \times \cancel{20}} =$$

Hence, $\frac{\overset{4}{\cancel{80}} \times \overset{\overset{2}{\cancel{6}}}{\cancel{60}} \times \overset{\cancel{5}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\cancel{7}}{\cancel{70}} \times \underset{\cancel{5}}{\cancel{50}} \times \underset{\cancel{3}}{\cancel{24}} \times \cancel{20}} = 4 \times 2 \times 2 \times 2 = 32.$ Ans.

(20) If the freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles the next week, then it ran (3 × 365 miles) − 246 miles, or 849 miles the second week. Thus,

$$\begin{array}{r} 365 \\ 3 \\ \hline 1095 \\ -\ 246 \\ \hline 849 \end{array} \text{ miles. Ans.}$$

(21) The distance from Philadelphia to Pittsburg is 354 miles. Since there are 5,280 feet in 1 mile, in 354 miles there are 354 × 5,280 feet, or 1,869,120 feet. If the driving wheel of the locomotive is 16 feet in circumference, in going from Philadelphia to Pittsburg, a distance of 1,869,120 feet, it will make 1,869,120 ÷ 16, or 116,820 revolutions.

```
16 ) 1869120 ( 116820 rev.  Ans.
     16
     --
      26
      16
     ---
     109
      96
     ---
      131
      128
      ---
        32
        32
        --
         0
```

ARITHMETIC.

(PART 2.)

(1) See Art. **1.**

(2) See Art. **6.**

(3) See Art. **3.**

(4) See Art. **3.**

(5) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(6) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(7) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms, we divide both numerator and denominator by 4 and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$.

(8) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6.

To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since the denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4, we will have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form.*

(9) In order to reduce a mixed number to an improper fraction, we must multiply the whole number by the denominator of the fraction and add the numerator of the fraction to the product. This result is the *numerator* of the improper fraction, of which the *denominator* is the denominator of the fractional part of the mixed number.

$7\frac{7}{8}$ means the same as $7 + \frac{7}{8}$. In 1 there are $\frac{8}{8}$; hence, in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$. Add the $\frac{7}{8}$ of the mixed number and we obtain $\frac{56}{8} + \frac{7}{8} = \frac{63}{8}$, which is the required improper fraction.

$$13\tfrac{5}{16} = \frac{(13 \times 16) + 5}{16} = \tfrac{213}{16};\ 10\tfrac{3}{4} = \frac{(10 \times 4) + 3}{4} = \tfrac{43}{4}.$$

(10) Each stroke of the engine is 18 inches in length. Since the piston makes 2 strokes for each revolution, it would pass over a distance of 2×18 inches $= 36$ inches, or 3 feet, in 1 minute, and in making 160 revolutions it would pass over 160×3, or 480 ft. Since 480 ft. are passed over in 1 minute, in 1 hour, or 60 minutes, the distance passed over equals $60 \times 480 = 28{,}800$ ft. Since the steam engine runs 6 days a week and $8\frac{1}{2}$ hours per day, the total number of hours it runs per week $= 6 \times 8\frac{1}{2}$, or 51 hours. If the piston passes over a distance of 28,800 ft. in 1 hour, in 51 hours it would pass over $51 \times 28{,}800$ ft., or 1,468,800 ft. Ans.

$$\begin{array}{r} 480 \text{ ft.} \\ \times \quad 60 \\ \hline 28800 \text{ ft.} \end{array}$$

$$\begin{array}{r} 28800 \\ \times \quad 51 \\ \hline 28800 \\ 144000 \\ \hline 1468800 \end{array}$$

(11) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1 + 2 + 5}{8} = \frac{8}{8} = 1.$ Ans.

When the denominators of the fractions to be added are alike, we know that the units are divided into the same number of parts (in this case eighths); we therefore add the numerators of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$, or 1, as the sum.

(12) When the denominators are not alike, we know that the units are divided into unequal parts, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators, and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to 16ths and then add their numerators.

$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3 \times 2}{8 \times 2} = \frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16} + \frac{6}{16} + \frac{5}{16} = \frac{4 + 6 + 5}{16} = \frac{15}{16}$. Ans.

(13) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers, we have $\frac{10}{16} + \frac{7}{16} = \frac{10 + 7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

$$\begin{array}{r} 42 \\ 31 \\ 9 \\ 1\frac{1}{16} \\ \hline 83\frac{1}{16} \end{array} \text{ Ans.}$$

(14) $29\frac{3}{4} + 50\frac{5}{8} + 41 + 69\frac{3}{16} = ?$ $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$;

$\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. $\frac{12}{16} + \frac{10}{16} + \frac{3}{16} = \frac{12 + 10 + 3}{16} = \frac{25}{16} = 1\frac{9}{16}$.

The problem now becomes $29 + 50 + 41 + 69 + 1\frac{9}{16} = ?$

$$\begin{array}{rl} 29 & \text{square inches.} \\ 50 & \text{square inches.} \\ 41 & \text{square inches.} \\ 69 & \text{square inches.} \\ 1\frac{9}{16} & \text{square inches.} \\ \hline 190\frac{9}{16} & \text{square inches. Ans.} \end{array}$$

(15) $\frac{7}{8} - \frac{7}{16} = ?$ When the denominators of fractions are not alike, it is evident that the units are divided into unequal parts; therefore, before subtracting, reduce the fractions to fractions having a common denominator. Then, subtract the numerators and place the remainder over the common denominator.

$$\frac{7 \times 2}{8 \times 2} = \frac{14}{16}. \quad \frac{14}{16} - \frac{7}{16} = \frac{14 - 7}{16} = \frac{7}{16}. \quad \text{Ans.}$$

$13 - 7\frac{7}{16} = ?$ This problem may be solved in two ways:

First: $13 = 12\frac{16}{16}$, since $\frac{16}{16} = 1$, and $12\frac{16}{16} = 12 + \frac{16}{16} = 12 + 1 = 13$.

$$\begin{array}{r} 12\frac{16}{16} \\ 7\frac{7}{16} \\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately and the fractions separately, and obtain $12 - 7 = 5$ and $\frac{16}{16} - \frac{7}{16} = \frac{16-7}{16} = \frac{9}{16}$. $5 + \frac{9}{16} = 5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$$13 = \frac{13}{1} = \frac{13 \times 16}{1 \times 16} = \frac{208}{16}. \quad 7\frac{7}{16} = \frac{(7 \times 16) + 7}{16} = \frac{112 + 7}{16} = \frac{119}{16}.$$

Subtracting, we have $\frac{208}{16} - \frac{119}{16} = \frac{208 - 119}{16} = \frac{89}{16}$, and

$$\frac{89}{16} = 16)89(5\frac{9}{16}$$
$$\begin{array}{r} 80 \\ \hline 9 \\ \hline 16 \end{array}$$

the same result that was obtained by the first method.

$312\frac{9}{16} - 229\frac{5}{32} = ?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this, we have $\frac{9}{16} = \frac{9 \times 2}{16 \times 2} = \frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312 - 229 = 83$ and $\frac{18}{32} - \frac{5}{32} = \frac{18-5}{32} = \frac{13}{32}$. $83 + \frac{13}{32} = 83\frac{13}{32}$. Ans.

$$\begin{array}{r} 312\frac{18}{32} \\ 229\frac{5}{32} \\ \hline 83\frac{13}{32} \end{array}$$

(16) In division of fractions, invert the divisor (or, in other words, turn it upside down) and proceed as in multiplication.

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

(*d*) $\frac{113}{64} \div \frac{7}{16} = \frac{113}{64} \times \frac{16}{7} = \frac{113 \times 16}{64 \times 7} = \frac{1808}{448} = 4\frac{1}{28}$. Ans.

$$\begin{array}{r} 448\,)\,1808\,(\,4\frac{1}{28} \\ \underline{1792} \\ \frac{16}{448} = \frac{1}{28} \end{array}$$

(*e*) $15\frac{3}{4} \div 4\frac{3}{8} = ?$ Before proceeding with the division, reduce both mixed numbers to improper fractions. Thus, $15\frac{3}{4} = \frac{(15 \times 4) + 3}{4} = \frac{60 + 3}{4} = \frac{63}{4}$, and $4\frac{3}{8} = \frac{(4 \times 8) + 3}{8} = \frac{32 + 3}{8} = \frac{35}{8}$. The problem is now $\frac{63}{4} \div \frac{35}{8} = ?$ As before, invert the divisor, multiply, and cancel; $\frac{63}{4} \div \frac{35}{8} = \frac{63}{4} \times \frac{8}{35} = \frac{\overset{9}{\cancel{63}} \times \overset{2}{\cancel{8}}}{\cancel{4} \times \underset{5}{\cancel{35}}} = \frac{18}{5} = 3\frac{3}{5}$. Ans.

(17) $\frac{7}{8}$ = value of the fraction, and 28 the numerator. We find that 4 multiplied by $7 = 28$, so multiplying 8, the denominator of the fraction, by 4, we have 32 for the required denominator, and $\frac{28}{32} = \frac{7}{8}$. Hence, 32 is the required denominator.

(18) Since these four bolts measure $2\frac{1}{2}$, $6\frac{7}{8}$, $3\frac{1}{16}$, and 4 inches, respectively, together they will measure $16\frac{7}{16}$ inches, since $2\frac{1}{2} + 6\frac{7}{8} + 3\frac{1}{16} + 4 = 16\frac{7}{16}$. Reducing the fractions of the mixed numbers to a common denominator, we have $\frac{1}{2} = \frac{1}{2} \times \frac{8}{8} = \frac{8}{16}$; $\frac{7}{8} = \frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$; $\frac{8}{16} + \frac{14}{16} + \frac{1}{16} = \frac{8 + 14 + 1}{16} = \frac{23}{16} = 1\frac{7}{16}$. $2 + 6 + 3 + 4 + 1\frac{7}{16} = 16\frac{7}{16}$. If $\frac{7}{16}$ of an inch is allowed for cutting and finishing each bolt, then the allowance for the 4 bolts would equal $4 \times \frac{7}{16} = \frac{28}{16} = 1\frac{12}{16}$ inches, which added to $16\frac{7}{16}$ inches equals $18\frac{3}{16}$ inches, the length of the piece of iron required. Ans.

$$\begin{array}{l} 16\frac{7}{16} \\ \underline{\ \ 1\frac{12}{16}} \\ 17\frac{19}{16}, \text{ or } 18\frac{3}{16}, \text{ since } \frac{19}{16} = 1\frac{3}{16}. \end{array}$$

ARITHMETIC.

(PART 3.)

(1) A fraction is one or more of the equal parts of a unit and is expressed by a numerator and a denominator, while a decimal fraction is a number of *tenths*, *hundredths*, *thousandths*, etc. of a unit and is expressed by placing a period (.), called a decimal point, to the left of the figures of the numerator and omitting the denominator.

(2) To reduce the fraction $\frac{1}{2}$ to a decimal, we annex one cipher to the numerator, which makes it 1.0. Dividing 1.0, the numerator, by 2, the denominator, gives a quotient of .5, the decimal point being placed before the *one* figure of the quotient, or .5, since only *one* cipher was annexed to the numerator.

```
7
-
8 ) 7.0 0 0
    -------
     .8 7 5

 5
---
3 2 ) 5.0 0 0 0 0 ( .1 5 6 2 5
      3 2
      -----
      1 8 0
      1 6 0
      -------
        2 0 0
        1 9 2
        -------
            8 0
            6 4
            -------
            1 6 0
            1 6 0
            -----
```

To express $\frac{65}{100}$ as a decimal, write the numerator and, beginning with the right-hand figure, point off towards the left as many decimal places as there are ciphers in the

denominator, prefixing ciphers to the numerator if necessary, and then insert the decimal point. Proceeding thus, $\frac{65}{100} = .65$ and $\frac{125}{1000} = .125$. Similarly, $\frac{24}{100000} = .00024$.

		tenths.	hundredths.	
(3)	.	0	8	= *Eight* **hundredths.**

	tenths.	hundredths.	thousandths.	
.	1	3	1	= *One hundred thirty-one* **thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
.	0	0	0	1	= *One* **ten-thousandths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	
.	0	0	0	0	2	7	= *Twenty-seven* **millionths.**

	tenths.	hundredths.	thousandths.	ten-thousandths.	
.	0	1	0	8	= *One hundred eight* **ten-thousandths.**

			tenths.	hundredths.	thousandths.	ten-thousandths.	
9	3	.	0	1	0	1	= Ninety-three and *one hundred one* **ten-thousandths.**

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the

decimal point towards the right, beginning with tenths, to as many places as there are figures, and the name of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example given, the simple reading of the figure is eight and the name of its position in the decimal scale is hundredths, so that the decimal reading is eight hundredths. Similarly, the figures in the fourth example are ordinarily read twenty-seven; the name of the position of the figure 7 in the decimal scale is millionths, giving, therefore, the decimal reading as twenty-seven millionths.

If there should be a whole number before the decimal point, read it as you would read any whole number and read the decimal as you would if the whole number were not there; or, read the whole number and then say "and" so many hundredths, thousandths, or whatever it may be, as "ninety-three and *one* hundred one ten-thousandths."

(4) In addition of decimals, the decimal points must be placed directly under each other, so that tenths will come under tenths, hundredths under hundredths, thousandths under thousandths, etc. The addition is then performed as in whole numbers, the decimal point of the sum being placed directly under the decimal points above.

$$
\begin{array}{r@{}l}
 & .125 \\
 & .7 \\
 & .089 \\
 & .4005 \\
 & .9 \\
 & .000027 \\
\hline
2 & .214527 \quad \text{Ans.}
\end{array}
$$

(5)

tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.
.0	1	7			
.2					
.0	0	0	0	4	7
.2	1	7	0	4	7

.2 1 7 0 4 7 = *Two hundred and seventeen thousand and forty-seven* **millionths.** Ans.

(6) (*a*) In subtraction of decimals, place the decimal points directly under each other and proceed as in the subtraction of whole numbers, placing the decimal point in the remainder directly under the decimal points above.

$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array} \text{ Ans.}$$

In the above example, we proceed as follows: We cannot subtract 4 ten-thousandths from 0 ten-thousandths, and as there are no thousandths, we take 1 hundredth from the 3 hundredths. 1 hundredth = 10 thousandths = 100 ten-thousandths. 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 thousandths + 6 ten-thousandths. Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths leaves but 2 hundredths now). Since we cannot do this, we take 1 tenth from 6 tenths. 1 tenth = 10 hundredths, and 10 hundredths + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this cannot be done, we take 1 unit from the 9 units. 1 unit = 10 tenths. 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (1 unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*) $$\begin{array}{r} 81.963 \\ 1.700 \\ \hline 80.263 \end{array} \text{ Ans.}$$

(*c*) $$\begin{array}{r} 18.00 \\ .18 \\ \hline 17.82 \end{array} \text{ Ans.}$$

(*d*) $$\begin{array}{r} 1.000 \\ .001 \\ \hline .999 \end{array} \text{ Ans.}$$

(*e*) $872.1 - (.8721 + .008) = ?$ In this problem we are to subtract $(.8721 + .008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{r} .8721 \\ .0080 \\ \hline .8801 \end{array} \textit{ sum.}$$

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

$$\begin{array}{r} 872.1000 \\ .8801 \\ \hline 871.2199 \end{array} \text{ Ans.}$$

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

$$\begin{array}{r} 5.0280 \\ .0073 \\ \hline 5.0353 \end{array} \textit{ sum.}$$

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

$$\begin{array}{r} 6.704 \\ 2.380 \\ \hline 4.324 \end{array} \textit{ diff.}$$

The sign between the parentheses indicates that the quantities obtained by performing the above operations are to be subtracted—namely, that 4.324 is to be subtracted from 5.0353. Performing this operation, we obtain .7113 as the final result.

$$\begin{array}{r} 5.0353 \\ 4.3240 \\ \hline .7113 \end{array} \text{ Ans.}$$

(7) If the cost of the coal consumed by a nest of steam boilers amounts to \$15.83 on Monday, to \$14.70 on Tuesday, to \$14.28 on Wednesday, to \$13.87 on Thursday, to \$14.98 on Friday, and to \$12.65 on Saturday, then we find the total cost of the week's supply by adding the different amounts together; hence, $\$15.83 + \$14.70 + \$14.28 + \$13.87 + \$14.98 + \$12.65 = \$86.31$.

$$\begin{array}{r} \$\,15.83 \\ 14.70 \\ 14.28 \\ 13.87 \\ 14.98 \\ 12.65 \\ \hline \$\,86.31 \end{array} \text{ Ans.}$$

(8) $482\frac{4}{5} + 316\frac{1}{3} + 390\frac{3}{4} = \text{what?}$

When mixed numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction. First, we will reduce the fractional parts $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to equivalent fractions having the least common denominator. In this case the least common denominator equals the product of the denominators 5, 3, and 4, since we cannot divide any two of them by any number (except 1) without having a remainder, as can be done in the examples in Art. **26**, Part 2. Hence, the least common denominator $= 5 \times 3 \times 4 = 60$. Reducing $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to fractions having this least common denominator, we have 60 divided by the first denominator, 5, equals 12. Then, $\frac{4}{5} \times \frac{12}{12} = \frac{48}{60}$. 60 divided by the second denominator, 3, equals 20. Then, $\frac{1}{3} \times \frac{20}{20} = \frac{20}{60}$. 60 divided by the third denominator, 4, equals 15. Then, $\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}$. The sum of these fractions equals

$$\frac{48}{60} + \frac{20}{60} + \frac{45}{60} = \frac{48 + 20 + 45}{60} = \frac{113}{60}, \text{ or } 1\frac{53}{60}.$$

The problem now becomes $482 + 316 + 390 + 1\frac{53}{60}$, the sum of which equals $1{,}189\frac{53}{60}$.

$$\begin{array}{r} 482 \\ 316 \\ 390 \\ 1\frac{53}{60} \\ \hline 1189\frac{53}{60} \end{array}$$

1 5 0 0 represents the actual horsepower required.

$1\ 1\ 8\ 9\ \frac{53}{60}$ represents the indicated horsepower of the engines in use.

$3\ 1\ 0\ \frac{7}{60}$, or $310.11\frac{2}{3}$ = the H. P. to be developed by the new engine. Ans.

$\frac{7}{60}$ reduced to its equivalent decimal = $\frac{7}{60}$) 7.00 (.11$\frac{2}{3}$

```
60) 7.0 0 (.1 1 2/3
    6 0
    ---
    1 0 0
      6 0
    -----
      4 0/6 0 = 2/3
```

(9) Since the inside diameter of the steam pipe is 6.06 inches and the outside diameter is 6.62 inches, there is a difference of 6.62 − 6.06, or .56 of an inch, in both diameters. But .56 of an inch is just twice the thickness of the pipe; hence, the pipe is $\frac{1}{2}$ of .56, or .28 of an inch thick.

(10) (*a*) There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3, or 6, decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary 6 decimal places.

```
      .1 0 7
      .0 1 3
      ------
       3 2 1
     1 0 7
  ----------
  .0 0 1 3 9 1   Ans.
```

(*b*)

```
         2 0 3
         2.0 3
        ------
         6 0 9
       0 0 0
     4 0 6
    ---------
     4 1 2.0 9
         .2 0 3
    -----------
   1 2 3 6 2 7
   0 0 0 0 0
 8 2 4 1 8
 -------------
 8 3.6 5 4 2 7   Ans.
```

There are 2 decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0, or 2, decimal places in the first product.

Since in the second multiplication there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2, or 5, decimal places in the second product.

When there are one or more ciphers in the multiplier, multiply just the same as with the other figures.

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parentheses, and then whatever may be required by the sign between the parentheses.

```
 31.85
   2.7
 -----
 22295
 6370
 -----
85.995
```

The first parenthesis shows that the numbers 2.7 and 31.85 are to be multiplied together.

The second parenthesis shows that .316 is to be taken from 3.16.

```
3.160
 .316
-----
2.844
```

The product obtained by performing the operation indicated by the signs within the first parenthesis is now multiplied by the remainder obtained by performing the operation indicated by the signs within the second parenthesis.

```
      85.995
       2.844
   ---------
      343980
     343980
    687960
   171990
 -----------
 244.569780  Ans.
```

(*d*) $(107.8 + 6.541 - 31.96) \times 1.742 = ?$

```
   107.8                  82.381
 +   6.541             ×   1.742
   -------             ---------
   114.341                164762
 -  31.96                329524
   -------              576667
    82.381              82381
                       -----------
                       143.507702  Ans.
```

(11) If one 3-inch tube measures $15\frac{1}{2}$ ft. in length, 60 of these tubes would measure $60 \times 15\frac{1}{2}$ ft., or 930 ft. in length. If 1 foot of tubing heats a surface of .728 sq. ft., then it is evident that 930 ft. of tubing would heat a surface of $930 \times .728$ ft., or 677.04 sq. ft.

```
15½
 60
---
900
 30
---
930 ft.
```

```
   .728
    930
  -----
  21840
 6552
-------
677.040 sq. ft.
```

(12) (*a*) $\dfrac{7}{\frac{3}{16}} = 7 \div \frac{3}{16} = 7 \times \frac{16}{3} = \dfrac{7 \times 16}{3} = \frac{112}{3} = 37\frac{1}{3}$. Ans.

The heavy line indicates that 7 is to be divided by $\frac{3}{16}$.

(*b*) $$\dfrac{\frac{15}{32}}{\frac{5}{8}} = \frac{15}{32} \div \frac{5}{8} = \frac{15}{32} \times \frac{8}{5} = \dfrac{\overset{3}{\cancel{15}} \times \cancel{8}}{\underset{4}{\cancel{32}} \times \cancel{5}} = \frac{3}{4} = .75. \quad \text{Ans.}$$

(*c*) $$\dfrac{1.25 \times 20 \times 3}{\dfrac{87 + 88}{459 + 32}} = ?$$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{rr} & 1.25 \\ \times & 20 \\ \hline & 25.00 \\ \times & 3 \\ \hline & 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\dfrac{87 + 88}{459 + 32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\dfrac{75}{\frac{175}{491}} = \dfrac{\frac{75}{1}}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \dfrac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \frac{1{,}473}{7} = 210\tfrac{3}{7}. \quad \text{Ans.}$$

(13) The pitch of the rivets is the distance between the centers of the rivets. Hence, since the distance around the cylindrical boiler is 166.85 in., and there are 72 rivets in one of the seams, the pitch of the rivets equals $166.85 \div 72 = 2.317+$ in. Ans.

$$\begin{array}{l} 72\,)\,166.850\,(\,2.317+ \\ \quad\;\; 144 \\ \quad\;\; \overline{\;\;228} \\ \quad\;\;\;\; 216 \\ \quad\;\;\;\; \overline{\;\;125} \\ \quad\;\;\;\;\;\;\; 72 \\ \quad\;\;\;\;\;\; \overline{\;530} \\ \quad\;\;\;\;\;\; 504 \\ \quad\;\;\;\;\;\; \overline{\;\;26} \end{array}$$

(14) If a keg containing 133 boiler rivets weighs 100 pounds, then each rivet must weigh as much as 133 is contained times in 100, or .75 of a pound.

$$
\begin{array}{r}
133\,)\,100.00\,(.75+ \quad \text{Ans.} \\
931\phantom{\,(.75+ \quad \text{Ans.}} \\
\hline
690\phantom{\,(.75+ \quad \text{Ans.}} \\
665\phantom{\,(.75+ \quad \text{Ans.}} \\
\hline
25\phantom{\,(.75+ \quad \text{Ans.}}
\end{array}
$$

Since there are 2 decimal places in the dividend and 0 decimal places in the divisor, we must point off $2 - 0 = 2$ decimal places in the quotient, or answer.

(15) $.875 = \dfrac{875}{1{,}000} = \frac{175}{200} = \frac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\tfrac{7}{8} \text{ of 1 foot} = \frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{21}{2} = 10\tfrac{1}{2} \text{ inches.} \quad \text{Ans.}$$

(16) 12 inches = 1 foot.

$$\tfrac{3}{16} \text{ of an inch} = \tfrac{3}{16} \div 12 = \frac{\cancel{3}}{16} \times \frac{1}{\underset{4}{\cancel{12}}} = \tfrac{1}{64} \text{ of a foot.}$$

$$
\begin{array}{l}
1 \\
\overline{64}\,)\,1.000000\,(.015625 \quad \text{Ans.} \\
\qquad 64 \\
\qquad \overline{360} \\
\qquad 320 \\
\qquad \overline{400} \\
\qquad 384 \\
\qquad \overline{160} \\
\qquad 128 \\
\qquad \overline{320} \\
\qquad 320 \\
\qquad \overline{0}
\end{array}
$$

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, $6 - 0 = 6$ places to be pointed off.

(17) The total horsepower developed equals 48.63 + 45.7 + 46.32 + 47.9 + 48.74 + 48.38 + 48.59 = 334.26.

Since the horsepower developed equals 334.26, then the average horsepower developed must equal 334.26 ÷ 7, or 47.75+ H. P.

$$
\begin{array}{r}
48.63 \\
45.7 \\
46.32 \\
47.9 \\
48.74 \\
48.38 \\
48.59 \\
\hline
7\,)\,334.26 \\
\hline
47.75+
\end{array}
\quad \text{Ans.}
$$

ARITHMETIC.

(PART 4.)

---

(1) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. $\frac{25}{100} = .25$. Hence, 8,428 lb. $\times$ .25 = 2,107 lb. Ans.

(2) $\frac{1}{2}\%$ means one-half of 1 per cent. Since 1% is .01, $\frac{1}{2}\%$ is .005, for $2\,\underline{)\,.0\,1\,0}$ $.0\,0\,5$. And \$35,000 $\times$ .005 = \$175. Ans.

$$\begin{array}{r} \$\,3\,5\,0\,0\,0 \\ .0\,0\,5 \\ \hline \$\,1\,7\,5.0\,0\,0 \end{array}$$

(3) If 2 is a certain per cent. of 50, then 50 multiplied by a certain rate gives a product of 2, and that rate is equal to 2 divided by 50. Dividing 2 by 50, the quotient is .04, which means that 2 is 4% of 50, or, since percentage = base $\times$ rate,

$$\begin{array}{r} 5\,0\,)\,2.0\,0\,(\,.0\,4 \text{ Ans.} \\ \underline{2\,0\,0} \end{array}$$

$$\begin{aligned} \text{rate} &= \text{percentage} \div \text{base} \\ &= 2 \div 50 = .04, \text{ or } 4\%. \quad \text{Ans.} \end{aligned}$$

(4) Since percentage = base $\times$ rate, rate = percentage $\div$ base.

As percentage = 10 and base = 10, we have rate = 10 $\div$ 10 = 1.

But $1 = \frac{100}{100}$ and $\frac{100}{100} = 100\%$; hence, the rate (1) means that 10 is 100% of 10.

(5) Since 5,500 lb. represent an increase of 15% over the consumption when the condenser is used, 5,500 lb. must be the amount, .15 the rate, and the number of pounds consumed when the condenser is running (to be found) the base.

$$\begin{aligned} \text{Base} &= \text{amount} \div (1 + \text{rate}) = 5{,}500 \div (1 + .15) \\ &= 5{,}500 \div 1.15 = 4{,}782.61 \text{ lb., nearly.} \quad \text{Ans.} \end{aligned}$$

```
1.15 ) 5500.0000 ( 4782.61
       460
       ---
        900
        805
        ---
         950
         920
         ---
          300
          230
          ---
           700
           690
           ---
            100
            115
            ---
```

Or, this problem could also have been solved as follows: 100% = the number of pounds consumed when the condenser is running. If there is a gain of 15%, then 100% + 15%, or 115% = 5,500 lb., the amount used when the condenser is not running. If 115% = 5,500 lb., $1\% = \frac{1}{115}$ of 5,500 = 47.8261 lb., and 100% = 100 × 47.8261 = 4,782.61 lb. Ans.

(6)

$$\begin{array}{lr} 24\ \% \text{ of } \$950 = 950 \times .24\ \ = & \$228 \\ 12\tfrac{1}{2}\% \text{ of } \$950 = 950 \times .125 = & 118.75 \\ 17\ \% \text{ of } \$950 = 950 \times .17\ \ = & 161.50 \\ \hline 53\tfrac{1}{2}\% \text{ of } \$950 \qquad\qquad\quad = & \$508.25 \end{array}$$

The total amount of his yearly expenses, then, is $508.25; hence, his savings are $950 − $508.25 = $441.75. Ans.

Or, as above, $24\% + 12\frac{1}{2}\% + 17\% = 53\frac{1}{2}\%$, the total percentage of expenditures; hence,

$\$950 \times .535 = \$508.25$, and
$\$950 - \$508.25 = \$441.75$ = his yearly savings. Ans.

(7) The percentage is 961.38 and the rate is $.37\frac{1}{2}$.

Base = percentage ÷ rate
$= 961.38 \div .375 = 2,563.68$, the number. Ans.

```
.375)961.38000(2563.68
     750
     ----
     2113
     1875
     ----
      2388
      2250
      ----
       1380
       1125
       ----
        2550
        2250
        ----
         3000
         3000
         ----
```

Another method of solving is the following:
If $37\frac{1}{2}\%$ of a number is 961.38, then $.37\frac{1}{2}$ times the number = 961.38 and the number = $961.38 \div .37\frac{1}{2}$, which, as above = 2,563.68. Ans.

(8) 298 revolutions per minute with the load = base, $.01\frac{1}{2}$ = rate, and the amount (to be found) will equal the speed of the engine when running unloaded.

Amount = base × (1 + rate)
$= 298 \times (1 + .015) = 302.47$ rev. per min. Ans.

```
      298
  × 1.015
  -------
     1490
     298
    000
   298
  -------
  302.470
```

(9) 4 yd. 2 ft. 10 in. to inches.

$$\begin{array}{rl}
 & 4 \\
\times & 3 \\ \hline
 & 12 \\
+ & 2 \\ \hline
 & 14 \text{ feet} \\
\times & 12 \\ \hline
 & 28 \\
 & 14 \\ \hline
 & 168 \\
+ & 10 \\ \hline
 & 178 \text{ inches. Ans.}
\end{array}$$

Since there are 3 feet in 1 yard, in 4 yards there are 4 × 3 feet, or 12 feet. 12 feet plus 2 feet = 14 feet.

There are 12 inches in 1 foot; therefore, in 14 feet there are 12 × 14, or 168 inches. 168 inches plus 10 inches = 178 inches.

(10)

$$\begin{array}{r}
12\,\underline{)\,3722} \text{ inches.} \\
3\,\underline{)\,310} + 2 \text{ inches.} \\
103 + 1 \text{ foot.}
\end{array}$$

Ans. = 103 yd. 1 ft. 2 in.

Explanation.—There are 12 inches in 1 foot; hence, in 3,722 inches there are as many feet as 12 is contained times in 3,722, or 310 feet and 2 inches remaining. Write 2 inches as a remainder. There are 3 feet in 1 yard; hence, in 310 yards there are as many feet as 3 is contained times in 310, or 103 yards and 1 foot remaining. Hence, in 3,722 inches there are 103 yd. 1 ft. 2 in.

(11)

$$\begin{array}{r}
1728\,\underline{)\,764325} \text{ cu. in.} \\
27\,\underline{)\,442} + 549 \text{ cu. in.} \\
16 \text{ cu. yd.} + 10 \text{ cu. ft.}
\end{array}$$

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

Explanation.—There are 1,728 cubic inches in 1 cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cubic feet and 549 cubic inches remaining. Write the 549 cubic inches as a remainder. There are 27 cubic feet in 1 cubic yard; hence, in 442 cubic feet there are as many cubic yards as 27

is contained times in 442 cubic feet, or 16 cubic yards and 10 cubic feet remaining. Then, in 764,325 cubic inches there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(12)

	T.	cwt.	lb.
	16	8	75
×	20		
	320		
+	8		
	328 cwt.		
×	100		
	32800		
+	75		
	32875 lb. Ans.		

Since in 1 ton there are 20 cwt., in 16 tons there are $16 \times 20 = 320$ cwt. 320 cwt. + 8 cwt. = 328 cwt. There are 100 lb. in 1 cwt.; hence, in 328 cwt. there are $328 \times 100 = 32,800$ lb. 32,800 lb. + 75 lb. = 32,875 lb. Ans.

(13)

$$\begin{array}{r} 100\,)\,\underline{25396} \text{ lb.} \\ 20\,)\,\underline{253} \text{ cwt.} + 96 \text{ lb.} \\ 12 \text{ T.} + 13 \text{ cwt.} \end{array}$$

There are 100 lb. in 1 cwt.; hence, in 25,396 lb. there are as many cwt. as 100 is contained times in 25,396, or 253 cwt. and 96 lb. remaining.

There are 20 cwt. in 1 ton, and in 253 cwt. there are as many tons as 20 is contained times in 253, or 12 tons and 13 cwt. remaining. Hence, 25,396 lb. = 12 T. 13 cwt. 96 lb. Ans.

(14) Arrange the different terms in columns, taking care to have like denominations in the same column. We begin to add at the right-hand column. $7 + 9 + 3 = 19$ in.; since 12 in. = 1 ft., 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column and reserve the 1 ft. to add to the sum of the feet. $2 + 1 + 2 + 1$ (reserved) = 6 ft. Since 3 ft. = 1 yd., 6 ft. = 2 yd. and 0 ft. remaining. Place the 0 in the feet column and reserve the 2 yd. to add to the sum of the yards. $4 + 2 + 2$ (reserved) = 8 yd., which we place in yards column. Ans. = 8 yd. 7 in.

yd.	ft.	in.	
2	2	3	
4	1	9	
	2	7	
8	0	7	Ans.

(15) Since 10 gal. 2 qt. 1 pt. of machine oil is sold at one time and 16 gal. 3 qt. at another time, together there was sold 27 gal. 1 qt. 1 pt. $0 + 1 = 1$ pt. We cannot reduce 1 pt. to any higher denomination, so place it under pints column. 3 qt. + 2 qt. = 5 qt.

gal.	qt.	pt.
1 0	2	1
1 6	3	0
2 7	1	1

Since 4 qt. = 1 gal., 5 qt. = 1 gal. and 1 qt. remaining. Place 1 qt. under quarts column and reserve the 1 gal. to add to the gallons. 16 gal. + 10 gal. + 1 gal. (reserved) = 27 gal.

gal.	qt.	pt.	
3 1	2	0	
2 7	1	1	
4	0	1	Ans.

Since the barrel contained $31\frac{1}{2}$, or 31.5 gal., and 27 gal. 1 qt. 1 pt. were sold, there remained the difference, or 4 gal. 1 pt. 31.5 gal. = 31 gal. 2 qt., since $.5 = \frac{1}{2}$, and $\frac{1}{2}$ of 1 gal. = $\frac{1}{2}$ of 4 qt. = 2 qt. 1 pt. cannot be taken from 0 pt., so we take 1 qt. from the 2 qt. The 1 qt. taken = 2 pt. 1 pt. from 2 pt. = 1 pt. Place 1 pt. under pints column. Since we took 1 qt. from the quarts column, there remains $2 - 1$, or 1 qt. 1 qt. from 1 qt. leaves 0 qt. Place 0 qt. under the quarts column. 27 gal. from 31 gal. leaves 4 gal. Place 4 gal. under the gallons column. We therefore find that 4 gal. 1 pt. of machine oil remained in the barrel.

(16) In multiplication of denominate numbers, we place the multiplier under the lowest denomination of the multiplicand, as

1 7 ft.	3 in.
	5 1
8 7 9 ft.	9 in.

and begin at the right to multiply. $51 \times 3 = 153$ in. Since there are 12 in. in 1 ft., in 153 in. there are as many feet as 12 is contained times in 153, or 12 ft. and 9 in. remaining. Place the 9 in. under the inches and reserve the 12 ft. 51×17 ft. = 867 ft.; 867 ft. + 12 ft. (reserved) = 879 ft. 879 ft.

can be reduced to higher denominations by dividing by 3 ft. to find the number of yards, and by $5\frac{1}{2}$ to find the number of rods.

$$\begin{array}{r|l} 3 & 8\ 7\ 9 \text{ ft. } 9 \text{ in.} \\ \hline 5.5 & 2\ 9\ 3 \text{ yd.} \\ \hline & 5\ 3 \text{ rd. } 1\tfrac{1}{2} \text{ yd.} \end{array}$$

Then, 879 ft. 9 in. = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in., or 53 rd. 1 yd. 2 ft. 3 in.

(17) Since 2 pt. = 1 qt., 3 qt. = 3×2, or 6 pt. 6 pt. + 1 pt. = 7 pt. $4.7 \times 7 = 32.9$ pt. Ans.

$$\begin{array}{rr} \text{qt.} & \text{pt.} \\ 3 & 1 \\ \times\ 2 & \\ \hline 6 & \\ +\ 1 & \\ \hline 7 \text{ pt.} & \end{array} \qquad\qquad \begin{array}{r} 7 \text{ pt.} \\ 4.7 \\ \hline 4\ 9 \\ 2\ 8 \\ \hline 3\ 2.9 \text{ pt.} \end{array}$$

(18) If there are four lengths, each 15 ft. 5 in., 15 ft. 5 in. $\times$ 4 = 60 ft. 20 in., or the length of the four pieces.

$$\begin{array}{rr} \text{ft.} & \text{in.} \\ 6\ 0 & 2\ 0 \\ 1\ 4 & 8 \\ 8 & 1\ 0 \\ \hline 8\ 2 & 3\ 8 \end{array} \qquad\qquad \begin{array}{rr} 1\ 5 \text{ ft.} & 5 \text{ in.} \\ & 4 \\ \hline 6\ 0 \text{ ft.} & 2\ 0 \text{ in.} \end{array}$$

82 38, or 85 ft. 2 in. = the length of the shaft.

From the length of the shaft we must subtract 8 in. $\times$ 2 = 16 in. to get the distance between the end hangers.

$$\begin{array}{rr} \text{ft.} & \text{in.} \\ 8\ 2 & 3\ 8 \\ & 1\ 6 \\ \hline 8\ 2 & 2\ 2 \end{array}$$

8 2 2 2, or 83 ft. 10 in.

Since there are six hangers, there are *five* spaces. The length of one space is 83 ft. 10 in $\div$ 5 = 16 ft. $9\frac{1}{5}$ in. Ans.

(19) Reducing 18 ft. $11\frac{1}{4}$ in. to inches, we have $227\frac{1}{4}$ in., or 227.25 in.

$$\begin{array}{rr}
\text{ft.} & \text{in.} \\
18 & 11\frac{1}{4} \\
12 & \\
\hline
36 & \\
18 & \\
\hline
216 & \\
11\frac{1}{4} & \\
\hline
227\frac{1}{4}\text{ in.} &
\end{array}$$

$1\frac{1}{8} \times 2$, or $2\frac{1}{4}$ in., for the two end rivets is deducted from the length, leaving 225 in., which is divided into equal spaces by the rivets.

$$\begin{array}{rl}
227.25 & \text{in.} \\
-\quad 2.25 & \text{in.} \\
\hline
225 & \text{in.}
\end{array}$$

The pitch of the rivets (or the distance between their centers) is $1\frac{1}{4}$ in., or 1.25 in.; hence, $225 \div 1.25 = 180$ spaces between the rivets. But since there will be one more rivet than the number of spaces, the number of rivets required for this boiler shell will be $180 + 1 = 181$. Ans.

$$\begin{array}{r}
1.25\,)\,225.00\,(\,180 \\
125 \\
\hline
1000 \\
1000 \\
\hline
0
\end{array}$$

ARITHMETIC.

(PART 5.)

(1) To find the second power of a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus, the second power of 108 is 108 $\times$ 108 = 11,664.

$$\begin{array}{r} 108 \\ \underline{108} \\ 864 \\ \underline{108} \\ 11664 \end{array} \text{ Ans.}$$

(2) $9^5 = 9 \times 9 \times 9 \times 9 \times 9 = 59{,}049.$ Ans.

$$\begin{array}{r} 9 \\ \underline{9} \\ 81 \\ \underline{9} \\ 729 \\ \underline{9} \\ 6561 \\ \underline{9} \\ 59049 \end{array}$$

(3) $.0133^3 = .0133 \times .0133 \times .0133 = .000002352637.$

$$\begin{array}{r} .0133 \\ \underline{.0133} \\ 399 \\ 399 \\ \underline{133} \\ .00017689 \\ \underline{.0133} \\ 53067 \\ 53067 \\ \underline{17689} \\ .000002352637 \end{array} \text{ Ans.}$$

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we should point off $4 + 4 = 8$ decimal places in the product; but as there are only 5 figures in the product, we prefix 3 ciphers to form the 8 necessary decimal places in the first product.

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we should point off $8 + 4 = 12$ decimal places in the product; but as there are only 7 figures in the product, we prefix 5 ciphers to make the necessary 12 decimal places in the final product.

(4) See page 9 of the table.

$$\begin{array}{rrcrr} 9.49^2 = & 90.0601 & \quad & \textit{given number} = & 90.0000 \\ 9.48^2 = & \underline{89.8704} & & 9.48^2 = & \underline{89.8704} \\ \textit{first difference} = & 1897 & & \textit{second difference} = & 1296 \end{array}$$

$$\begin{array}{l} 1897)1296.000(.683 \text{ or } .68 \\ \quad\quad\;\; \underline{11382} \\ \quad\quad\quad 15780 \\ \quad\quad\quad \underline{15176} \\ \quad\quad\quad\quad 6040 \\ \quad\quad\quad\quad \underline{5691} \\ \quad\quad\quad\quad\;\; 349 \end{array}$$

Therefore, $\sqrt{90} = 9.4868$. Ans.

(5) Given number = 92,416 = 92′416.

Altered number = 92.416.

See page 4 of the table.

$$\begin{array}{rrcrr} 4.53^2 = & 92.9597 & \quad & \textit{altered number} = & 92.4160 \\ 4.52^2 = & \underline{92.3454} & & 4.52^2 = & \underline{92.3454} \\ \textit{first difference} = & 6143 & & \textit{second difference} = & 706 \end{array}$$

$$\begin{array}{l} 6143)706.000(.114 \text{ or } .11 \\ \quad\quad\;\; \underline{6143} \\ \quad\quad\quad 9170 \\ \quad\quad\quad \underline{6143} \\ \quad\quad\quad 30270 \\ \quad\quad\quad \underline{24572} \\ \quad\quad\quad\;\; 5698 \end{array}$$

Therefore, $\sqrt[3]{92.416} = 4.5211$, and $\sqrt[3]{92,416} = 45.211$. Ans.

(6) Given number = 502,681 = 50′26′81.

Altered number = 50.2681.

Referring to page 7 of the table, $7.09^2 = 50.2681$; hence, $\sqrt{50.2681} = 7.09$. Since there are three periods in the integral part of the given number, there are three figures in the integral part of the root; therefore, $\sqrt{502,681} = 709$. Ans.

(7) Since $\frac{3}{8} = .375$, $\sqrt[3]{\frac{3}{8}} = \sqrt[3]{.375}$.

Given number = .375.

Altered number = 375.000.

See page 7 of the table.

$$
\begin{array}{rrrr}
7.22^3 = & 376.367 & \textit{altered number} = & 375.000 \\
7.21^3 = & \underline{374.805} & 7.21^3 = & \underline{374.805} \\
\textit{first difference} = & 1562 & \textit{second difference} = & 195
\end{array}
$$

$$
\begin{array}{l}
1562)195.000(.124 \text{ or } .12 \\
\quad\quad\;\; \underline{1562} \\
\quad\quad\quad 3880 \\
\quad\quad\quad \underline{3124} \\
\quad\quad\quad\; 7560 \\
\quad\quad\quad\; \underline{6248} \\
\quad\quad\quad\; 1312
\end{array}
$$

Therefore, $\sqrt[3]{375} = 7.2112$, and $\sqrt[3]{.375} = \sqrt[3]{\frac{3}{8}} = .72112$. Ans.

(8) Given number = .3364 = .33′64.

Altered number = 33.6400.

Referring to page 5 of the table, $5.80^2 = 33.6400$; therefore, $\sqrt{33.6400} = 5.80$, and $\sqrt{.3364} = .58$. Ans.

(9) Given number = .7854 = .78′54.

Altered number = 78.5400.

See page 8 of the table.

$$
\begin{array}{rrrr}
8.87^2 = & 78.6769 & \textit{altered number} = & 78.5400 \\
8.86^2 = & \underline{78.4996} & 8.86^2 = & \underline{78.4996} \\
\textit{first difference} = & 1773 & \textit{second difference} = & 404
\end{array}
$$

```
1773)404.000(.227 or .23
     3546
      4940
      3546
      13940
      12411
       1529
```

Therefore, $\sqrt{78.54} = 8.8623$, and $\sqrt{.7854} = .88623$. Ans.

(10) The number is evidently the square root of 114.9184.
Given number = 114.9184 = 1′14′.91′84.
Altered number = 1.149184 = 1.1492.
See page 1 of the table.

$$
\begin{array}{lrlr}
 & 1.08^2 = 1.1664 & \textit{altered number} = & 1.1492 \\
 & 1.07^2 = \underline{1.1449} & 1.07^2 = & \underline{1.1449} \\
\textit{first difference} = & 215 & \textit{second difference} = & 43
\end{array}
$$

```
215)43.0(.2
     430
```

Therefore, $\sqrt{1.1492} = 1.072$, and $\sqrt{114.9184} = 10.72$. Ans.

(11) Given number = 3,486,784 = 3′48′67′84.
Altered number = 3.486784 = 3.4868.
See page 1 of the table.

$$
\begin{array}{lrlr}
 & 1.87^2 = 3.4969 & \textit{altered number} = & 3.4868 \\
 & 1.86^2 = \underline{3.4596} & 1.86^2 = & \underline{3.4596} \\
\textit{first difference} = & 373 & \textit{second difference} = & 272
\end{array}
$$

```
373)272.000(.729 or .73
    2611
     1090
      746
      3440
      3357
        83
```

Therefore, $\sqrt{3.4868} = 1.8673$, and $\sqrt{3,486,784} = 1,867.3$.
Ans.

(12) Given number = .00041209 = .00′04′12′09.
Altered number = 4.1209.
See page 2 of the table.
$2.03^2 = 4.1209$; therefore, $\sqrt{4.1209} = 2.03$, and $\sqrt{.00041209} = .0203$. Ans.

(13) $11.7 : 13 :: 20 : x$. The product of the means equals the product of the extremes.

$$11.7\,x = 13 \times 20$$
$$11.7\,x = 260$$
$$x = \frac{260}{11.7}$$

```
11.7)260.000(22.22+   Ans.
     234
      260
      234
       260
       234
        260
        234
         26
```

(14) (*a*) $20 + 7 : 10 + 8 :: 3 : x$.

$$27 : 18 :: 3 : x$$
$$27\,x = 18 \times 3$$
$$27\,x = 54$$
$$x = \frac{54}{27} = 2. \quad \text{Ans.}$$

(*b*) $12^2 : 100^2 :: 4 : x$.

$$144 : 10{,}000 :: 4 : x$$
$$144\,x = 10{,}000 \times 4$$
$$144\,x = 40{,}000$$
$$x = \frac{40{,}000}{144}$$

```
 12       100
 12       100
144     10000
```

```
144)40000.0(277.7+   Ans.
     288
     1120
     1008
      1120
      1008
       1120
       1008
```

(15) (*a*) $\frac{4}{x} = \frac{7}{21}$ is equivalent to $4 : x :: 7 : 21$. The product of the means equals the product of the extremes. Hence,

$$7x = 4 \times 21$$
$$7x = 84$$
$$x = \frac{84}{7}, \text{ or } 12. \quad \text{Ans.}$$

In like manner,

(*b*) $\frac{x}{24} = \frac{8}{16}$ is equivalent to $x : 24 :: 8 : 16$.

$$16x = 24 \times 8$$
$$16x = 192$$
$$x = \frac{192}{16} = 12. \quad \text{Ans.}$$

(*c*) $\frac{2}{10} = \frac{x}{100}$ is equivalent to $2 : 10 :: x = 100$.

$$10x = 2 \times 100$$
$$10x = 200$$
$$x = \frac{200}{10} = 20. \quad \text{Ans.}$$

(*d*) $\frac{15}{45} = \frac{60}{x}$ is equivalent to $15 : 45 :: 60 : x$.

$$15x = 45 \times 60$$
$$15x = 2{,}700$$
$$x = \frac{2{,}700}{15} = 180. \quad \text{Ans.}$$

(*e*) $\frac{10}{150} = \frac{x}{600}$ is equivalent to $10 : 150 :: x : 600$.

$$150x = 10 \times 600$$
$$150x = 6{,}000$$
$$x = \frac{6{,}000}{150} = 40. \quad \text{Ans.}$$

(16) $3\frac{1}{2}$ ft. = 3.5 ft., since $\frac{1}{2} = .5$.
$6\frac{3}{4}$ ft. = 6.75 ft., since $\frac{3}{4} = .75$.

Consider the question "What do we wish to find?" In this case it is "pounds." We know that a piece of shafting 3.5 ft. long weighs 37.45 lb., and we wish to know the weight of a piece of shafting $6\frac{3}{4}$ ft. long. It is evident that the weight of a piece of shafting 6.75 ft. long bears the same relation to the weight of a piece 3.5 ft. long that 6.75 ft. bears to 3.5 ft. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(*a*) 3.5 ft. : 6.75 ft. :: 37.45 lb. : x lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*b*) 6.75 ft. : 3.5 ft. :: x lb. : 37.45 lb.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*c*) x lb. : 37.45 lb. :: 6.75 ft. : 3.5 ft.,

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(*d*) 37.45 lb. : x lb. :: 3.5 ft. : 6.75 ft.

$$\text{or } x = \frac{6.75 \times 37.45}{3.5} = \frac{252.7875}{3.5} = 72.225 \text{ lb. Ans.}$$

(17) We will first reduce 8 hr. 40 min. to minutes. 8 hr. + 40 min. = (8 × 60 min.) + 40 min. = 520 min. In this problem we are required to find "time." We know that a railway train runs 444 miles in 520 minutes, and we want to know how long it will take it to run 1,060 miles at the same rate of speed. It is evident that the time it requires to run 1,060 miles bears the same relation to the time it takes to run 444 miles that 1,060 miles bears to 444 miles. Letting x occupy any place in the proportion, we have the following, the value of x being the same in each. Thus,

(a) 1,060 miles : 444 miles :: x min. : 520 min.,

$$\text{or } x = \frac{1{,}060 \times 520}{444} = \frac{551{,}200}{444} =$$

```
   1060         444)551200.00(1241.44+ min.
    520             444
  -----             ----
  21200             1072
 5300                888
 ------              ----
 551200              1840
                     1776
                     ----
                       640
                       444
                       ----
                       1960
                       1776
                       ----
                        1840
                        1776
                        ----
                          64
```

Reducing 1,241.44 min. to hours by dividing by 60, we have

```
60)1241.44(20 hr. 41.44 min.   Ans.
   120
   ---
     41
```

(b) 444 miles : 1,060 miles :: 520 min. : x min.

$$x = \frac{1{,}060 \times 520}{444} = 1{,}241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min.} \quad \text{Ans.}$$

(c) x min. : 520 min. :: 1,060 miles : 444 miles.

$$x = \frac{1{,}060 \times 520}{444} = 1{,}241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min.} \quad \text{Ans.}$$

(d) 520 min. : x min. :: 444 miles : 1,060 miles.

$$x = \frac{1{,}060 \times 520}{444} = 1{,}241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min.} \quad \text{Ans.}$$

(18) A pump discharging 135 gal. per min. fills the tank in 38 min. Therefore, a pump discharging 1 gal. per

min. fills it in 135×38 min. Hence, a pump discharging 85 gal. per min. fills it in $\frac{135 \times 38}{85} = 60\frac{6}{17}$ min. Ans.

$$
\begin{array}{r}
135 \\
\times \quad 38 \\
\hline
1080 \\
405 \\
\hline
85)\,5130\,(60\tfrac{6}{17} \\
510 \\
\hline
30 \\
\hline
85
\end{array}
\qquad \frac{30}{85} = \frac{6}{17}
$$

(19) If a wheel measuring 12.56 ft. around it turns 520 times, a wheel measuring 1 ft. around it turns 520×12.56 times. Hence, a wheel measuring 15.7 ft. around it turns $\frac{520 \times 12.56}{15.7} = 416$ times. Ans.

$$
\begin{array}{r}
520 \\
12.56 \\
\hline
3120 \\
2600 \\
1040 \\
520 \\
\hline
15.7)\,6531.20\,(416 \text{ times} \\
628 \\
\hline
251 \\
157 \\
\hline
942 \\
942 \\
\hline
\end{array}
$$

(20) If a cistern 28 ft. by 12 ft. by 10 ft. holds 798 bbl. of water,

a cistern 1 ft. by 12 ft. by 10 ft. holds $\frac{798}{28}$ bbl.;

a cistern 1 ft. by 1 ft. by 10 ft. holds $\frac{798}{28 \times 12}$ bbl.;

a cistern 1 ft. by 1 ft. by 1 ft. holds $\frac{798}{28 \times 12 \times 10}$ bbl.

Therefore, by a similar course of reasoning, a cistern 20 ft. by 17 ft. by 6 ft. holds

$$\frac{798 \times 20 \times 17 \times 6}{28 \times 12 \times 10} = \frac{\overset{\overset{57}{\cancel{114}}}{\cancel{798}} \times \overset{\cancel{2}}{\cancel{20}} \times 17 \times \cancel{6}}{\underset{\underset{2}{\cancel{4}}}{\cancel{28}} \times \underset{\cancel{2}}{\cancel{12}} \times \cancel{10}} = \frac{969}{2} = 484\tfrac{1}{2} \text{ bbl.}$$

Ans.

$$\begin{array}{r} 57 \\ 17 \\ \hline 399 \\ 57 \\ \hline 2\,)\,969 \\ \hline 484\tfrac{1}{2} \end{array}$$

MENSURATION AND USE OF LETTERS IN FORMULAS.

(1) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line is to be divided by the one below the line.

(2) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2\,x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(3) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}}$$

$$= \sqrt{16.4383} = 4.05+. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(4) Substituting for A, B, D, and h their values,

$$g = \frac{(B - A)^2 - \sqrt{h + 2\,B + A}}{A^3 - (1 + D)} = \frac{(10 - 5)^2 - \sqrt{200 + 2 \times 10 + 5}}{5^3 - (1 + 120)}$$

$$= \frac{5^2 - \sqrt{225}}{125 - 121} = \frac{25 - 15}{4} = \tfrac{10}{4} = 2\tfrac{1}{2}. \quad \text{Ans.}$$

(5) When one straight line meets another straight line, two angles are formed which together equal 180°. Hence, if one of the angles = 152° 3′, the other angle = 180° − 152° 3′, or

$$\begin{array}{rl} 180^\circ = & 179^\circ\ 60' \\ \text{subtracting,} & \underline{152^\circ\ \ 3'} \\ & 27^\circ\ 57' \quad \text{Ans.} \end{array}$$

(6) See Arts. **26-28.**

(7) See Art. **41.** A rectangle with the same area would have the same base and altitude.

(8) Since the area is to be found in square inches, the $2\frac{1}{2}$ feet must be reduced to inches. $2\frac{1}{2}$ ft. = 30 in. Area = $30 \times 11\frac{1}{2} = 345$ sq. in. Ans.

(9) It will take $1\frac{1}{2}$ boards to reach lengthways of the room. Since the room is 15 feet wide and each board is 5 *inches* wide, it will take $15 \div \frac{5}{12} = 36$ boards, laid side by side, to extend across the width of the room. Hence, number of boards required = $36 \times 1\frac{1}{2} = 54$. Ans.

(10) The total area of the floor of the station = 55×58 ft. = 3,190 sq. ft. − 25×26 ft. = 650 sq. ft., the area represented by the lower right-hand corner of the figure. Hence, total area of floor = 3,190 − 650 = 2,540 sq. ft.

From this we have to deduct the following areas:

$$\begin{array}{lcrl} \text{2 boilers} & = 2 \times 8 \times 19 = & 304 & \text{sq. ft.} \\ \text{Feed-pump} & = \quad 2\frac{1}{2} \times 5 \quad = & 12.5 & \text{sq. ft.} \\ \text{2 engines} & = 2 \times 4\frac{1}{2} \times 10 = & 90 & \text{sq. ft.} \\ \text{2 dynamos} & = 2 \times 5\frac{1}{2} \times 6\frac{1}{2} = & 71.5 & \text{sq. ft.} \\ \text{Switchboard} = & \dfrac{10 \times 3.5}{12} \quad = & 2.92 & \text{sq. ft.} \\ \hline & & 480.92 & \text{sq. ft.} \end{array}$$

The unoccupied floor space, therefore, equals

$$2{,}540 - 480.92 = 2{,}059.08 \text{ sq. ft.} \quad \text{Ans.}$$

(11) A triangle with three equal angles has three equal sides, and is therefore an equilateral triangle.

(12) A triangle with two equal angles has two equal sides, and is therefore an isosceles triangle.

(13) The sum of the three angles in any triangle = 2 right angles, or 180°. In the given triangle, the sum of two angles $= 23° + 32° \ 32' = 55° \ 32'$, and the third angle $= 180° - 55° \ 32'$, or

$$\begin{array}{rr} 180° = & 179° \ 60' \\ \text{subtracting,} & 55° \ 32' \\ \hline & 124° \ 28' \end{array} \quad \text{Ans.}$$

(14) In Fig. I we have the proportion $A D : D E :: A B : B C$, in which $A D = 10$ in., $A B = 24$ in., and $B C = 13\frac{1}{2}$ in., to find $D E$.

Substituting the given values,

$$10 : D E :: 24 : 13\tfrac{1}{2}, \text{ or}$$

$$D E = \frac{10 \times 13.5}{24} = 5.625 \text{ in.} \quad \text{Ans.}$$

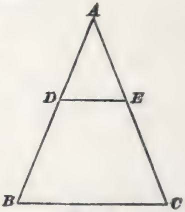


FIG. I.

(15) A line drawn diagonally from one corner to the opposite one would form the hypotenuse of a right triangle, whose two sides are 39 and 52 feet. By rule **6**, Art. **58**, the length of the diagonal $= \sqrt{52^2 + 39^2} = 65$ ft. Ans.

(16) See example, Art. **64.** The process is simply to find one of the angles of the polygon, and then to divide it by 2. By rule **10**, Art. **64**, one of the interior angles $= \frac{180 \times (8 - 2)}{8} = 135°$. This divided by $2 = 67\frac{1}{2}°$. Ans.

(17) Since this is a regular hexagon, it may be inscribed in a circle (Fig. II), and the radius of the inscribing circle will be equal to one side of the hexagon. Since the diameter $E F = 2$ inches, the radii $A B$ and $A C$, and the side $B C$ each = 1 inch, and the triangle $A B C$ is equilateral. Draw the line $A D$ perpendicular to the side $B C$; it will bisect $B C$. Then, in the right-angled triangle $A D B$, $A B = 1''$ and $B D = \frac{1}{2}''$,

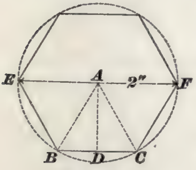


FIG. II.

to find $A D$. According to rule **7**, Art. **59**, $A D = \sqrt{1^2 - .5^2} = \sqrt{.75} = .866''$. Hence, the distance between two opposite sides of the hexagon $= A D \times 2 = .866 \times 2 = 1.732''$. Ans.

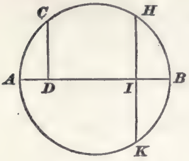


FIG. III.

(18) In Fig. III, we have the proportion $B I : H I :: H I : I A$, in which $B I = 6$ and $H I = \frac{1}{2}$ of $H K = \frac{18}{2} = 9$. Substituting, $6 : 9 :: 9 : I A$, or $I A = \frac{81}{6} = 13.5$ in. Hence, the diameter $A B = I A + B I = 13.5 + 6 = 19.5$ in. Ans.

(19) One mile = 5,280 feet. The circumference of the wheel in feet $= \frac{72 \times 3.1416}{12} = 18.8496$. (See rule **12**, Art. **77.**) Number of revolutions in going 1 mile $= 5,280 \div 18.8496 = 280.112$. Ans.

(20) Using rule **15**, Art. **80**, area = diameter squared $\times .7854$. $6.06^2 = 36.7236$; $36.7236 \times .7854 = 28.8427$ sq. in. Ans.

(21) Since the radius of the circle = 6 in., its diameter = 12 in., and its circumference $= 12 \times 3.1416 = 37.6992$ in. There are 360° in the circumference, and the length of an arc of $12° = 37.6992 \times \frac{12}{360} = 1.25664$ in. Ans.

(22) The area of a circle 15 in. in diameter $= 15^2 \times .7854 = 176.715$ sq. in. Hence, the area of a sector of this circle whose angle is $12\frac{1}{2}° = 176.715 \times \frac{12\frac{1}{2}}{360} = \frac{2,208.937}{360} = 6.1359$ sq. in. Ans. (See rule **17**, Art. **82.**)

(23) (*a*) The side of a square whose area = 103.8691 sq. in. $= \sqrt{103.8691} = 10.1916$ in. Ans.

(*b*) By rule **16**, Art. **81**, the diameter of a circle having the same area $= \sqrt{\frac{103.8691}{.7854}} = 11\frac{1}{2}$ in. Ans.

(*c*) Perimeter of the square $= 10.1916 \times 4 = 40.7664$ in.; circumference of the circle $= 11.5 \times 3.1416 = 36.1284$ in.; difference $= 40.7664 - 36.1284 = 4.638$ in. Ans.

(24) The perimeter of the base $= 4 \times 6 = 24$ in. $= 2$ ft. Convex area $= 2 \times 12 = 24$ sq. ft. The area of the bases is found as follows: In Fig. IV, $AB = 4$ in. and $AC = 2$ in.; since this is a regular hexagon, $AO = AB = 4$ in. By rule **7**, Art. **59**, $OC = \sqrt{4^2 - 2^2} = \sqrt{12} = 3.4641$ in.; area of triangle $AOB = \frac{4 \times 3.4641}{2} = 6.9282$ sq. in.; area of base $= 6.9282 \times 6 = 41.5692$; and the area of both bases $= 41.5692 \times 2 = 83.1384$ sq. in. This reduced to square feet $= \frac{83.1384}{144} = .5774$. Hence, the area of the entire surface of the column is $24 + .5774 = 24.5774$ sq. ft. Ans.

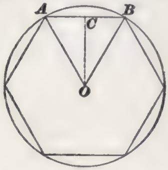


FIG. IV.

(25) The cubical contents in cubic inches = area of base in square inches $\times$ altitude in inches. The area of the base in the last example was found to be 41.5692 sq. in.; altitude $= 12 \times 12 = 144$ in. Hence, the cubical contents $= 41.5692 \times 144 = 5{,}985.9648$ cu. in. Ans.

(26) This example is solved by combining the rules for the circular ring (see example, Art. **81**) and for the cylinder. To obtain the area of one end of the tube, we have $4^2 \times .7854 = 12.5664$ = area of a circle 4 inches in diameter; $3.73^2 \times .7854 = 10.9272$ = area of a circle 3.73 inches in diameter; difference $= 12.5664 - 10.9272 = 1.6392$ = area of one end of the tube. The cubical contents $= 1.6392 \times 12 = 19.6704$ cu. in.; the weight $= 19.6704 \times .28 = 5.5$, or $5\frac{1}{2}$ lb. Ans.

(27) This example is done exactly like the one in Art. **92**, and the solution is given here without explanation.

(*a*) In the formula of rule **18**, Art. **83**, $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h in this case $= 18$, and $D = 60$.

Substituting, area =

$$\frac{4 \times 18^2}{3}\sqrt{\frac{60}{18} - .608} = \frac{4 \times 324}{3}\sqrt{3.333 - .608} = 432 \times \sqrt{2.725}$$

$= 432 \times 1.65 = 712.8$ sq. in. This reduced to square feet $= 712.8 \div 144 = 4.95$. Hence, the steam space $= 4.95 \times 16 = 79.2$ cu. ft. Ans.

(*b*) Total area of one end of boiler in square inches $= 60^2 \times .7854 = 2{,}827.44$. From this is to be subtracted the area of the tube ends and of the segment found above.

Area of ends of tubes	$= 3.5^2 \times .7854 \times 64 =$	615.75 sq. in.
Area of segment	$=$	712.8 sq. in.
		1,328.55 sq. in.

Area of water space $= 2{,}827.44 - 1{,}328.55 = 1{,}498.89$ sq. in.

Contents of water space $= 1{,}498.89 \times 16 \times 12 = 287{,}786.88$ cu. in., and $287{,}786.88 \div 231 = 1{,}245.83$, number of gallons, or say 1,246 gal. Ans.

(28) The area of the convex surface = circumference of base $\times \frac{1}{2}$ slant height $= 18.8496 \times \frac{10}{2} = 94.248$ sq. in. (See rule **21**, Art. **97**.) The area of the entire surface $= 94.248$ sq. in. + the area of the base. The diameter of the base $= \frac{18.8496}{3.1416} = 6$ in.; hence, the area of the base $= 6^2 \times .7854 = 28.2744$ (rules **13** and **15**, Arts. **78** and **80**); therefore, the area of the entire surface $= 94.248 + 28.2744 = 122.5224$ sq. in. Ans.

(29) Using rule **22**, Art. **98**, volume = area of base $\times \frac{1}{3}$ altitude $= 28.2744 \times \frac{9}{3} = 84.8232$ cu. in. Ans.

(30) The vat has the form of an inverted frustum of a pyramid. Area of larger base $= 15^2 = 225$ sq. ft.; area of smaller base $= 12^2 = 144$ sq. ft. Hence, by rule **24**, Art. **102**, the contents of the vat in cubic feet =

$$(225 + 144 + \sqrt{225 \times 144})\frac{11}{3} = (369 + 180) \times \frac{11}{3} = 549 \times \frac{11}{3}$$

$= 2{,}013$ cu. ft. This should be reduced to cubic inches by

multiplying by 1,728, the number of cubic inches in a cubic foot. $2,013 \times 1,728 = 3,478,464$ cu. in. Since there are 231 cubic inches in a gallon, the number of gallons that the vat will hold $= \frac{3,478,464}{231} = 15,058.29$. Ans.

(31) The pail is in the form of a frustum of a cone. Area of larger base $= 12^2 \times .7854 = 113.0976$ sq. in. Area of smaller base $= 63.6174$ sq. in. Hence, the contents in cubic inches =

$$(113.0976 + 63.6174 + \sqrt{113.0976 \times 63.6174}) \times \frac{11}{3}$$

$$= (176.715 + \sqrt{7,194.9753})\frac{11}{3} = (176.715 + 84.8232) \times \frac{11}{3}$$

$$= 261.5382 \times \frac{11}{3} = 958.9734.$$

The contents of the vat in cubic inches were found in the last example to be 3,478,464. Hence, the number of pails of water required to fill the vat $= 3,478,464 \div 958.9734 = 3,627.28$. Ans.

(32) (*a*) By rule **25,** Art. **104,** area of the surface $= 22.5^2 \times 3.1416 = 506.25 \times 3.1416 = 1,590.435$ sq. in. Ans.

(*b*) Using rule **26,** Art. **105,** the cubical contents = the cube of the diameter $\times .5236 = 11,390.625 \times .5236 = 5,964.1313$ cu. in. Ans.

(33) (*a*) Given $OB = \frac{16}{2}$, or 8 inches, and $OA = \frac{13}{2}$, or $6\frac{1}{2}$ inches, to find the volume, area, and weight (see Fig. V):

Radius of center circle equals $\frac{8 + 6.5}{2}$, or $7\frac{1}{4}$ inches.

Length of center line $= 2 \times 3.1416 \times 7\frac{1}{4} = 45.5532$ inches.

The radius of the inner circle is $6\frac{1}{2}$ inches and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, according to rule **27,** Art. **106,**

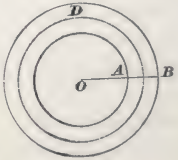


FIG. V.

the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 = 214.665$ square inches. Ans.

Diameter of cross-section of ring $= 1\frac{1}{2}$ inches.

Area of cross-section of ring $= (1\frac{1}{2})^2 \times .7854 = 1.76715$ sq. in. Ans.

By rule **28**, Art. **107**, volume of ring $= 1.76715 \times 45.553 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.499 \times .261 = 21$ lb. Ans.

PRINCIPLES OF MECHANICS.

(1) Broadly speaking, mechanics is the science that treats of forces and their effects on material bodies. See Art. **1.**

(2) See Arts. **2, 3,** and **4.**

(3) Solid, liquid, and gaseous. See Arts. **6, 7,** and **8.**

(4) General properties are such as are common to all substances; special properties are such as are possessed by certain substances only. See Arts. **12** and **24.**

(5) (*a*) Motion is that condition of a body that causes it to change its position in relation to some other body. See Art. **30.**

(*b*) Velocity is the rate of motion, that is, the distance passed through in a unit of time. See Art. **32.**

(*c*) Uniform velocity means passing over equal distances in equal intervals of time. See Art. **32.**

(*d*) Variable velocity means passing over equal distances in unequal intervals of time, or passing over unequal distances in equal intervals of time. See Art. **33.**

(6) (*a*) Acceleration is the rate of change of velocity. See Arts. **35** and **57.**

(*b*) Retardation denotes the rate of decrease of velocity. See Arts. **36** and **58.**

(*c*) The average velocity is that uniform velocity that carries the body over the same distance in the same time as the variable velocity. See Art. **37.**

(7) Since 1 day contains 24 hours, the total number of hours consumed in the trip was $6 \times 24 + 16 = 160$ hours. The average speed per hour was, therefore, $3,240 \div 160 = 20\frac{1}{4}$ mi. Ans. See Art. **38.**

(8) Reducing the 25,000 miles to feet, we have $25,000 \times 5,280 = 132,000,000$ feet. The time in minutes would, therefore, be $132,000,000 \div 3,000 = 44,000$ minutes. Reducing the time to days, hours, and minutes, we get 30 da. 13 hr. 20 min. Ans. See Art. **40.**

(9) By the effects it produces on matter. See Art. **41.**

(10) The point of application, the direction of action, and the magnitude of each force must be known. See Art. **44.**

(11) See Art. **46.**

(12) Inertia is that property of a body by virtue of which the body always tends to remain in the particular state of rest or motion that it has at the moment considered. See Art. **48.**

(13) See Arts. **52** and **53.**

(14) The weight of a body is proportional to the force of gravity; and since the force of gravity is different for different localities, the weight of any body will differ likewise. See Art. **56.**

(15) By rule **4,** Art. **60,**

$$m = \frac{W}{g} = \frac{346}{32.174} = 10.75+. \quad \text{Ans.}$$

(16) See Art. **62.**

(17) The constant opposing force is $30 \times 15 = 450$ pounds. 30 tons $= 30 \times 2,000 = 60,000$ pounds. The force f to produce the required acceleration will, by rule **5,** Art. **63,** be $\frac{W}{g}a = \frac{60,000}{32.16} \times 3 = 5,597+$ pounds. Hence, the total force required will be $5,597 + 450 = 6,047$ lb. Ans.

(18) 1 mile per hour = 5,280 feet per hour. $\frac{5{,}280}{60 \times 60} = \frac{22}{15} = 1\frac{7}{15}$ feet per second. Momentum, according to Art. **67**, $= \frac{60{,}000}{32.16} \times \frac{22}{15} = 2{,}736.3$ lb. Ans.

(19) (*a*) Work is the overcoming of a resistance through a certain distance. See Arts. **69, 70,** and **71.**

(*b*) Power is the rate of doing work. See Arts. **72** and **73.**

(*c*) Energy denotes ability to do work. See Arts. **74** and **75.**

(20) $\frac{66{,}000 \times 80}{33{,}000 \times 30} = 5\frac{1}{3}$ H. P. Ans. See Art. **73.**

(21) The kinetic energy of the body by rule **7,** Art. **76,** is $\frac{w v^2}{2 g} = \frac{6{,}432 \times 60^2}{2 \times 32.16} = 360{,}000$ foot-pounds. This amount of work must be done in 3 minutes. Hence, the horsepower required to stop the body will be $\frac{360{,}000}{3} \div 33{,}000 = 3\frac{7}{11}$ H. P. Ans.

(22) Zero. See Art. **86.**

(23) Friction is the resistance that a moving body encounters from the surface of another body along which or through which it moves. See Art. **88.**

(24) According to Art. **89,** the coefficient of friction will be $\frac{300}{5000} = .06$. Ans.

(25) (*a*) From Table I we find the coefficient of friction to be .16. Hence, the total friction will be $1{,}000 \times .16 = 160$ lb. Ans. See Art. **92.**

(*b*) $\frac{160}{100} = 1.6$ lb. per sq. in. Ans.

(26) The center of gravity of a body is a point at which the whole weight of the body may be considered as concentrated. This point may be either inside or outside the body. See Art. **98.**

(27) See Art. **99.**

(28) Suspend the body successively from two different points and find the point of intersection of two plumb-lines from the points of suspension. The center of gravity will be on the line passing through the point of intersection and perpendicular to the plumb-lines. See Art. **103.**

(29) (*a*) Centrifugal force is that force that tends to pull a revolving body away from the point about which the body revolves. See Art. **104.**

(*b*) Centripetal force is that force that tends to pull a revolving body toward the point about which it revolves. See Art. **105.**

(30) The centrifugal and centripetal forces of a given revolving body are always equal and opposite. See Art. **104.**

(31) Applying rule **14,** given in Art. **106,**

$$F = .00034 \times 10 \times 6 \times 60^2 = 73.44 \text{ lb.} \quad \text{Ans.}$$

(32) See Arts. **110, 111,** and **112.**

(33) The forces acting on a body at rest are in equilibrium. See Art. **109.**

(34) As long as the line of direction falls within the base, the body will stand; if it falls outside the base, the body will fall. In the former case the forces will be in equilibrium; in the latter they will not be in equilibrium. See Art. **113.**

MACHINE ELEMENTS.

(1) (*a*) A lever is a rigid bar or rod capable of being turned about a pivot.

(*b*) The weight arm is that part of the lever between the fulcrum and the weight.

(*c*) The force arm is that part of the lever between the fulcrum and the force.

(*d*) The fulcrum is the point or pivot about which the lever turns. See Arts. **1** and **2.**

(2) The product of the weight and the weight arm must be equal to the product of the force and the force arm. See Art. **2.**

(3) See rule **1,** Art. **3.**

(4) $\frac{12 \times 20}{100} = 2.4$ in. Ans. See Art. **3.**

(5) Solid pulleys and split pulleys. See Art. **9.**

(6) Split pulleys can be more easily put on and removed from shafts than solid pulleys. See Art. **9.**

(7) As explained in Art. **11,** the belt climbs toward the highest part of the pulley; and since in a crowned pulley the highest portion is in the center, the belt will tend to stay on the pulley.

(8) "Balancing pulleys" is the operation of making opposite sides of pulleys equal in weight, so that the centrifugal

forces of the opposite sides of a revolving pulley will be equal. A pulley is balanced by first finding which side is the lighter and then adding weights to the lighter side until it is as heavy as the other. See Art. **12.**

(9) The "driver" is the pulley that imparts motion to the belt. The "driven" is the pulley that receives motion from the belt. See Art. **14.**

(10) Applying rule **3,** Art. **15,**

$$D = \frac{6 \times 1{,}800}{600} = 18 \text{ in. Ans.}$$

(11) Applying rule **6,** Art. **18,**

$$N = \frac{5 \times 1{,}600}{12} = 666\tfrac{2}{3} \text{ rev. per min. Ans.}$$

(12) Applying rule **7,** Art. **23,**

$$B = 3\tfrac{1}{4}\left(\frac{6+2}{2}\right) + 2 \times 40 = 93 \text{ ft. Ans.}$$

(13) From Table I we find that the allowable effective pull C for an arc of contact of 120° is 28.8 pounds. Applying rule **8,** Art. **27,**

$$W = \frac{33{,}000 \times 40}{1{,}650 \times 28.8} = 27.7, \text{ say } 28 \text{ in. Ans.}$$

(14) Applying rule **9,** Art. **28,**

$$H = \frac{28.8 \times 28 \times 1{,}200}{33{,}000} = 29.3+ \text{ H. P. Ans.}$$

(15) Applying rule **10,** Art. **31,**

$$W = \frac{23{,}100 \times 40}{1{,}650 \times 28.8} = 19.44, \text{ say } 20 \text{ in. Ans.}$$

(16) The hair or grain side is commonly considered to be the proper side to be in contact with the pulley face. For reasons see Art. **32.**

(17) No; rosin makes the belt gummy and causes it to crack. See Art. **32.**

(18) Pulleys run out of true; they should be turned true. Pulleys out of line; they should be lined up. Belts sometimes flap if they are run at speeds over 4,000 feet per minute; perforating the belt with a series of small holes is said to remedy the defect in this case. Lack of steadiness in running; take steps to insure steady running. A defective joint; unlace the joint and relace it properly. Too great a distance between pulleys; reduce the distance if possible or substitute a wider belt. See Art. **33.**

(19) Lacing, sewing, riveting, and cementing. The cemented joint is considered the best.

(20) See Art. **38.**

(21) A wheel that imparts motion to another is called a "driver." A "follower" is a wheel that receives motion from another.

(22) Letting W represent the weight we have, by rule **12,** Art. **39,**

$$50 \times 90 \times 30 \times 36 = W \times 30 \times 12 \times 20,$$

or

$$4{,}860{,}000 = 7{,}200\ W.$$

Hence, $W = \dfrac{4{,}860{,}000}{7{,}200} = 675$ lb. Ans.

(23) (*a*) Circular pitch is the distance measured along the pitch circle from a point on one tooth to the corresponding point on the next tooth. See Art. **47.**

(*b*) Diametral pitch is the number of teeth per inch of pitch diameter. See Art. **47.**

(24) The epicycloidal and the involute. See Arts. **51** and **52.**

(25) Involute teeth are stronger and their action is more satisfactory. See Art. **52.**

(26) Applying rule **13,** Art. **54,**

$$D = \frac{1.152 \times 60}{3.1416} = 22 \text{ in.} \quad \text{Ans.}$$

(27) Applying rule **15**, Art. **56**,

$$P = \frac{3.1416 \times 30}{60} = 1.57 \text{ in. Ans.}$$

(28) Applying rule **17**, Art. **59**,

$$D_o = \frac{80 + 2}{8} = 10\tfrac{1}{4} \text{ in. Ans.}$$

(29) Applying rule **21**, Art. **63**,

$$N = 7.5 \times 8 - 2 = 58 \text{ teeth. Ans.}$$

(30) A fixed pulley has a stationary block. A movable pulley has a movable block. See Arts. **71** and **72.**

(31) Calling W the weight that can be raised, we have, by applying rule **27**, Art. **74**,

$$W = 150 \times 14 = 2{,}100 \text{ lb. Ans.}$$

(32) The force that must be applied is greater than if there were no frictional losses. See Art. **75.**

(33) Applying rule **28**, Art. **75**,

$$f_a = \frac{1{,}500}{8 \times .60} = 312.5 \text{ lb. Ans.}$$

(34) With the Weston differential pulley block a much greater weight can be raised with a given force than with the ordinary pulley; and the load can be stopped anywhere by ceasing to pull on the chain. See Art. **78.**

(35) Applying rule **30**, Art. **82**,

$$P = \frac{600 \times 12}{70} = 102.8+, \text{ say } 103 \text{ lb. Ans.}$$

(36) Applying rule **31**, Art. **83**,

$$W = \frac{36 \times 50}{15} = 120 \text{ lb. Ans.}$$

(37) First applying rule **34**, Art. **89**, we find the factor by which the theoretical weight is to be multiplied.

Thus, $$E = \frac{\frac{1}{6}}{\frac{1}{6} + 2\frac{1}{2}} = \frac{1}{16}.$$

Then applying rule **35**, Art. **90**, to obtain the theoretical weight, we find

$$W = \frac{6.2832 \times 50 \times 20}{\frac{1}{6}} = 37{,}699.2 \text{ pounds.}$$

Finally applying the principle of Art. **92**, we find the probable actual weight to be

$$37{,}699.2 \times \tfrac{1}{16} = 2{,}356.2 \text{ lb.} \quad \text{Ans.}$$

(38) (*a*) The velocity ratio is the ratio between the distance through which the force acts and that through which the weight moves. See Art. **103**.

(*b*) The efficiency is the ratio of the actual work to the theoretical work. See Art. **105**.

(39) By Art. **105**, the efficiency would be $\frac{2{,}356.2}{37{,}699.2} = .0625$, or $6\frac{1}{4}$ per cent. Ans.

MECHANICS OF FLUIDS.

(1) Hydrostatics is that branch of mechanics that treats of liquids at rest. See Art. **1.**

(2) See Art. **4.**

(3) See Art. **5.**

(4) The pressure due to the weight of water acts, like any other pressure to which the water may be subjected, in *all* directions. See Art. **5.**

(5) First find the area of the base. This equals $.7854 \times 3^2 = 7.0686$ square inches. Multiplying this by the height of the water, we get $7.0686 \times 8 = 56.5488$ cubic inches as the volume of a column of water whose weight is equal to the pressure on the base. Therefore, the pressure on the base $= 56.5488 \times .03617 = 2.045+$ lb. Ans. See Arts. **7, 8,** and **9.**

(6) See Art. **12.**

(7) The water stands at the same level in each tube, because it is then in equilibrium. See Art. **16.**

(8) Because of the resistance of the air and the friction in the pipe. See Art. **17.**

(9) Specific gravity is the ratio between weights of equal volumes of any substance and water. Thus, the specific gravity of iron is the ratio between the weight of a given volume of iron and the weight of an equal volume of water. See Art. **23.**

(10) Using the principle of Art. **25** and taking the specific gravity from Table I, we have for the weight of the aluminum $62.42 \times 2.50 \times 100 = 15,605$ lb. Ans.

(11) A body immersed in a liquid will be buoyed up with a force equal to the weight of the liquid displaced. See Art. **26.**

(12) See Art. **29.**

(13) (*a*) Hydrometers are instruments for determining the specific gravity of liquids and of some forms of solids.

(*b*) Hydrometers of constant weight and hydrometers of constant volume. See Art. **30.**

(14) Applying rule **5,** we have

$$n = \frac{.7854 \times 1^2 \times 400}{19.25} = 16.32 \text{ gallons.}$$

As there are 60 minutes in 1 hour, the amount of water that can be pumped in 1 hour will be $16.32 \times 60 = 979.2$ gal. Ans.

(15) The amount of water to be delivered per minute will be $\frac{936}{60} = 15.6$ gallons. The velocity of flow for this case should not exceed 500 feet per minute. Applying rule **4,** we have

$$A = \frac{19.25 \times 15.6}{500} = .601 \text{ square inch, nearly.}$$

From Table VI we see that the nearest larger size pipe is the 1-inch pipe, which has an actual area of .863 square inch. Hence, a 1-inch pipe should be used.

(16) Bends in a pipe greatly increase the resistance to the flow of water, and, hence, also the power required to move the water through the pipe. As the power available for the suction pipe of a pump is quite small, any increase in the resistance to the flow will materially interfere with the satisfactory working of the pump. See Art. **39.**

(17) See Art. **41.**

(18) About 30 in., as explained in Art. **43.**

(19) By means of a vacuum gauge. See Arts. **45** to **47**, inclusive.

(20) Applying the principle of Art. **48**, we have $\frac{30 \times 13.6}{2.5} = 163.2$ in. Ans.

(21) The pressure of the atmosphere is measured with an instrument called a barometer. See Arts. **49** to **52**, inclusive.

(22) The higher the elevation the shorter is the column of air and the lower the density of the air. Consequently, the pressure exerted by the air becomes less the higher we go above sea level. See Art. **53.**

(23) Like the pressure exerted by a liquid, the pressure of the atmosphere acts in all directions with the same intensity. See Art. **54.**

(24) The pressure of a gas varies inversely as the volume, the temperature remaining constant. Thus, if the volume is reduced one-half, the pressure is doubled; if the volume is reduced to one-third the original volume, the pressure is increased to three times the original pressure. See Art. **55.**

(25) (*a*) Gauge pressure is pressure measured above the atmospheric pressure. See Art. **56.**

(*b*) Absolute pressure is pressure measured from vacuum and is equal to gauge pressure plus atmospheric pressure. See Art. **56.**

(26) Call the original volume, or volume of the tube, 1; then the volume after the piston has moved $\frac{4}{5}$ the length of the tube will be $1 - \frac{4}{5} = \frac{1}{5}$. Applying rule **7**, we have

$$p_1 = \frac{14.7 \times 1}{\frac{1}{5}} = 73.5 \text{ lb. per sq. in. Ans}$$

(27) Applying rule **8**,

$$v_1 = \frac{14.7 \times 1}{147} = .1 = \tfrac{1}{10} \text{ the original volume. Ans.}$$

(28) An air pump is an apparatus for removing air from any vessel; in other words, it is an apparatus for producing a vacuum in any vessel. See Art. **60.**

(29) No. See Art. **61.**

(30) See Art. **62.**

(31) See Arts. **63** and **64.**

(32) When the water reaches the same level in both vessels, the flow of water from one vessel into the other will cease. This follows from Arts. **63** and **64.**

(33) The highest point of the siphon should not be more than 28 feet above the level of the water that is being siphoned, or the siphon will not work. See Art. **65.**

(34) (*a*) A pump is an apparatus for raising water. See Art. **68.**

(*b*) Pumps may be divided into three classes, depending on their mode of operation. See Art. **69.**

(*c*) (1) Suction pumps; (2) lifting pumps; and (3) force pumps. See Art. **69.**

(35) A suction pump acts by producing a vacuum in the suction pipe, which causes the water into which the lower end of the suction pipe is immersed to rise in the suction pipe. See Art. **70.**

(36) A lifting pump will raise water to a higher point than a suction pump. See Art. **72.**

(37) A force pump piston does not have any valve like the piston of a lifting pump. See Art. **73.**

(38) A single-acting force pump discharges water at every second stroke; a double-acting force pump discharges water during each stroke. See Art. **76.**

STRENGTH OF MATERIALS.

---

(1) (*a*) A stress is a force that acts in the interior of a body and tends to produce a deformation in the body. See Art. **1.**

(*b*) Tensile, compressive, shearing, transverse, and torsional. See Art. **2.**

(2) By Art. **3,** we have

$$\text{unit stress} = \tfrac{8000}{20} = 400 \text{ lb. per sq. in.} \quad \text{Ans.}$$

(3) (*a*) Strain is the amount of deformation produced in a body by the action of a stress. See Art. **4.**

(*b*) Elasticity is that property of a body by virtue of which it will tend to return to its original shape after the force producing the deformation is removed. See Art. **5.**

(*c*) The elastic limit is that unit stress at which permanent set begins. See Art. **7.**

(4) Since the ultimate tensile strength per square inch is 55,000 pounds, the force at which a bar having an area of 4 square inches will rupture is $4 \times 55{,}000 = 220{,}000$ lb. Ans. See Art. **10.**

(5) Annealing makes the chain tougher, more ductile, and less liable to break from sudden jerks. See Art. **16.**

(6) Untwist a portion of the rope and examine the condition of the inner surfaces. See Art. **20.**

(7) (*a*) The safe load to be lifted depends on the size of the rope and on its manner of attachment to the hook of the tackle block or to the work. See Art. **23.**

(*b*) See Art. **23.**

(8) (*a*) Manila rope is chiefly used for hoisting and for power transmission. See Art. **26.**

(*b*) The fibers are oiled to reduce internal wear. See Art. **26.**

(9) The diameter of the pulley should be at least 40 times the diameter of the rope. See Art. **28.**

(10) Applying rule 4, Art. **30,**

$$W = 600 \times (1\tfrac{1}{2})^2 = 1,350 \text{ lb.} \quad \text{Ans.}$$

(11) Applying rule 6, Art. **30,**

$$C = \sqrt{16,000} \times .0316 = 4 \text{ in., nearly.} \quad \text{Ans.}$$

(12) A column both of whose ends are fixed is 4 times as strong as one that has both ends movable and $1\frac{7}{9}$ times as strong as one that has one fixed end and one movable end. See Art. **34.**

(13) See Art. **34.**

(14) Applying rule 7, Art. **37,** and taking the proper values from Tables V and VII, we have

$$W = \frac{14,000 \times .7854 \times 14^2}{1 + \frac{(16 \times 12)^2}{281.25 \times 14^2}} = 1,291,480 \text{ lb.} \quad \text{Ans.}$$

(15) By rule 8, Art. **42,** the area of the piston rod should be

$$\frac{.7854 \times 40^2 \times 110}{3,600} = 38.397 \text{ square inches.}$$

The corresponding diameter is $\sqrt{\frac{38.397}{.7854}} = 7$ inches, nearly. Hence a 6″ piston rod is too small. Ans.

(16) From Table X, S for this case is found to be 1,460. From Table XI, $R = \frac{14 \times 14^2}{6} = 457.33$. Now, applying the proper formula from Table IX, we have

$$W = \frac{4 \times 1{,}460 \times 457.33}{12 \times 12} = 18{,}547 \text{ lb.} \quad \text{Ans.}$$

(17) By rule **9**, Art. **50**, and Table XII, the safe load would be $4 \times 6 \times 500 = 12{,}000$ pounds. Hence, a load of 10,000 pounds is safe. Ans.

(18) From Table XII, $S = 4{,}400$. Applying rule **10**, Art. **51**, we have

$$a = \frac{400{,}000}{4{,}400} = 91 \text{ sq. in., nearly.} \quad \text{Ans.}$$

(19) Double shear means that the body subjected to the shearing stress resists this stress at two planes, at both of which shear must occur to produce failure. See Art. **48.**

(20) Countershafts serve to effect changes in speed and to stop and start the machinery. See Art. **54.**

(21) Cold-rolled shafting is made cylindrically true by a special rolling process. Bright shafting is turned up in a lathe. See Art. **55.**

(22) The diameter by which bright shafting is designated is that of the bar from which it is turned. See Art. **56.**

(23) Pulleys should be placed as near the bearings as possible so that the deflection of the shaft may be as small as possible. See Art. **58.**

(24) Applying rule **11**, Art. **60**, after finding from Table XIV that C for this case is 85, we have

$$H = \frac{8^3 \times 80}{85} = 481.8, \text{ nearly, say 482 H. P.} \quad \text{Ans.}$$

(25) From Table XIV we find C for this case to be 65. Applying rule **12**, Art. **61**,

$$R = \frac{65 \times 75}{4^3} = 76 \text{ rev.} \quad \text{Ans.}$$

(26) From Table XIV, $C = 95$. Applying rule **13**, Art. **62**,

$$D = \sqrt[3]{\frac{95 \times 84}{300}} = \sqrt[3]{26.6}, \text{ say 3 in.} \quad \text{Ans.}$$

(27) The horsepower of a shaft will vary directly with the speed. See Art. **63.**

(28) See Art. **11.**

(29) A steel rope wears better than an iron rope. See Art. **29.**

ELEMENTS OF ELECTRICITY AND MAGNETISM.

(1) The end b; because in looking at that end the current circulates around the helix in an opposite direction to the hands of a watch. See Art. **29.**

(2) (*a*) Negative. See Art. **7.**
(*b*) Negative. See Art. **7.**

(3) Let A represent the first branch and B the second; then $r_1 = 16.2$ ohms, $r_2 = 14.1$ ohms, and $C = 6.37$ amperes.

The current c_1 in branch A is found by using formula **10**; substituting gives

$$c_1 = \frac{C r_2}{r_1 + r_2} = \frac{6.37 \times 14.1}{16.2 + 14.1} = 2.9643 \text{ amperes. Ans.}$$

The current c_2 in branch B is found by using formula **11**; substituting gives

$$c_2 = \frac{C r_1}{r_1 + r_2} = \frac{6.37 \times 16.2}{16.2 + 14.1} = 3.4057 \text{ amperes. Ans.}$$

(4) (*a*) From Art. **64** and formula **6**, $C = \frac{E}{R}$, where C is the current in amperes, E is the difference of potential in volts between two points, and R is the resistance in ohms between them. In this example, $E = 58.4$ volts and $R = 2.3$ ohms; hence, $C = \frac{E}{R} = \frac{58.4}{2.3} = 25.3913$ amperes. Ans.

(*b*) From formula **21**, $W = \frac{E^2}{R}$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and R is the resistance in ohms. In this example, $E = 58.4$ volts and $R = 2.3$ ohms; hence,

$$W = \frac{E^2}{R} = \frac{58.4^2}{2.3} = \frac{3{,}410.56}{2.3} = 1{,}482.8521 \text{ watts. Ans.}$$

(*c*) By formula **22**, $\text{H. P.} = \frac{W}{746}$; by formula **21**, $W = \frac{E^2}{R}$; therefore (see Art. **81**), $\text{H. P.} = \frac{E^2}{746\,R}$, where H. P. is the horsepower, E the E. M. F., or difference of potential in volts, and R the resistance in ohms.

$$\text{Hence, H. P.} = \frac{58.4^2}{746 \times 2.3} = \frac{3{,}410.56}{1{,}715.8} = 1.9877 \text{ horsepower. Ans.}$$

(5) By formula **8**, $E = C\,R$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes that is flowing, and R is the total resistance in ohms of the circuit. In this example, $C = .75$ ampere and $R = 17.2 + 8.2 + 11.3 = 36.7$ ohms; hence, $E = C\,R = .75 \times 36.7 = 27.525$ volts, the total E. M. F. developed in the battery.

By derivation from formula **8**, $E' = C\,R'$, where E' is the difference of potential in volts between two points, C is the current in amperes flowing, and R' is the resistance in ohms between the two points.

Between *a* and *b*, $R' = 11.3$ ohms and $C = .75$ ampere; hence, $E' = C\,R' = .75 \times 11.3 = 8.475$ volts. Ans.

Between *b* and *c*, $R' = 8.2$ ohms and $C = .75$ ampere; hence, $E' = C\,R' = .75 \times 8.2 = 6.15$ volts. Ans.

Between *a* and *c*, the difference of potential is the difference of potential between *a* and *b* plus that between *b* and *c*, which is $8.475 + 6.15 = 14.625$ volts. Or, since the difference of potential between *a* and *c* is the available E. M. F. of the battery, when a current of .75 ampere is flowing, it

can be calculated by using formula **9,** $E' = E - C r_i$, where E' is the available E. M. F., E is the total E. M. F. developed in the battery, C is the current that is flowing, and r_i is the internal resistance of the battery. In this case, $E = 27.525$ volts, $C = .75$ ampere, and $r_i = 17.2$ ohms; hence, $E' = E - C r_i = 27.525 - (.75 \times 17.2) = 14.625$ volts. Ans.

(6) By formula **8,** $E = C R$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes flowing, and R is the total resistance in ohms of the circuit. In this example, $C = .127$ ampere and $R = 36.2 + 21.7 = 57.9$ ohms. Hence, by substituting, $E = C R = .127 \times 57.9 = 7.3533$ volts. Ans.

(7) By formula **14,** $Q = C t$, where Q is the quantity of electricity in coulombs that passes through a circuit, C is the current in amperes flowing in that circuit, and t is the time in seconds during which the current flows. In this example, $C = 8.32$ amperes and $t = 2.25 \times 60 \times 60 = 8{,}100$ seconds. Hence, by substituting, $Q = C t = 8.32 \times 8{,}100 = 67{,}392$ coulombs. Ans.

(8) By formula **19,** $W = C E$, where W is the power in watts, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 112.5$ volts and $C = 12.2$ amperes. Hence, by substituting, $W = C E = 12.2 \times 112.5 = 1{,}372.5$ watts. Ans.

(9) By formula **4,** $r_2 = r_1 (1 + t k)$, where r_1 is the original resistance of a conductor, r_2 is the resistance after a rise in temperature, k is the temperature coefficient, and t is the rise of temperature in degrees F. In this example, $r_1 = 43.2$ ohms, $t = 85 - 60 = 25°$ F., and $k = .002155$, from Table I. Hence, $r_2 = r_1 (1 + t k) = 43.2 (1 + 25 \times .002155) = 43.2 \times 1.053875 = 45.5274$ ohms. Ans.

(10) By formula **13,** the joint resistance of three conductors in parallel $R''' = \dfrac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}$, where r_1, r_2, and r_3 are the separate resistance of the three conductors.

respectively. In this example, let $r_1 = 37$ ohms, the resistance of A; $r_2 = 45$ ohms, the resistance of B; and $r_3 = 72$ ohms, the resistance of C. Substituting gives

$$\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{37 \times 45 \times 72}{45 \times 72 + 37 \times 72 + 37 \times 45} = \frac{119{,}880}{7{,}569}$$

$= 15.8383$ ohms, the joint resistance of the three conductors A, B, and C connected in parallel. Ans.

(11) From Art. **43,** the joint resistance of several conductors connected in series is equal to the sum of their separate resistances; hence, in this example the joint resistance of the four conductors A, B, C, and D in series is $3 + 19 + 72 + 111 = 205$ ohms. Ans.

(12) We here use formula **16,** $J = C^2 R t$, where J is the work in joules, C is the current in amperes, R is the resistance in ohms, and t is the time in seconds. In this case $C = 14.2$ amperes, $R = 8$ ohms, $t = 4{,}500$ seconds. Then, the work done $= 14.2 \times 14.2 \times 8 \times 4{,}500 = 7{,}259{,}040$ joules. Ans.

(13) From Art. **75,** the separate resistance of any branch of a derived circuit is equal to the difference of potential between where all the branches divide and where they unite, divided by the current in that branch.

Hence, the separate resistance of branch A is $\frac{11.6}{6.7} = 1.7313$ ohms. Ans.

The separate resistance of branch B is $\frac{11.6}{4.9} = 2.3673$ ohms. Ans.

(14) By formula **7,** $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed in the circuit, and C is the current in amperes flowing in the circuit. In this example, $E = 22.4$ volts, and $C = .43$ ampere; hence, $R = \frac{E}{C} = \frac{22.4}{.43} = 52.093$ ohms, the total resistance of the circuit. Since the total resistance of a closed circuit is equal to the sum of

the external and internal resistances, the external resistance must be the difference between the total resistance and the internal resistance. Hence, the external resistance $= 52.093 - 13.4 = 38.693$ ohms. Ans.

(15) By transposition of terms in formula **14,** $C = \frac{Q}{t}$, where C is the current in amperes, Q is the quantity of electricity in coulombs, and t is the time in seconds. In this example, $Q = 368{,}422$ coulombs and $t = 4.5 \times 60 \times 60 = 16{,}200$ seconds; hence, $C = \frac{Q}{t} = \frac{368{,}422}{16{,}200} = 22.7421$ amperes. Ans.

(16) By formula **16,** $J = C^2 R t$, where J is the work done in joules, C is the current in amperes, R is the resistance in ohms, and t is the time in seconds. In this example, $C = 2.4$ amperes, $R = 45$ ohms, $t = 3{,}000$ seconds. Then the electrical work done $= 2.4 \times 2.4 \times 45 \times 3{,}000 = 777{,}600$ joules. By formula **18,** the mechanical work done $=$ F. P. $= .7373\, J = .7373 \times 777{,}600 = 573{,}324.48$ foot-pounds. Ans.

(17) By formula **22,** H. P. $= \frac{W}{746}$; by formula **19,** $W = C E$; therefore (see Art. **81**), H. P. $= \frac{E C}{746}$, where H. P. is the horsepower, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 525$ volts, and $C = 12.5$ amperes; hence,

$$\text{H. P.} = \frac{E C}{746} = \frac{525 \times 12.5}{746} = 8.7969 \text{ horsepower. Ans.}$$

(18) (*a*) By formula **20,** $W = C^2 R$, where W is the power in watts, C is the current in amperes, and R is the resistance in ohms. In this example, $C = 110$ amperes and $R = 4.2$ ohms; hence, $W = C^2 R = 110^2 \times 4.2 = 50{,}820$ watts. Ans.

(*b*) $$\text{H. P.} = \frac{W}{746} = \frac{50{,}820}{746} = 68.123 \text{ horsepower. Ans.}$$

(19) By formula **1**, the changed resistance for variation in length, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance, l_1 is the original length, and l_2 is the changed length. In this case, $r_1 = 1$ ohm, $l_1 = 1,000$ feet, and $l_2 = 2,000$ feet. Then, the changed resistance $r_2 = \frac{1 \times 2,000}{1,000} = 2$ ohms. The next operation is to determine the resistance of the wire when its sectional area is changed. A round wire 1 inch in diameter has a sectional area of $.1^2 \times .7854 = .007854$ square inch, and a square wire .1 inch on a side has a sectional area of $.1 \times .1 = .01$ square inch. By formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its sectional area is changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = 2$ ohms, $a_1 = .007854$ square inch, and $a_2 = .01$ square inch. Hence, $r_2 = \frac{r_1 a_1}{a_2} = \frac{2 \times .007854}{.01} = 1.5708$ ohms. Ans.

(20) By formula **22**, H. P. $= \frac{W}{746}$, where H. P. is the horsepower and W is the power in watts. In this example, $W = 54,200$ watts; hence, H. P. $= \frac{W}{746} = \frac{54,200}{746} = 72.6541$ horsepower. Ans.

(21) The sectional area of a round column .04 inch in diameter is $.04^2 \times .7854 = .00125664$ square inch, or .001257 square inch, nearly.

Reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000, Art. **44**, which gives $\frac{37.15}{1,000,000} = .00003715$ ohm; or, in other words, the resistance of a quantity of mercury 1 inch long and whose sectional area is 1 square inch is .00003715 ohm. Next, from this resistance and length, calculate the resistance of a column of mercury 72.3 inches high, with a sectional area

of 1 square inch, by using formula **1,** $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is its original length, and l_2 is its changed length. In this example, $r_1 = .00003715$ ohm, $l_1 = 1$ inch, and $l_2 = 72.3$ inches. Hence, $r_2 = \frac{r_1 l_2}{l_1}$ $= \frac{.00003715 \times 72.3}{1} = .002685945$, or .002686 ohm, nearly; or, in other words, the resistance of a column of mercury 72.3 inches high, having a sectional area of 1 square inch, is .002686 ohm. From this result calculate the resistance of the column when its sectional area is .001257 square inch, by using formula **2,** $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance, r_2 is the resistance after the sectional area has been changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = .002686$ ohm, $a_1 = 1$ square inch, and $a_2 = .001257$ square inch. Hence,

$$r_2 = \frac{r_1 a_1}{a_2} = \frac{.002686 \times 1}{.001257} = 2.1368 \text{ ohms. Ans.}$$

(22) By formula **6,** $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts generated, and R is the total resistance in ohms of the circuit. Since the total resistance of a closed circuit is the sum of the external and internal circuits, $R = 33 + 30 = 63$ ohms, and $E = 45$ volts; hence, $C = \frac{E}{R} = \frac{45}{63} = .7143$ ampere. Ans.

(23) By formula **19,** $W = C E$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and C is the current in amperes. In this example, $E = 510$ volts and $C = 24.3$ amperes; hence, $W = 510 \times 24.3 = 12{,}393$ watts. Ans.

(24) Referring to Art. **56,** the total E. M. F. developed by connecting several cells in series is equal to the E. M. F. of one cell multiplied by the number of cells; hence, the E. M. F. of one of the groups of 6 cells is $6 \times 1.5 = 9$ volts. In the same article it is stated that connecting cells in multiple, or parallel, does not change the E. M. F. between the main conductors. In this case each group of 6 cells can be considered as one large cell developing an E. M. F. of 9 volts, and, consequently, the E. M. F. of the four groups connected in multiple, or parallel, is 9 volts, which would be the E. M. F. indicated by a voltmeter, connected to the main conductors c and c'. Ans.

(25) By formula **22,** H. P. $= \frac{W}{746}$; by formula **19,** $W = C E$; therefore (see Art. **81**), H. P. $= \frac{E C}{746}$, where H. P. is the horsepower, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 250$ volts and $C = 65.7$ amperes; hence, H. P. $= \frac{E C}{746} = \frac{250 \times 65.7}{746} = \frac{16,425}{746}$ $= 22.0174$ horsepower. Ans.

(26) End a; since, Art. **29,** in looking at the face of the end a, the current circulates around the core in the same direction as the movement of the hands of a watch.

(27) From Art. **21,** iron and its alloys, nickel, cobalt, manganese, oxygen, cerium, and chromium.

(28) Toward the south pole, since, from Art. **20,** unlike poles attract one another.

(29) Toward the north pole, since, from Art. **20,** unlike poles attract one another.

(30) The current should enter the wire at the end b; since, Art. **29,** in looking at the face of the south pole of the magnet, the current should circulate around the core in the direction of motion of the hands of a watch.

(31) By formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is the original length, and l_2 is its changed length. In this example, $r_1 = 100.8$ ohms, $l_1 = (112 \times 12) + 6 = 1{,}350$ inches, $l_2 = 11.7$ inches. Hence,

$$r_2 = \frac{r_1 l_2}{l_1} = \frac{100.8 \times 11.7}{1{,}350} = .8736 \text{ ohm. Ans.}$$

(32) By formula **4**, $r_2 = r_1 (1 + t k)$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has risen, k is the temperature coefficient, and t is the number of degrees rise Fahrenheit. In this example, $r_1 = 91.8$ ohms, $t = 72 - 45 = 27°$, and $k = .000244$, from Table I. Hence, $r_2 = r_1 (1 + t k) = 91.8 (1 + 27 \times .000244) = 91.8 \times 1.006588 = 92.4048$ ohms. Ans.

(33) By formula **5**, $r_2 = \frac{r_1}{1 + t k}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has fallen, t is the number of degrees fall Fahrenheit, and k is the temperature coefficient of the material. In this example, $r_1 = .144$ ohm, $t = 87 - 41 = 46°$ F., and $k = .002155$, from Table I. Hence,

$$r_2 = \frac{r_1}{1 + t k} = \frac{.144}{1 + 46 \times .002155} = \frac{.144}{1.09913} = .131 \text{ ohm. Ans.}$$

(34) From Art. **53**, the fundamental equation of the Wheatstone's bridge is $X = \frac{M}{N} \times P$, where X is the unknown resistance, M is the resistance of the upper balance arm, N is the resistance of the lower balance arm, and P is the resistance of the adjustable arm. It will be seen from the connections of the battery and galvanometer circuits in the diagram that the coils lying between c and a form the upper balance arm of the bridge, and, hence, in this example, $M = 1$ ohm; the coils between a and d form the lower balance arm, and, hence, $N = 100$ ohms; the coils between

d and b form the adjustable arm, and, hence, $P = 500 + 200 + 20 + 2 + 1 = 723$ ohms. Substituting these values in the fundamental equation gives

$$X = \frac{M}{N} \times P = \frac{1}{100} \times 723 = 7.23 \text{ ohms. Ans.}$$

(35) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts developed in the circuit, and R is the total resistance in ohms of the circuit. In this example, $E = 36$ volts and $R = 24 + 18 = 42$ ohms; since, according to Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence, $C = \frac{E}{R} = \frac{36}{42} = .8571$ ampere. Ans.

(36) By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed in the circuit, and C is the current in amperes flowing through the circuit. In this example, $E = 12.6$ volts and $C = 2.7$ amperes; hence, $R = \frac{E}{C} = \frac{12.6}{2.7} = 4.6667$ ohms. Ans.

(37) By formula **8**, $E = C R$, where E is the total E.M.F. in volts developed in a closed circuit, C is the current in amperes flowing through the circuit, and R is the total resistance of the circuit. In this example, $C = .8$ ampere and $R = 31.5 + 11 = 42.5$ ohms, since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence, $E = C R = .8 \times 42.5 = 34$ volts. Ans.

(38) (*a*) By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms between two points in a circuit, E is the drop or loss of potential in volts between the two points, and C is the current in amperes flowing in the circuit. In

this example, the two conductors leading to and from the receptive device can be considered as in series, forming one single conductor 1,200 feet in length, in which the drop or loss of potential is 10 per cent. of 250 volts, or $.10 \times 250 = 25$ volts; that is, $E = 25$ volts. Since $C = 80$ amperes, then, $R = \frac{E}{C} = \frac{25}{80} = .3125$ ohm; or, in other words, the sum of the resistances of two conductors that transmit a current of 80 amperes to and from the receptive device with a loss of 25 volts is .3125 ohm. Ans.

(*b*) The resistance per foot of any conductor is found by dividing its total resistance by its length in feet. Assume the two conductors leading to and from the receptive device to be one single conductor 1,200 feet in length and offering a resistance of .3125 ohm; hence, its resistance per foot is

$$\frac{.3125}{1,200} = .00026 \text{ ohm. Ans.}$$

(39) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts developed in the circuit, and R is the total resistance in ohms of the circuit. In this example, $E = 24$ volts and $R = 8.1 + 15.9 = 24$ ohms, since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence,

$$C = \frac{E}{R} = \frac{24}{24} = 1 \text{ ampere.}$$

By formula **9**, $E' = E - C r_i$, where E' is the available, or external, E. M. F. in volts of a battery or other electric source in a closed circuit, E is the total E. M. F. in volts developed in the source, C is the current in amperes flowing through the circuit, and r_i is the internal resistance of the battery or electric source. In this example, $E = 24$ volts, $C = 1$ ampere, and $r_i = 8.1$ ohms. Hence, $E' = E - C r_i = 24 - (8.1 \times 1) = 15.9$ volts. Ans.

(40) Let A represent the first branch and B the second; then $r_1 = 1.2$ ohms, $r_2 = 2.2$ ohms, and $C = 45$ amperes.

The current c_1 in branch A will then be found by substituting these values in formula **10,** which gives

$$c_1 = \frac{C r_2}{r_1 + r_2} = \frac{45 \times 2.2}{1.2 + 2.2} = \frac{99}{3.4} = 29.1176 \text{ amperes. Ans.}$$

Since the sum of the currents in the two branches is 45 amperes, the current in branch B is, therefore, the difference between 45 amperes and the current in branch A, or $45 - 29.1176 = 15.8824$ amperes. Ans.

(41) (*a*) A kilowatt is 1,000 watts or about $1\frac{1}{3}$ horsepower.

(*b*) 150 kilowatt = 150,000 watts. By formula **22,**

$$\text{H. P.} = \frac{150,000}{746} = 201.072 \text{ horsepower. Ans.}$$

(42) 300 kilowatt = 300,000 watts. By transposing formula **19,** we get

$$C = \frac{W}{E} = \frac{300,000}{250} = 1,200 \text{ amperes. Ans.}$$

(43) (*a*) A kilowatt-hour is a unit of electrical work and is equal to the total amount of work done when one kilowatt is supplied for one hour.

(*b*) 110 volts $\times$ 40 amperes = 4,400 watts. $\frac{4400}{1000} = 4.4$ kilowatts. The time is 9.5 hours. The kilowatt-hours $= 4.4 \times 9.5$, or 41.8 kilowatt-hours. Ans. See Art. **83.**

HEAT AND STEAM.

(1) It causes a rise in temperature, causes an increase in volume, and causes a body to change from a solid to a liquid or from a liquid to a gas, according to the state it is in when the heat is applied. See Art. **8.**

(2) When we speak of the temperature of a body, we refer to the condition of the molecules that compose it. If they are in a state of rapid vibration, the body is said to have a high temperature; if they are moving slowly, it is said to have a low temperature. See Art. **2.**

(3) (*a*) A British thermal unit is the quantity of heat required to raise 1 pound of water through 1 degree Fahrenheit. See Art. **10.**

(*b*) One B. T. U. is equivalent to 778 foot-pounds of work. See Art. **13.**

(4) If two blocks of wood are rubbed briskly together, they will develop heat. There are many other similar instances. See Art. **12.**

(5) (*a*) and (*b*) See Art. **16.**

(*c*) See Art. **14.**

(6) To raise 10 pounds of water from 74° to 75° requires 10 B. T. U. Then, the specific heat of glass, according to Art. **15,** is the ratio of 1.77 to 10, or .177 equals the specific heat of glass. Ans. See Art. **15.**

(7) Since it takes 1 B. T. U. to raise 1 pound of water through 1 degree of temperature, and when 1 pound of water falls 1 degree of temperature, it gives off 1 B. T. U., 1,000 pounds of water falling 1 degree of temperature will give off 1,000 B. T. U., and falling from 212° to 192°, or 20°, will give off $1{,}000 \times 20 = 20{,}000$ B. T. U. Each pound of steam in being transformed into water gives off its latent heat, 966 B. T. U.; 1,000 pounds give off $1{,}000 \times 966 = 966{,}000$ B. T. U.; so the whole amount of heat liberated by 1,000 pounds of steam at 212° in being transformed to water at 192° is $966{,}000 + 20{,}000 = 986{,}000$ B. T. U. In order to raise 1 pound of water from 32° to 112°, it takes $112° - 32° = 80$ B. T. U. Therefore, 986,000 B. T. U. will raise $986{,}000 \div 80 = 12{,}325$ lb. of water from 32° to 112°. Ans. See Arts. **9, 10,** and **17.**

(8) Applying rule **2,** Art. **19,** the product of the weight, specific heat, and temperature of the water is $10 \times 1 \times 90 = 900$; of the mercury, $15 \times .0333 \times 60 = 29.97$; of the alcohol, $20 \times .60 \times 40 = 480$; and the sum of these products is $900 + 29.97 + 480 = 1{,}409.97$. The product of the weight and specific heat of the water is $10 \times 1 = 10$; of the mercury, $15 \times .0333 = .4995$; of the alcohol, $20 \times .60 = 12$; and the sum of these products is $10 + .4995 + 12 = 22.4995$. Then the temperature of the mixture is $1{,}409.97 \div 22.4995 = 62.66°$. See Art. **19.**

(9) The quantity of heat contained in a mixture is the same as the quantity of heat contained in the substances before mixing. See Art. **19.**

(10) Using rule **3,** Art. **20,** and substituting, we have

$$T = \frac{20\,(966 + 212) + 400 \times 55}{20 + 400} = 108.47°. \quad \text{Ans.}$$

(11) The boiling point of a liquid varies with the pressure on its surface. See Art. **22.**

(12) (*a*) Saturated steam is steam having the temperature corresponding to its pressure. See Art. **23.**

(*b*) Superheated steam is steam that is not in contact with water and is at a temperature higher than the boiling point corresponding to its pressure. See Art. **25.**

(13) See Art. **27.**

(14) According to rule **1,** Art. **15,** the heat required to raise the temperature of 10 pounds of ice from 25° to 32° is $.504 \times 10\ (32 - 25) = 35.28$ B. T. U. It requires 144 B. T. U. to change 1 pound of ice to water at the same temperature. To change 10 pounds of ice at 32° to water at 32° requires, therefore, $10 \times 144 = 1{,}440$ B. T. U. Then, to raise the water from 32° to 212° requires, according to rule **1,** Art. **15,** $1 \times 10\ (212 - 32) = 1{,}800$ B. T. U. To change 1 pound of water into steam at the same temperature requires 966 B. T. U., and to change 10 pounds of water at 212° to steam at 212° requires $10 \times 966 = 9{,}660$ B. T. U. Therefore, to change 10 pounds of ice at 25° to steam at 212° requires

$$
\begin{array}{r}
35.28 \\
1440.00 \\
1800.00 \\
9660.00 \\
\hline
12935.28
\end{array}
\text{ B. T. U. Ans.}
$$

(15) See Art. **29.**

(16) 90 pounds gauge pressure $= 90 + 14.7 = 104.7$ pounds absolute pressure. The heat required to change water at the boiling temperature into steam at the same temperature, called the "latent heat of vaporization," is denoted in the table by L. Now, referring to the table, we find that for $p = 104.7$, $L = 881.368$. That is, it takes 881.368 B. T. U. to change 1 pound of water at 90 pounds gauge pressure into steam at the same pressure. To change 40 pounds requires $40 \times 881.368 = 35{,}254.72$ B. T. U. $35{,}254.72 \times 778 = 27{,}428{,}172$ ft.-lb. Ans.

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
1.00	1.0000	1.00000	1.33	1.7689	2.35264	1.66	2.7556	4.57430
1.01	1.0201	1.03030	1.34	1.7956	2.40610	1.67	2.7889	4.65746
1.02	1.0404	1.06121	1.35	1.8225	2.46038	1.68	2.8224	4.74163
1.03	1.0609	1.09273	1.36	1.8496	2.51546	1.69	2.8561	4.82681
1.04	1.0816	1.12486	1.37	1.8769	2.57135	1.70	2.8900	4.91300
1.05	1.1025	1.15763	1.38	1.9044	2.62807	1.71	2.9241	5.00021
1.06	1.1236	1.19102	1.39	1.9321	2.68562	1.72	2.9584	5.08845
1.07	1.1449	1.22504	1.40	1.9600	2.74400	1.73	2.9929	5.17772
1.08	1.1664	1.25971	1.41	1.9881	2.80322	1.74	3.0276	5.26802
1.09	1.1881	1.29503	1.42	2.0164	2.86329	1.75	3.0625	5.35938
1.10	1.2100	1.33100	1.43	2.0449	2.92421	1.76	3.0976	5.45178
1.11	1.2321	1.36763	1.44	2.0736	2.98598	1.77	3.1329	5.54523
1.12	1.2544	1.40493	1.45	2.1025	3.04863	1.78	3.1684	5.63975
1.13	1.2769	1.44290	1.46	2.1316	3.11214	1.79	3.2041	5.73534
1.14	1.2996	1.48154	1.47	2.1609	3.17652	1.80	3.2400	5.83200
1.15	1.3225	1.52088	1.48	2.1904	3.24179	1.81	3.2761	5.92974
1.16	1.3456	1.56090	1.49	2.2201	3.30795	1.82	3.3124	6.02857
1.17	1.3689	1.60161	1.50	2.2500	3.37500	1.83	3.3489	6.12849
1.18	1.3924	1.64303	1.51	2.2801	3.44295	1.84	3.3856	6.22950
1.19	1.4161	1.68516	1.52	2.3104	3.51181	1.85	3.4225	6.33163
1.20	1.4400	1.72800	1.53	2.3409	3.58158	1.86	3.4596	6.43486
1.21	1.4641	1.77156	1.54	2.3716	3.65226	1.87	3.4969	6.53920
1.22	1.4884	1.81585	1.55	2.4025	3.72388	1.88	3.5344	6.64467
1.23	1.5129	1.86087	1.56	2.4336	3.79642	1.89	3.5721	6.75127
1.24	1.5376	1.90662	1.57	2.4649	3.86989	1.90	3.6100	6.85900
1.25	1.5625	1.95313	1.58	2.4964	3.94431	1.91	3.6481	6.96787
1.26	1.5876	2.00038	1.59	2.5281	4.01968	1.92	3.6864	7.07789
1.27	1.6129	2.04838	1.60	2.5600	4.09600	1.93	3.7249	7.18906
1.28	1.6384	2.09715	1.61	2.5921	4.17328	1.94	3.7636	7.30138
1.29	1.6641	2.14669	1.62	2.6244	4.25153	1.95	3.8025	7.41488
1.30	1.6900	2.19700	1.63	2.6569	4.33075	1.96	3.8416	7.52954
1.31	1.7161	2.24809	1.64	2.6896	4.41094	1.97	3.8809	7.64537
1.32	1.7424	2.29997	1.65	2.7225	4.49213	1.98	3.9204	7.76239
1.33	1.7689	2.35264	1.66	2.7556	4.57430	1.99	3.9601	7.88060

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
2.00	4.0000	8.00000	**2.33**	5.4289	12.6493	**2.66**	7.0756	18.8211
2.01	4.0401	8.12060	**2.34**	5.4756	12.8129	**2.67**	7.1289	19.0342
2.02	4.0804	8.24241	**2.35**	5.5225	12.9779	**2.68**	7.1824	19.2488
2.03	4.1209	8.36543	**2.36**	5.5696	13.1443	**2.69**	7.2361	19.4651
2.04	4.1616	8.48966	**2.37**	5.6169	13.3121	**2.70**	7.2900	19.6830
2.05	4.2025	8.61513	**2.38**	5.6644	13.4813	**2.71**	7.3441	19.9025
2.06	4.2436	8.74182	**2.39**	5.7121	13.6519	**2.72**	7.3984	20.1236
2.07	4.2849	8.86974	**2.40**	5.7600	13.8240	**2.73**	7.4529	20.3464
2.08	4.3264	8.99891	**2.41**	5.8081	13.9975	**2.74**	7.5076	20.5708
2.09	4.3681	9.12933	**2.42**	5.8564	14.1725	**2.75**	7.5625	20.7969
2.10	4.4100	9.26100	**2.43**	5.9049	14.3489	**2.76**	7.6176	21.0246
2.11	4.4521	9.39393	**2.44**	5.9536	14.5268	**2.77**	7.6729	21.2539
2.12	4.4944	9.52813	**2.45**	6.0025	14.7061	**2.78**	7.7284	21.4850
2.13	4.5369	9.66360	**2.46**	6.0516	14.8869	**2.79**	7.7841	21.7176
2.14	4.5796	9.80034	**2.47**	6.1009	15.0692	**2.80**	7.8400	21.9520
2.15	4.6225	9.93838	**2.48**	6.1504	15.2530	**2.81**	7.8961	22.1880
2.16	4.6656	10.0777	**2.49**	6.2001	15.4382	**2.82**	7.9524	22.4258
2.17	4.7089	10.2183	**2.50**	6.2500	15.6250	**2.83**	8.0089	22.6652
2.18	4.7524	10.3602	**2.51**	6.3001	15.8133	**2.84**	8.0656	22.9063
2.19	4.7961	10.5035	**2.52**	6.3504	16.0030	**2.85**	8.1225	23.1491
2.20	4.8400	10.6480	**2.53**	6.4009	16.1943	**2.86**	8.1796	23.3937
2.21	4.8841	10.7939	**2.54**	6.4516	16.3871	**2.87**	8.2369	23.6399
2.22	4.9284	10.9410	**2.55**	6.5025	16.5814	**2.88**	8.2944	23.8879
2.23	4.9729	11.0896	**2.56**	6.5536	16.7772	**2.89**	8.3521	24.1376
2.24	5.0176	11.2394	**2.57**	6.6049	16.9746	**2.90**	8.4100	24.3890
2.25	5.0625	11.3906	**2.58**	6.6564	17.1735	**2.91**	8.4681	24.6422
2.26	5.1076	11.5432	**2.59**	6.7081	17.3740	**2.92**	8.5264	24.8971
2.27	5.1529	11.6971	**2.60**	6.7600	17.5760	**2.93**	8.5849	25.1538
2.28	5.1984	11.8524	**2.61**	6.8121	17.7796	**2.94**	8.6436	25.4122
2.29	5.2441	12.0090	**2.62**	6.8644	17.9847	**2.95**	8.7025	25.6724
2.30	5.2900	12.1670	**2.63**	6.9169	18.1914	**2.96**	8.7616	25.9343
2.31	5.3361	12.3264	**2.64**	6.9696	18.3997	**2.97**	8.8209	26.1981
2.32	5.3824	12.4872	**2.65**	7.0225	18.6096	**2.98**	8.8804	26.4636
2.33	5.4289	12.6493	**2.66**	7.0756	18.8211	**2.99**	8.9401	26.7309

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
3.00	9.0000	27.0000	**3.33**	11.0889	36.9260	**3.66**	13.3956	49.0279
3.01	9.0601	27.2709	**3.34**	11.1556	37.2597	**3.67**	13.4689	49.4309
3.02	9.1204	27.5436	**3.35**	11.2225	37.5954	**3.68**	13.5424	49.8360
3.03	9.1809	27.8181	**3.36**	11.2896	37.9331	**3.69**	13.6161	50.2434
3.04	9.2416	28.0945	**3.37**	11.3569	38.2728	**3.70**	13.6900	50.6530
3.05	9.3025	28.3726	**3.38**	11.4244	38.6145	**3.71**	13.7641	51.0648
3.06	9.3636	28.6526	**3.39**	11.4921	38.9582	**3.72**	13.8384	51.4788
3.07	9.4249	28.9344	**3.40**	11.5600	39.3040	**3.73**	13.9129	51.8951
3.08	9.4864	29.2181	**3.41**	11.6281	39.6518	**3.74**	13.9876	52.3136
3.09	9.5481	29.5036	**3.42**	11.6964	40.0017	**3.75**	14.0625	52.7344
3.10	9.6100	29.7910	**3.43**	11.7649	40.3536	**3.76**	14.1376	53.1574
3.11	9.6721	30.0802	**3.44**	11.8336	40.7076	**3.77**	14.2129	53.5826
3.12	9.7344	30.3713	**3.45**	11.9025	41.0636	**3.78**	14.2884	54.0102
3.13	9.7969	30.6643	**3.46**	11.9716	41.4217	**3.79**	14.3641	54.4399
3.14	9.8596	30.9591	**3.47**	12.0409	41.7819	**3.80**	14.4400	54.8720
3.15	9.9225	31.2559	**3.48**	12.1104	42.1442	**3.81**	14.5161	55.3063
3.16	9.9856	31.5545	**3.49**	12.1801	42.5085	**3.82**	14.5924	55.7430
3.17	10.0489	31.8550	**3.50**	12.2500	42.8750	**3.83**	14.6689	56.1819
3.18	10.1124	32.1574	**3.51**	12.3201	43.2436	**3.84**	14.7456	56.6231
3.19	10.1761	32.4618	**3.52**	12.3904	43.6142	**3.85**	14.8225	57.0666
3.20	10.2400	32.7680	**3.53**	12.4609	43.9870	**3.86**	14.8996	57.5125
3.21	10.3041	33.0762	**3.54**	12.5316	44.3619	**3.87**	14.9769	57.9606
3.22	10.3684	33.3862	**3.55**	12.6025	44.7389	**3.88**	15.0544	58.4111
3.23	10.4329	33.6983	**3.56**	12.6736	45.1180	**3.89**	15.1321	58.8639
3.24	10.4976	34.0122	**3.57**	12.7449	45.4993	**3.90**	15.2100	59.3190
3.25	10.5625	34.3281	**3.58**	12.8164	45.8827	**3.91**	15.2881	59.7765
3.26	10.6276	34.6460	**3.59**	12.8881	46.2683	**3.92**	15.3664	60.2363
3.27	10.6929	34.9658	**3.60**	12.9600	46.6560	**3.93**	15.4449	60.6985
3.28	10.7584	35.2876	**3.61**	13.0321	47.0459	**3.94**	15.5236	61.1630
3.29	10.8241	35.6129	**3.62**	13.1044	47.4379	**3.95**	15.6025	61.6299
3.30	10.8900	35.9370	**3.63**	13.1769	47.8321	**3.96**	15.6816	62.0991
3.31	10.9561	36.2647	**3.64**	13.2496	48.2285	**3.97**	15.7609	62.5708
3.32	11.0224	36.5944	**3.65**	13.3225	48.6271	**3.98**	15.8404	63.0448
3.33	11.0889	36.9260	**3.66**	13.3956	49.0279	**3.99**	15.9201	63.5212

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
4.00	16.0000	64.0000	4.33	18.7489	81.1827	4.66	21.7156	101.195
4.01	16.0801	64.4812	4.34	18.8356	81.7465	4.67	21.8089	101.848
4.02	16.1604	64.9648	4.35	18.9225	82.3129	4.68	21.9024	102.503
4.03	16.2409	65.4508	4.36	19.0096	82.8819	4.69	21.9961	103.162
4.04	16.3216	65.9393	4.37	19.0969	83.4535	4.70	22.0900	103.823
4.05	16.4025	66.4301	4.38	19.1844	84.0277	4.71	22.1841	104.487
4.06	16.4836	66.9234	4.39	19.2721	84.6045	4.72	22.2784	105.154
4.07	16.5649	67.4191	4.40	19.3600	85.1840	4.73	22.3729	105.824
4.08	16.6464	67.9173	4.41	19.4481	85.7661	4.74	22.4676	106.496
4.09	16.7281	68.4179	4.42	19.5364	86.3509	4.75	22.5625	107.172
4.10	16.8100	68.9210	4.43	19.6249	86.9383	4.76	22.6576	107.850
4.11	16.8921	69.4265	4.44	19.7136	87.5284	4.77	22.7529	108.531
4.12	16.9744	69.9345	4.45	19.8025	88.1211	4.78	22.8484	109.215
4.13	17.0569	70.4450	4.46	19.8916	88.7165	4.79	22.9441	109.902
4.14	17.1396	70.9579	4.47	19.9809	89.3146	4.80	23.0400	110.592
4.15	17.2225	71.4734	4.48	20.0704	89.9154	4.81	23.1361	111.285
4.16	17.3056	71.9913	4.49	20.1601	90.5188	4.82	23.2324	111.980
4.17	17.3889	72.5117	4.50	20.2500	91.1250	4.83	23.3289	112.679
4.18	17.4724	73.0346	4.51	20.3401	91.7339	4.84	23.4256	113.380
4.19	17.5561	73.5601	4.52	20.4304	92.3454	4.85	23.5225	114.084
4.20	17.6400	74.0880	4.53	20.5209	92.9597	4.86	23.6196	114.791
4.21	17.7241	74.6185	4.54	20.6116	93.5767	4.87	23.7169	115.501
4.22	17.8084	75.1514	4.55	20.7025	94.1964	4.88	23.8144	116.214
4.23	17.8929	75.6870	4.56	20.7936	94.8188	4.89	23.9121	116.930
4.24	17.9776	76.2250	4.57	20.8849	95.4440	4.90	24.0100	117.649
4.25	18.0625	76.7656	4.58	20.9764	96.0719	4.91	24.1081	118.371
4.26	18.1476	77.3088	4.59	21.0681	96.7026	4.92	24.2064	119.095
4.27	18.2329	77.8545	4.60	21.1600	97.3360	4.93	24.3049	119.823
4.28	18.3184	78.4028	4.61	21.2521	97.9722	4.94	24.4036	120.554
4.29	18.4041	78.9536	4.62	21.3444	98.6111	4.95	24.5025	121.287
4.30	18.4900	79.5070	4.63	21.4369	99.2528	4.96	24.6016	122.024
4.31	18.5761	80.0630	4.64	21.5296	99.8973	4.97	24.7009	122.763
4.32	18.6624	80.6216	4.65	21.6225	100.545	4.98	24.8004	123.506
4.33	18.7489	81.1827	4.66	21.7156	101.195	4.99	24.9001	124.251

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
5.00	25.0000	125.000	**5.33**	28.4089	151.419	**5.66**	32.0356	181.321
5.01	25.1001	125.752	**5.34**	28.5156	152.273	**5.67**	32.1489	182.284
5.02	25.2004	126.506	**5.35**	28.6225	153.130	**5.68**	32.2624	183.250
5.03	25.3009	127.264	**5.36**	28.7296	153.991	**5.69**	32.3761	184.220
5.04	25.4016	128.024	**5.37**	28.8369	154.854	**5.70**	32.4900	185.193
5.05	25.5025	128.788	**5.38**	28.9444	155.721	**5.71**	32.6041	186.169
5.06	25.6036	129.554	**5.39**	29.0521	156.591	**5.72**	32.7184	187.149
5.07	25.7049	130.324	**5.40**	29.1600	157.464	**5.73**	32.8329	188.133
5.08	25.8064	131.097	**5.41**	29.2681	158.340	**5.74**	32.9476	189.119
5.09	25.9081	131.872	**5.42**	29.3764	159.220	**5.75**	33.0625	190.109
5.10	26.0100	132.651	**5.43**	29.4849	160.103	**5.76**	33.1776	191.103
5.11	26.1121	133.433	**5.44**	29.5936	160.989	**5.77**	33.2929	192.100
5.12	26.2144	134.218	**5.45**	29.7025	161.879	**5.78**	33.4084	193.101
5.13	26.3169	135.006	**5.46**	29.8116	162.771	**5.79**	33.5241	194.105
5.14	26.4196	135.797	**5.47**	29.9209	163.667	**5.80**	33.6400	195.112
5.15	26.5225	136.591	**5.48**	30.0304	164.567	**5.81**	33.7561	196.123
5.16	26.6256	137.388	**5.49**	30.1401	165.469	**5.82**	33.8724	197.137
5.17	26.7289	138.188	**5.50**	30.2500	166.375	**5.83**	33.9889	198.155
5.18	26.8324	138.992	**5.51**	30.3601	167.284	**5.84**	34.1056	199.177
5.19	26.9361	139.798	**5.52**	30.4704	168.197	**5.85**	34.2225	200.202
5.20	27.0400	140.608	**5.53**	30.5809	169.112	**5.86**	34.3396	201.230
5.21	27.1441	141.421	**5.54**	30.6916	170.031	**5.87**	34.4569	202.262
5.22	27.2484	142.237	**5.55**	30.8025	170.954	**5.88**	34.5744	203.297
5.23	27.3529	143.056	**5.56**	30.9136	171.880	**5.89**	34.6921	204.336
5.24	27.4576	143.878	**5.57**	31.0249	172.809	**5.90**	34.8100	205.379
5.25	27.5625	144.703	**5.58**	31.1364	173.741	**5.91**	34.9281	206.425
5.26	27.6676	145.532	**5.59**	31.2481	174.677	**5.92**	35.0464	207.475
5.27	27.7729	146.363	**5.60**	31.3600	175.616	**5.93**	35.1649	208.528
5.28	27.8784	147.198	**5.61**	31.4721	176.558	**5.94**	35.2836	209.585
5.29	27.9841	148.036	**5.62**	31.5844	177.504	**5.95**	35.4025	210.645
5.30	28.0900	148.877	**5.63**	31.6969	178.454	**5.96**	35.5216	211.709
5.31	28.1961	149.721	**5.64**	31.8096	179.406	**5.97**	35.6409	212.776
5.32	28.3024	150.569	**5.65**	31.9225	180.362	**5.98**	35.7604	213.847
5.33	28.4089	151.419	**5.66**	32.0356	181.321	**5.99**	35.8801	214.922

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
6.00	36.0000	216.000	**6.33**	40.0689	253.636	**6.66**	44.3556	295.408
6.01	36.1201	217.082	**6.34**	40.1956	254.840	**6.67**	44.4889	296.741
6.02	36.2404	218.167	**6.35**	40.3225	256.048	**6.68**	44.6224	298.078
6.03	36.3609	219.256	**6.36**	40.4496	257.259	**6.69**	44.7561	299.418
6.04	36.4816	220.349	**6.37**	40.5769	258.475	**6.70**	44.8900	300.763
6.05	36.6025	221.445	**6.38**	40.7044	259.694	**6.71**	45.0241	302.112
6.06	36.7236	222.545	**6.39**	40.8321	260.917	**6.72**	45.1584	303.464
6.07	36.8449	223.649	**6.40**	40.9600	262.144	**6.73**	45.2929	304.821
6.08	36.9664	224.756	**6.41**	41.0881	263.375	**6.74**	45.4276	306.182
6.09	37.0881	225.867	**6.42**	41.2164	264.609	**6.75**	45.5625	307.547
6.10	37.2100	226.981	**6.43**	41.3449	265.848	**6.76**	45.6976	308.916
6.11	37.3321	228.099	**6.44**	41.4736	267.090	**6.77**	45.8329	310.289
6.12	37.4544	229.221	**6.45**	41.6025	268.336	**6.78**	45.9684	311.666
6.13	37.5769	230.346	**6.46**	41.7316	269.586	**6.79**	46.1041	313.047
6.14	37.6996	231.476	**6.47**	41.8609	270.840	**6.80**	46.2400	314.432
6.15	37.8225	232.608	**6.48**	41.9904	272.098	**6.81**	46.3761	315.821
6.16	37.9456	233.745	**6.49**	42.1201	273.359	**6.82**	46.5124	317.215
6.17	38.0689	234.885	**6.50**	42.2500	274.625	**6.83**	46.6489	318.612
6.18	38.1924	236.029	**6.51**	42.3801	275.894	**6.84**	46.7856	320.014
6.19	38.3161	237.177	**6.52**	42.5104	277.168	**6.85**	46.9225	321.419
6.20	38.4400	238.328	**6.53**	42.6409	278.445	**6.86**	47.0596	322.829
6.21	38.5641	239.483	**6.54**	42.7716	279.726	**6.87**	47.1969	324.243
6.22	38.6884	240.642	**6.55**	42.9025	281.011	**6.88**	47.3344	325.661
6.23	38.8129	241.804	**6.56**	43.0336	282.300	**6.89**	47.4721	327.083
6.24	38.9376	242.971	**6.57**	43.1649	283.593	**6.90**	47.6100	328.509
6.25	39.0625	244.141	**6.58**	43.2964	284.890	**6.91**	47.7481	329.939
6.26	39.1876	245.314	**6.59**	43.4281	286.191	**6.92**	47.8864	331.374
6.27	39.3129	246.492	**6.60**	43.5600	287.496	**6.93**	48.0249	332.813
6.28	39.4384	247.673	**6.61**	43.6921	288.805	**6.94**	48.1636	334.255
6.29	39.5641	248.858	**6.62**	43.8244	290.118	**6.95**	48.3025	335.702
6.30	39.6900	250.047	**6.63**	43.9569	291.434	**6.96**	48.4416	337.154
6.31	39.8161	251.240	**6.64**	44.0896	292.755	**6.97**	48.5809	338.609
6.32	39.9424	252.436	**6.65**	44.2225	294.080	**6.98**	48.7204	340.068
6.33	40.0689	253.636	**6.66**	44.3556	295.408	**6.99**	48.8601	341.532

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
7.00	49.0000	343.000	**7.33**	53.7289	393.833	**7.66**	58.6756	449.455
7.01	49.1401	344.472	**7.34**	53.8756	395.447	**7.67**	58.8289	451.218
7.02	49.2804	345.948	**7.35**	54.0225	397.065	**7.68**	58.9824	452.985
7.03	49.4209	347.429	**7.36**	54.1696	398.688	**7.69**	59.1361	454.757
7.04	49.5616	348.914	**7.37**	54.3169	400.316	**7.70**	59.2900	456.533
7.05	49.7025	350.403	**7.38**	54.4644	401.947	**7.71**	59.4441	458.314
7.06	49.8436	351.896	**7.39**	54.6121	403.583	**7.72**	59.5984	460.100
7.07	49.9849	353.393	**7.40**	54.7600	405.224	**7.73**	59.7529	461.890
7.08	50.1264	354.895	**7.41**	54.9081	406.869	**7.74**	59.9076	463.685
7.09	50.2681	356.401	**7.42**	55.0564	408.518	**7.75**	60.0625	465.484
7.10	50.4100	357.911	**7.43**	55.2049	410.172	**7.76**	60.2176	467.289
7.11	50.5521	359.425	**7.44**	55.3536	411.831	**7.77**	60.3729	469.097
7.12	50.6944	360.944	**7.45**	55.5025	413.494	**7.78**	60.5284	470.911
7.13	50.8369	362.467	**7.46**	55.6516	415.161	**7.79**	60.6841	472.729
7.14	50.9796	363.994	**7.47**	55.8009	416.833	**7.80**	60.8400	474.552
7.15	51.1225	365.526	**7.48**	55.9504	418.509	**7.81**	60.9961	476.380
7.16	51.2656	367.062	**7.49**	56.1001	420.190	**7.82**	61.1524	478.212
7.17	51.4089	368.602	**7.50**	56.2500	421.875	**7.83**	61.3089	480.049
7.18	51.5524	370.146	**7.51**	56.4001	423.565	**7.84**	61.4656	481.890
7.19	51.6961	371.695	**7.52**	56.5504	425.259	**7.85**	61.6225	483.737
7.20	51.8400	373.248	**7.53**	56.7009	426.958	**7.86**	61.7796	485.588
7.21	51.9841	374.805	**7.54**	56.8516	428.661	**7.87**	61.9369	487.443
7.22	52.1284	376.367	**7.55**	57.0025	430.369	**7.88**	62.0944	489.304
7.23	52.2729	377.933	**7.56**	57.1536	432.081	**7.89**	62.2521	491.169
7.24	52.4176	379.503	**7.57**	57.3049	433.798	**7.90**	62.4100	493.039
7.25	52.5625	381.078	**7.58**	57.4564	435.520	**7.91**	62.5681	494.914
7.26	52.7076	382.657	**7.59**	57.6081	437.245	**7.92**	62.7264	496.793
7.27	52.8529	384.241	**7.60**	57.7600	438.976	**7.93**	62.8849	498.677
7.28	52.9984	385.828	**7.61**	57.9121	440.711	**7.94**	63.0436	500.566
7.29	53.1441	387.420	**7.62**	58.0644	442.451	**7.95**	63.2025	502.460
7.30	53.2900	389.017	**7.63**	58.2169	444.195	**7.96**	63.3616	504.358
7.31	53.4361	390.618	**7.64**	58.3696	445.994	**7.97**	63.5209	506.262
7.32	53.5824	392.223	**7.65**	58.5225	447.697	**7.98**	63.6804	508.170
7.33	53.7289	393.833	**7.66**	58.6756	449.455	**7.99**	63.8401	510.082

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
8.00	64.0000	512.000	**8.33**	69.3889	578.010	**8.66**	74.9956	649.462
8.01	64.1601	513.922	**8.34**	69.5556	580.094	**8.67**	75.1689	651.714
8.02	64.3204	515.850	**8.35**	69.7225	582.183	**8.68**	75.3424	653.972
8.03	64.4809	517.782	**8.36**	69.8896	584.277	**8.69**	75.5161	656.235
8.04	64.6416	519.718	**8.37**	70.0569	586.376	**8.70**	75.6900	658.503
8.05	64.8025	521.660	**8.38**	70.2244	588.480	**8.71**	75.8641	660.776
8.06	64.9636	523.607	**8.39**	70.3921	590.590	**8.72**	76.0384	663.055
8.07	65.1249	525.558	**8.40**	70.5600	592.704	**8.73**	76.2129	665.339
8.08	65.2864	527.514	**8.41**	70.7281	594.823	**8.74**	76.3876	667.628
8.09	65.4481	529.475	**8.42**	70.8964	596.948	**8.75**	76.5625	669.922
8.10	65.6100	531.441	**8.43**	71.0649	599.077	**8.76**	76.7376	672.221
8.11	65.7721	533.412	**8.44**	71.2336	601.212	**8.77**	76.9129	674.526
8.12	65.9344	535.387	**8.45**	71.4025	603.351	**8.78**	77.0884	676.836
8.13	66.0969	537.368	**8.46**	71.5716	605.496	**8.79**	77.2641	679.151
8.14	66.2596	539.353	**8.47**	71.7409	607.645	**8.80**	77.4400	681.472
8.15	66.4225	541.343	**8.48**	71.9104	609.800	**8.81**	77.6161	683.798
8.16	66.5856	543.338	**8.49**	72.0801	611.960	**8.82**	77.7924	686.129
8.17	66.7489	545.339	**8.50**	72.2500	614.125	**8.83**	77.9689	688.465
8.18	66.9124	547.343	**8.51**	72.4201	616.295	**8.84**	78.1456	690.807
8.19	67.0761	549.353	**8.52**	72.5904	618.470	**8.85**	78.3225	693.154
8.20	67.2400	551.368	**8.53**	72.7609	620.650	**8.86**	78.4996	695.506
8.21	67.4041	553.388	**8.54**	72.9316	622.836	**8.87**	78.6769	697.864
8.22	67.5684	555.412	**8.55**	73.1025	625.026	**8.88**	78.8544	700.227
8.23	67.7329	557.442	**8.56**	73.2736	627.222	**8.89**	79.0321	702.595
8.24	67.8976	559.476	**8.57**	73.4449	629.423	**8.90**	79.2100	704.969
8.25	68.0625	561.516	**8.58**	73.6164	631.629	**8.91**	79.3881	707.348
8.26	68.2276	563.560	**8.59**	73.7881	633.840	**8.92**	79.5664	709.732
8.27	68.3929	565.609	**8.60**	73.9600	636.056	**8.93**	79.7449	712.122
8.28	68.5584	567.664	**8.61**	74.1321	638.277	**8.94**	79.9236	714.517
8.29	68.7241	569.723	**8.62**	74.3044	640.504	**8.95**	80.1025	716.917
8.30	68.8900	571.787	**8.63**	74.4769	642.736	**8.96**	80.2816	719.323
8.31	69.0561	573.856	**8.64**	74.6496	644.973	**8.97**	80.4609	721.734
8.32	69.2224	575.930	**8.65**	74.8225	647.215	**8.98**	80.6404	724.151
8.33	69.3889	578.010	**8.66**	74.9956	649.462	**8.99**	80.8201	726.573

n	n^2	n^3	n	n^2	n^3	n	n^2	n^3
9.00	81.0000	729.000	**9.33**	87.0489	812.166	**9.66**	93.3156	901.429
9.01	81.1801	731.433	**9.34**	87.2356	814.781	**9.67**	93.5089	904.231
9.02	81.3604	733.871	**9.35**	87.4225	817.400	**9.68**	93.7024	907.039
9.03	81.5409	736.314	**9.36**	87.6096	820.026	**9.69**	93.8961	909.853
9.04	81.7216	738.763	**9.37**	87.7969	822.657	**9.70**	94.0900	912.673
9.05	81.9025	741.218	**9.38**	87.9844	825.294	**9.71**	94.2841	915.499
9.06	82.0836	743.677	**9.39**	88.1721	827.936	**9.72**	94.4784	918.330
9.07	82.2649	746.143	**9.40**	88.3600	830.584	**9.73**	94.6729	921.167
9.08	82.4464	748.613	**9.41**	88.5481	833.238	**9.74**	94.8676	924.010
9.09	82.6281	751.089	**9.42**	88.7364	835.897	**9.75**	95.0625	926.859
9.10	82.8100	753.571	**9.43**	88.9249	838.562	**9.76**	95.2576	929.714
9.11	82.9921	756.058	**9.44**	89.1136	841.232	**9.77**	95.4529	932.575
9.12	83.1744	758.551	**9.45**	89.3025	843.909	**9.78**	95.6484	935.441
9.13	83.3569	761.048	**9.46**	89.4916	846.591	**9.79**	95.8441	938.314
9.14	83.5396	763.552	**9.47**	89.6809	849.278	**9.80**	96.0400	941.192
9.15	83.7225	766.061	**9.48**	89.8704	851.971	**9.81**	96.2361	944.076
9.16	83.9056	768.575	**9.49**	90.0601	854.670	**9.82**	96.4324	946.966
9.17	84.0889	771.095	**9.50**	90.2500	857.375	**9.83**	96.6289	949.862
9.18	84.2724	773.621	**9.51**	90.4401	860.085	**9.84**	96.8256	952.764
9.19	84.4561	776.152	**9.52**	90.6304	862.801	**9.85**	97.0225	955.672
9.20	84.6400	778.688	**9.53**	90.8209	865.523	**9.86**	97.2196	958.585
9.21	84.8241	781.230	**9.54**	91.0116	868.251	**9.87**	97.4169	961.505
9.22	85.0084	783.777	**9.55**	91.2025	870.984	**9.88**	97.6144	964.430
9.23	85.1929	786.330	**9.56**	91.3936	873.723	**9.89**	97.8121	967.362
9.24	85.3776	788.889	**9.57**	91.5849	876.467	**9.90**	98.0100	970.299
9.25	85.5625	791.453	**9.58**	91.7764	879.218	**9.91**	98.2081	973.242
9.26	85.7476	794.023	**9.59**	91.9681	881.974	**9.92**	98.4064	976.191
9.27	85.9329	796.598	**9.60**	92.1600	884.736	**9.93**	98.6049	979.147
9.28	86.1184	799.179	**9.61**	92.3521	887.504	**9.94**	98.8036	982.108
9.29	86.3041	801.765	**9.62**	92.5444	890.277	**9.95**	99.0025	985.075
9.30	86.4900	804.357	**9.63**	92.7369	893.056	**9.96**	99.2016	988.048
9.31	86.6761	806.954	**9.64**	92.9296	895.841	**9.97**	99.4009	991.027
9.32	86.8624	809.558	**9.65**	93.1225	898.632	**9.98**	99.6004	994.012
9.33	87.0489	812.166	**9.66**	93.3156	901.429	**9.99**	99.8001	997.003
						10.00	100.000	1000.00

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729

INDEX

NOTE.—All items in this index refer first to the section (see the Preface) and then to the page of the section. Thus, "Acceleration 4 7" means that acceleration will be found on page 7 of section 4.

Q

R